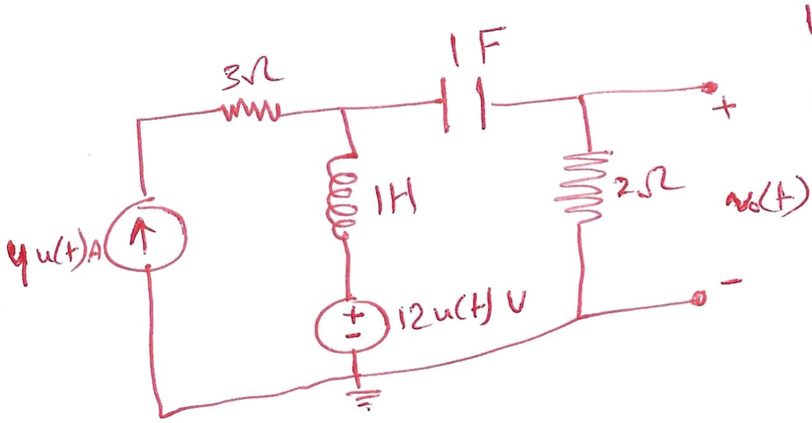
Analysis techniques

All the analysis techniques are applicable in the s-domain

For the circuit shown, find $V_o(t)$ using node Analysis, mesh analysis, superposition, source transformation, Thevenin's theorem and Norton theorem



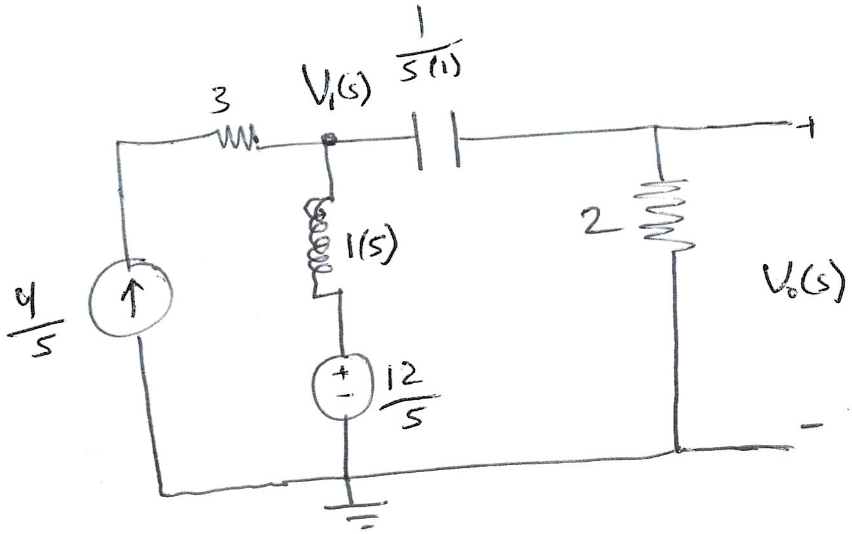
$v_c(0) = 0$

$i_L(0) = 0$

1) Node analysis :-

1 node :-

at node ①



$$-\frac{4}{s} + \frac{V_1 - \frac{12}{s}}{s} + \frac{V_1 - 0}{2 + \frac{1}{s}} = 0$$

$$-\frac{4}{s} + \frac{V_1}{s} - \frac{12}{s^2} + \frac{V_1}{\frac{2s+1}{s}} = 0$$

$$V_1\left(\frac{1}{s} + \frac{s}{2s+1}\right) = \frac{12+4s}{s^2}$$

$$V_1\left(\frac{s^2+2s+1}{2s^2+s}\right) = \frac{12+4s}{s^2}$$

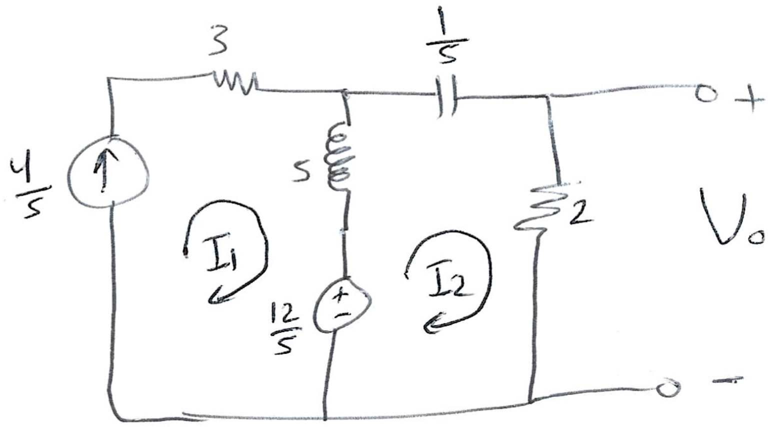
$$V_1 = \frac{\frac{12+4s}{s^2}}{\frac{s^2+2s+1}{2s^2+s}} = \frac{12+4s}{s^{\cancel{2}}} \cdot \frac{2s^{\cancel{2}} + \cancel{s}}{s^2+2s+1} = \frac{(12+4s)(2s+1)}{s(s^2+2s+1)}$$

but $$V_o = \frac{V_1(2)}{2+\frac{1}{s}} = \frac{2V_1}{\frac{2s+1}{s}} = \frac{2s V_1}{2s+1}$$

$$\therefore\ V_o = \frac{2\cancel{s}}{2s+1} \cdot \frac{(12+4s)\cancel{(2s+1)}}{\cancel{s}(s^2+2s+1)}$$

$$= \frac{2(12+4s)}{s^2+2s+1} = \frac{8(s+3)}{(s+1)^2}$$

2 Mesh Analysis :-



Mesh (1)

$$I_1 = \frac{4}{s}$$

Mesh (2)

$$-\frac{12}{s} + I_2\left(s + \frac{1}{s} + 2\right) - I_1(s) = 0$$

$$I_2\left(\frac{s^2+2s+1}{s}\right) = \frac{12}{s} + 4$$

$$I_2\left(\frac{s^2+2s+1}{s}\right) = \frac{12+4s}{s}$$

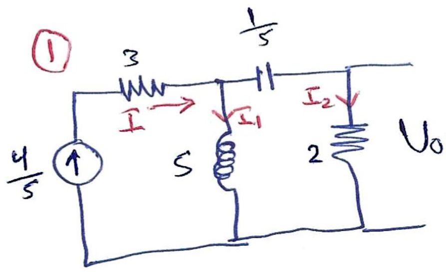
$$I_2 = \frac{12+4s}{s^2+2s+1}$$

but $V_o = I_2(2)$

$$V_o = \frac{2(12+4s)}{s^2+2s+1} = \frac{8(s+3)}{(s+1)^2}$$

3 Superposition

applying Current Source

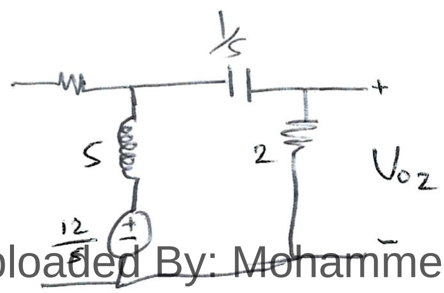


$$V_{o1} = I_2(2) = \frac{I(s)}{s+\frac{1}{s}+2}(2) \quad \text{current divider}$$

$$= \frac{4}{\cancel{s}} \times 2 \times \frac{\cancel{s}}{s+\frac{1}{s}+2}$$

$$= \frac{8s}{s^2+2s+1}$$

applying Voltage Source



$$V_{o2} = \frac{\frac{12}{s} \times 2}{2+\frac{1}{s}+s} = \frac{24}{s^2+2s+1}$$

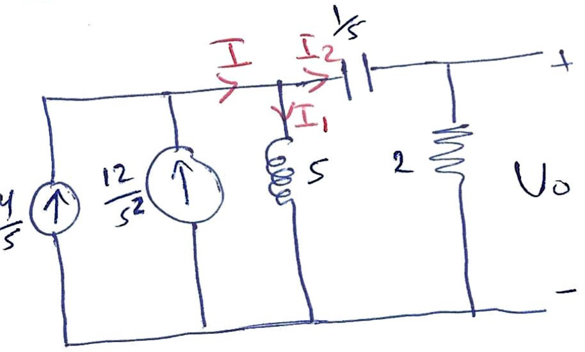
$$\therefore V_o = V_{o1} + V_{o2}$$

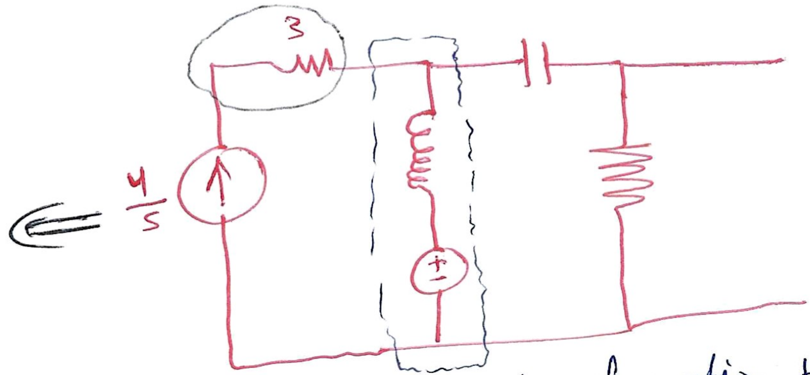
$$= \frac{8s}{s^2+2s+1} + \frac{24}{s^2+2s+1}$$

$$= \frac{8(s+3)}{(s+1)^2}$$

(4) Source transformation

resistor in series with current source so the resistance is redundant





source transformation to current source

$$V_o = I_2(2)$$

$$I_2 = \frac{I(s)}{s+\frac{1}{s}+2}$$

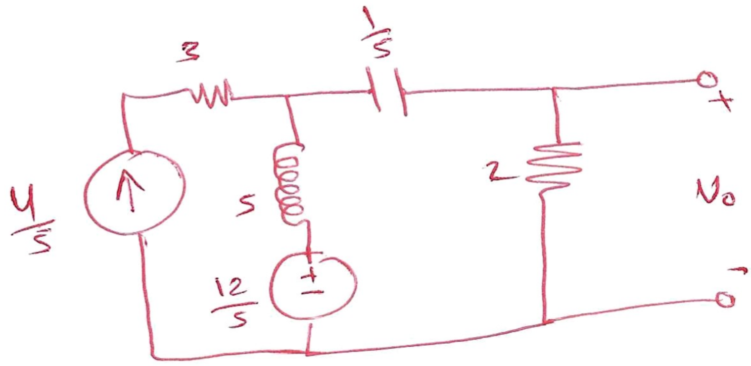
$$I = \frac{4}{s} + \frac{12}{s^2} = \frac{4s^2+12s}{s^3}$$

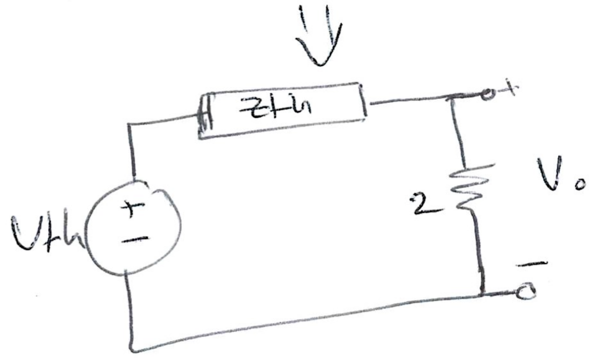
$$V_o = \frac{4s^2+12s}{s^3}\left(\frac{s}{s+\frac{1}{s}+2}\right)(2)$$

$$= \frac{4s^2+12s}{s^3} \cdot \frac{s^2}{s^2+2s+1}(2)$$

$$= \frac{8(s+3)}{(s+1)^2}$$

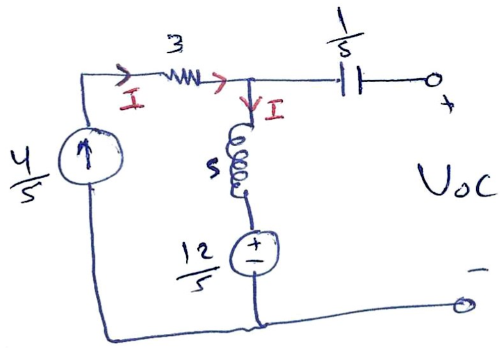
5 Thevenin's theorem



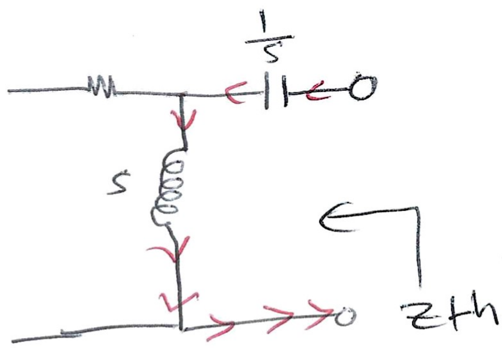


$I = \frac{4}{s}$

$\therefore V_{oc} = \frac{12}{s} + \frac{4}{s}(s) = \frac{12 + 4s}{s}$



$Z_{th} = \frac{1}{s} + s = \frac{s^2 + 1}{s}$

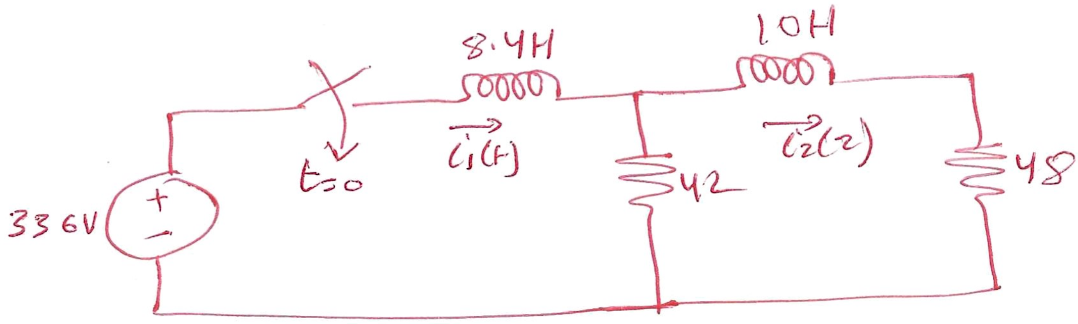


$$\therefore V_o = \frac{V_{th}(2)}{Z_{th} + 2}$$

$$= \frac{\frac{12 + 4s}{s}(2)}{\frac{s^2+1}{s} + 2}$$

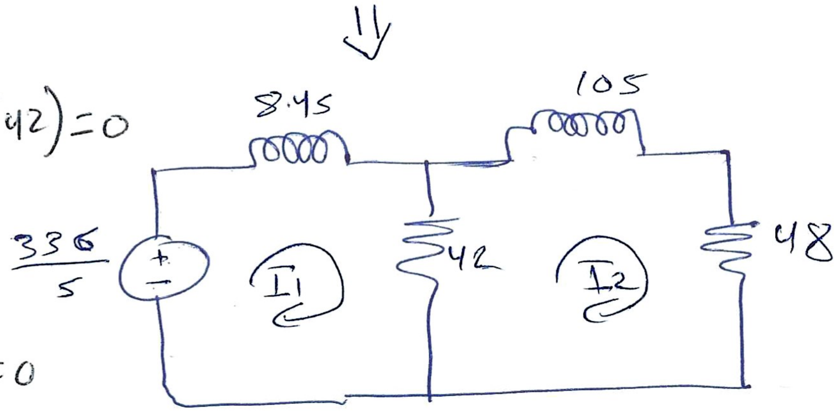
$$= \frac{8(s+3)}{(s+1)^2}$$

Example :- find $i_1(t)$ and $i_2(t)$



mesh (1)

$$-\frac{336}{s}+I_1(8.4s+42)-I_2(42)=0$$



Mesh (2)

$$I_2(42+48+10s)-I_1(42)=0$$

$$\begin{bmatrix} 42+8.4s & -42 \\ -42 & 90+10s \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}=\begin{bmatrix} \frac{336}{s} \\ 0 \end{bmatrix}$$

$$D=(42+8.4s)(90+10s)-(42)(42)$$

$$=3780+756s+420s+84s^2-1764$$

$$=84s^2+1176s+2016$$

$$D_{I_1}=\begin{bmatrix} \frac{336}{s} & -42 \\ 0 & 90+10s \end{bmatrix}$$

$$D_{I_1}=\frac{30240+3360s}{s}$$

$$D_{I_2}=\begin{bmatrix} 42+8.4s & \frac{336}{s} \\ -42 & 0 \end{bmatrix}$$

$$D_{I_2}=\frac{14112}{s}$$

$$I_1 = \frac{3360s + 30240}{s(84s^2 + 1176s + 2016)}$$

$$I_1 = \frac{40s + 360}{s(s^2 + 14s + 24)} = \frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+12}$$

$$I_1 = \frac{15}{s} - \frac{14}{s+2} - \frac{1}{s+12}$$

$$i_1(t) = (15 - 14e^{-2t} - e^{-12t})u(t)$$

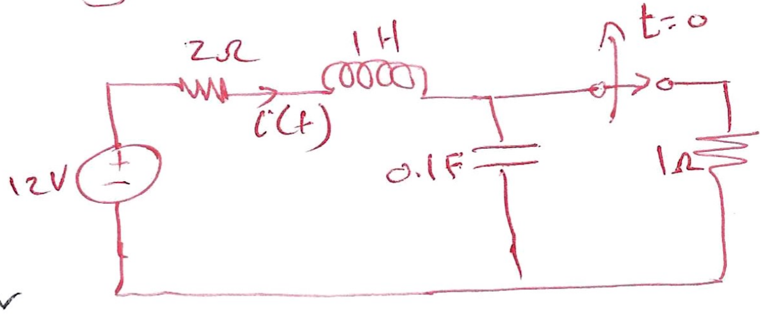
$$I_2 = \frac{14112}{s(84s^2 + 1176s + 2016)}$$

$$I_2 = \frac{168}{s(s^2 + 14s + 24)} = \frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+12}$$

$$I_2 = \frac{7}{s} - \frac{8.4}{s+2} + \frac{1.4}{s+12}$$

$$i_2(t) = (7 - 8.4e^{-2t} + 1.4e^{-12t})u(t)$$

Example:- Given the following circuit, find $i(t)$ for $t>0$

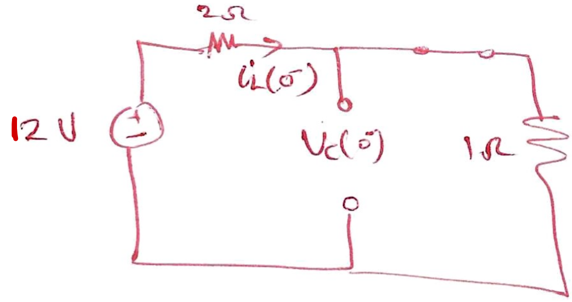


to calculate the initial conditions for the capacitor and the inductor for $t = 0^-$

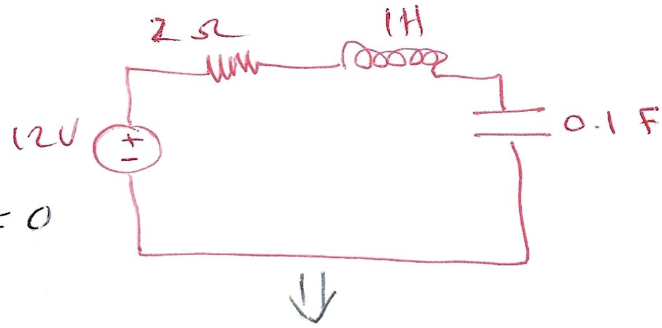
for $t<0$

$$V_c(0) = \frac{1}{1+2}(12) = 4V$$

$$i_L(0) = \frac{12}{1+2} = 4A$$



for $t>0$

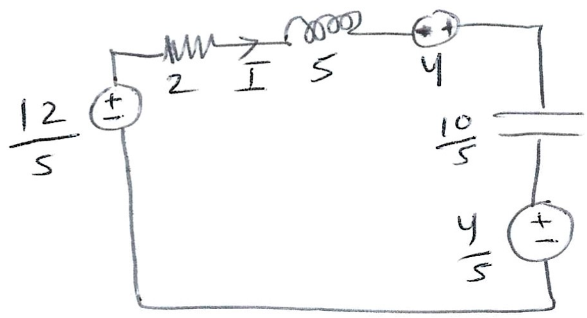


$$-\frac{12}{s} - 4 + \frac{4}{s} + I\left(2+s+\frac{10}{s}\right) = 0$$

$$I\left(\frac{s^2+2s+10}{s}\right) = \frac{12}{s} + 4 - \frac{4}{s}$$

$$= \frac{4s+8}{s}$$

$$I = \frac{4(s+2)}{s^2+2s+10} = \frac{4(s+2)}{(s+1-j3)(s+1+3j)}$$



$$= \frac{K_1}{s+1-j3} + \frac{K_1^*}{s+1+j3}$$

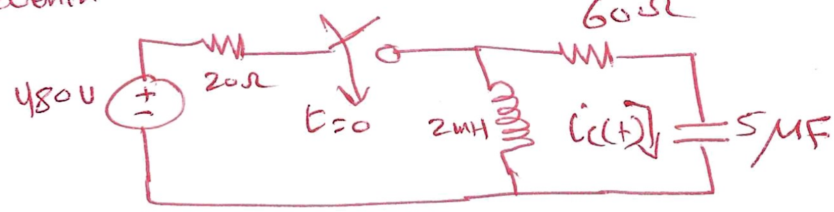
$$= \frac{2.11\angle -18.4}{s+1-j3} + \frac{2.11\angle 18.4}{s+1+j3}$$

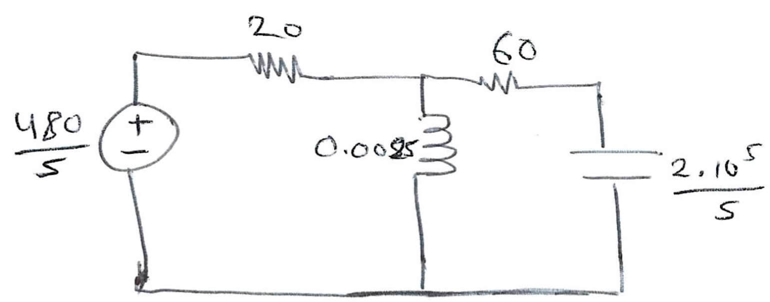
$$i(t) = 2(2.11)\,e^{-t}\cos(3t-18.4)$$

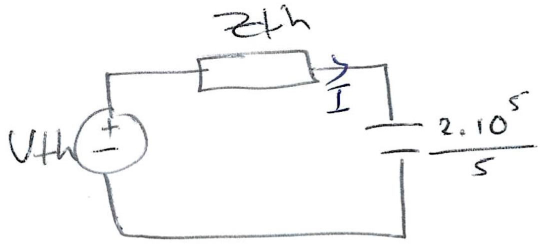
Find $i_c(t)$ for $t>0$ using Thevenin

$V_c(0) = 0$

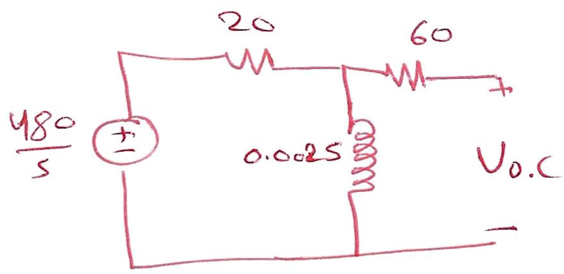
$i_L(0) = 0$





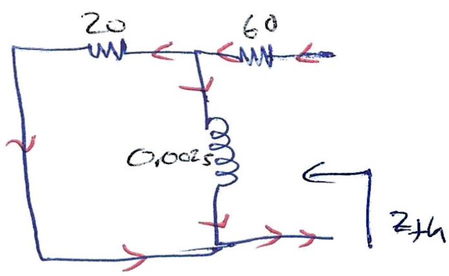


$$I = \frac{V_{th}}{Z_{th} + \frac{2.10^5}{s}}$$



$$V_{th} = V_{o.c} = \frac{480}{s} * \frac{0.0025}{0.0025s+20}$$

$$= \frac{480}{s+10^4}$$



$$Z_{th} = 60 + (20 // 0.0025s)$$

$$= 60 + \frac{20 \times 0.0025s}{20 + 0.0025s}$$

$$= \frac{1200 + 0.16s}{20 + 0.0025s}$$

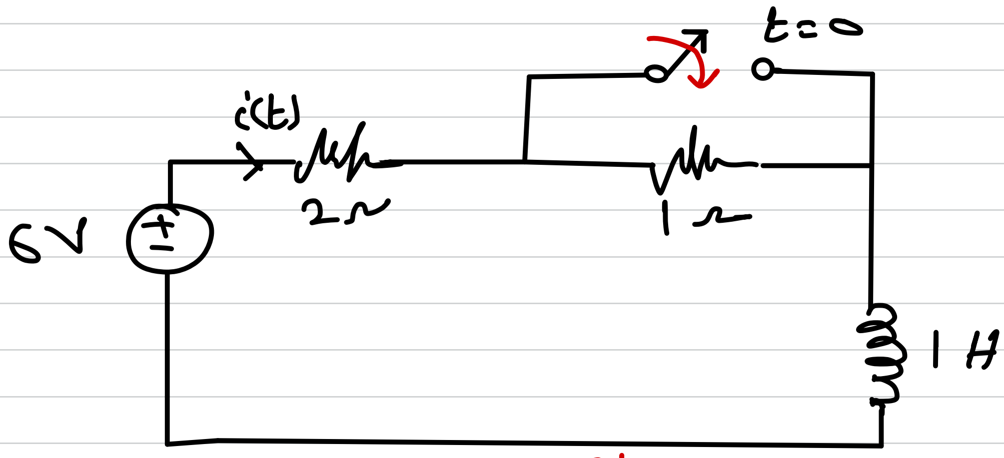
$$I = \frac{\frac{480}{s+10^4}}{\frac{1200 + 0.16s}{20 + 0.0025s}} = \frac{480}{s+10^4} \cdot \frac{20 + 0.0025s}{1200 + 0.16s}$$

$$= \frac{6s}{(s+5000)^2} = \frac{K_1}{(s+5000)^2} + \frac{K_2}{(s+5000)}$$

$$= \frac{-30{,}000}{(s+5000)^2} + \frac{6}{(s+5000)} \Rightarrow i(t) = (-30{,}000 t e^{-5000t} + 6 e^{-5000t}) u(t)$$

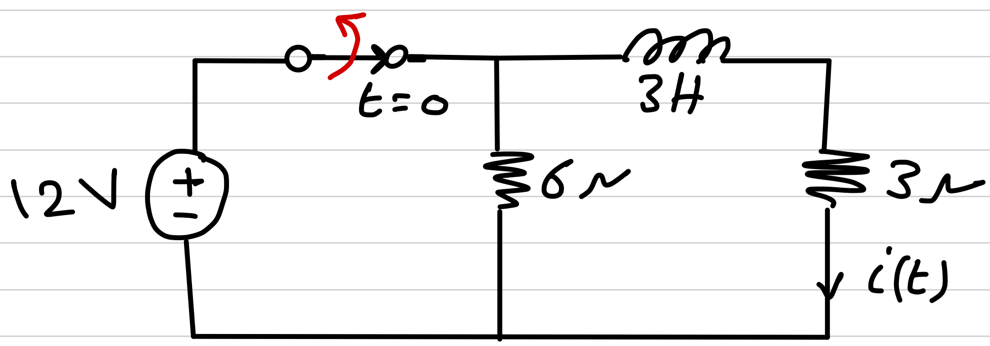
Quiz # 5

① For the circuit shown, obtain $i(t)$ for $t>0$



Solution: $i(t) = (3 - e^{-2t})\, u(t)$

② For the circuit shown, obtain $i(t)$ for $t>0$



Solution: $i(t) = 4e^{-3t}$