Chapter 7 :- Kinetic Energy & Work

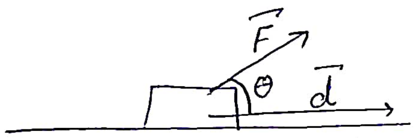
→ Kinetic Energy (K) : is energy associated with the state of motion of an object.

$$K = \frac{1}{2} m v^2 \quad , \quad [K] = \text{Joule } (J) = kg \cdot m^2/s^2$$

→ Work : is energy transferred to or from an object by means of a force acting on the object.

- Energy transferred to the object → positive work
- Energy transferred from the object → negative work

$\vec{F}$: Constant



work done by force $\vec{F}$: $W = \vec{F} \cdot \vec{d}$

$$= (F\cos\theta)\, d$$

$$= F_x\, d$$

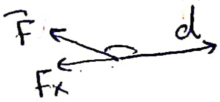
F_y doesn't make work !!

* $[W] = J \equiv N \cdot m$

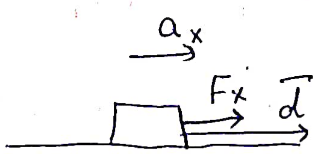
→ $W = 0$, $\vec{F} \perp \vec{d} \equiv (\theta = 90)$

→ $W < 0$ $\theta > 90$

→ $W > 0$ $\theta < 90$



Work - Energy Theorem :-



$$v_f^2 = v_i^2 + 2 a_x \Delta x \quad , \quad \Delta x = d$$

$$\frac{m}{2}\left[v_f^2 - v_i^2 = 2 a_x d\right]$$

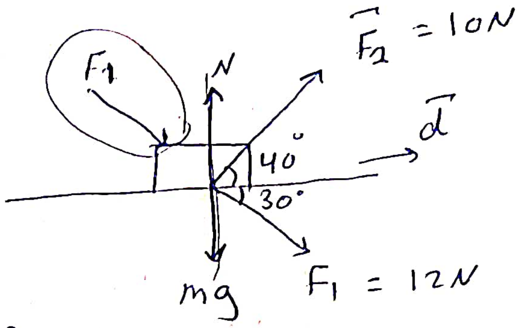
$$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = (m a_x) d$$

$K_f - K_i = F_x d$ ↳ net force

$$\boxed{\Delta K = W_{net}}$$

$$\boxed{W = \vec{F} \cdot \vec{d}}$$

Sample problem 7.02:



$m = 225$ kg

$\vec{d} = 8.5$ m (+X)

Find the work done by each force?

$W_1 = \vec{F_1} \cdot \vec{d} = F_1 \cos 30\, d$

$= 12 \times 8.5 \cos 30$

$= 88.3$ J

$W_2 = F_2 \cdot \vec{d} = F_2 \cos 40\, d$

$= 10 \times 8.5 \cos 40$

$= 65.11$ J

$W_{mg} = F_{mg} \cdot d = F_{mg} \cos 90\, d$

$= 0$

$W_N = N \cdot d = N \cos 90\, d$

$= 0$

$W_{net} = W_1 + W_2 + W_{mg} + W_N$

$= 88.3 + 65.11$

$= 153.4$ J

or $\boxed{W_{net} = \vec{F}_{net} \cdot d}$

b) Find v_f ?

$$W_{net} = \Delta K$$
$$= K_f - K_i$$
$$= \frac{1}{2} m v_f^2 - \cancel{\frac{1}{2} m v_i^2}^{\,0} \quad , (v_i = 0)$$

$$153.4 = \frac{1}{2}(225) v_f^2$$

$$v_f = 1.17 \text{ m/s}$$

problem 8 :

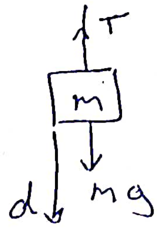
$$\vec{d} = (20\,\hat{i} - 16\,\hat{j})\text{ m}$$
$$\vec{F} = (210\,\hat{i} - 150\,\hat{j})\text{ N}$$

Find W ?

$$W = \vec{F} \cdot \vec{d}$$
$$= (210\,\hat{i} - 150\,\hat{j}) \cdot (20\,\hat{i} - 16\,\hat{j})$$
$$= (210)(20) + (-150)(-16)$$
$$= 6.6 \times 10^3 \text{ J} = 6.6 \text{ kJ}$$

Work done by gravitational Force (mg)

Case 1



$a = 0$

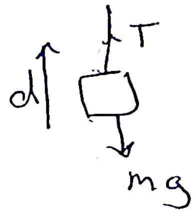
$$W_g = mg \cdot d$$
$$= mgd$$

$$W_{net} = \Delta K$$

$$W_g + W_T = \cancel{K_f - K_i}^{\,0} \qquad (V_f = V_i) \; a = 0$$

$$W_T = -W_g$$
$$= -mgd \qquad , \quad W_T = W_{applied\ force}.$$

Case 2

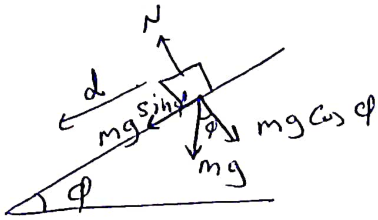


$a = 0$

$$W_g = mg \cdot d$$
$$= mgd \cos 180$$
$$= -mgd$$

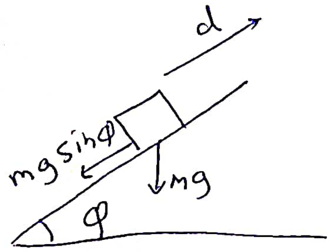
$$W_T = -W_g$$
$$= mgd$$

Case 3



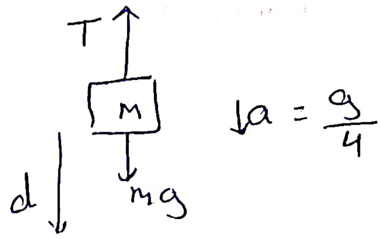
$$W_g = mg\sin\varphi \cdot d$$
$$= mg\sin\varphi \, d \cos 0$$

Case 4



$$W_g = mg\sin\varphi \cdot d$$
$$= mg\sin\varphi \, d \cos 180$$
$$= -mgd\sin\varphi$$

Problem 21



→ $W_g = mgd\cos 0$

$W_g = mgd$

→ W_T ? , we want to find T!

$$\sum \vec{F} = m\vec{a}$$

$$T - mg = m\left(\frac{-g}{4}\right)$$

$$T = mg - m\frac{g}{4}$$

$$= \frac{3}{4}mg$$

$$W_T = \left(\frac{3mg}{4}\right) d \cos 180$$

$$= -\frac{3}{4}mgd$$

→ W_{net} ?

$$W_{net} = W_g + W_T$$

$$= mgd + \frac{-3}{4}mgd$$

$$= \frac{1}{4}mgd$$

→ V_f ?

$$W_{net} = \Delta K$$

$$= K_f - \cancel{K_i}^{\,0}$$

$$\frac{1}{4}mgd = \frac{1}{2}mV_f^2 \Rightarrow V_f = \sqrt{\frac{gd}{2}}$$

problem 11

$\vec{d} = 5\hat{i} - 3\hat{j} + 4\hat{k}$

$F = 22\,N$

Find θ bt. $\vec{d}$ & $\vec{F}$ if:

① $\Delta K = 45\,J$?

$\Delta K = W = \vec{F}\cdot\vec{d}$

$= F d \cos\theta$

$d = \sqrt{5^2 + 3^2 + 4^2} = 7.07\,m$

$\Rightarrow 45 = 22 \times 7.07 \cos\theta$

$\cos\theta = 0.289$

$\theta = 73.2^\circ$

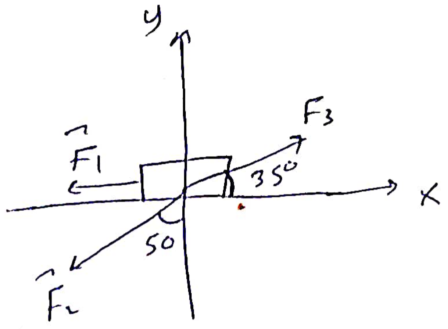
② $\Delta K = -45\,J$

$\Delta K = F d \cos\theta$

$-45 = 22 \times 7.07 \cos\theta$

$\theta = 106.8^\circ$

Problem 14



$\vec{F_1} = 3N$ in. $-x$

$\vec{F_2} = 4N$, at $\theta = 50^\circ$ with $-y$ clockwise

$\vec{F_3} = 10N$, $\theta = 35^\circ$ with $+x$ Counter Clockwise

$d = 4.m$ (in the direction of F_{net})

Find W_{net}?

$$W_{net} = \vec{F}_{net} \cdot d$$
$$= F_{net} d \cos\theta$$

To find F_{net}:

$$\Sigma F_x = -3 + 10 \cos 35 - 4 \cos 40$$
$$= 2.13 N$$

$$\Sigma F_y = 10 \sin 35 - 4 \sin 40$$
$$= 3.17 N$$

$$F_{net} = \sqrt{(2.13)^2 + (3.17)^2}$$
$$= 3.82 N$$

$$W_{net} = (3.82)(4) \cos 0$$
$$= 15.3 J$$

Ch:7 , Lec. 2

$\rightarrow K = \frac{1}{2} m v^2$

$\rightarrow W = \vec{F} \cdot \vec{d} = (F \cos \theta) d$ (work done by constant $\vec{F}$)

$\rightarrow W_{net} = \Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$

* Work done by variable $\vec{F}$:

$$W = \int_{r_i}^{r_f} \vec{F} \cdot d\vec{r}$$ = Area under the curve of F vs. r

$$= \int_{r_i}^{r_f} (F_x \hat{i} + F_y \hat{j} + F_z \hat{k}) \cdot (dx \hat{i} + dy \hat{j} + dz \hat{k})$$

$$\boxed{W = \int_{x_i}^{x_f} F_x \, dx + \int_{y_i}^{y_f} F_y \, dy + \int_{z_i}^{z_f} F_z \, dz}$$

Sample problem 8 :

$\vec{F} = 3x^2 \hat{i} + 4\hat{j}$ N , if the particle moves from $\underset{x_i \; y_i}{(2,3)}$ m $\longrightarrow$ $\underset{x_f \; y_f}{(3,0)}$ m

$$W = \int_{x_i}^{x_f} F_x \, dx + \int_{y_i}^{y_f} F_y \, dy$$

$$= \int_{2}^{3} 3x^2 \, dx + \int_{3}^{0} 4 \, dy$$

$$= \cancel{3} \frac{x^3}{\cancel{3}} \Big|_{2}^{3} + 4y \Big|_{3}^{0}$$

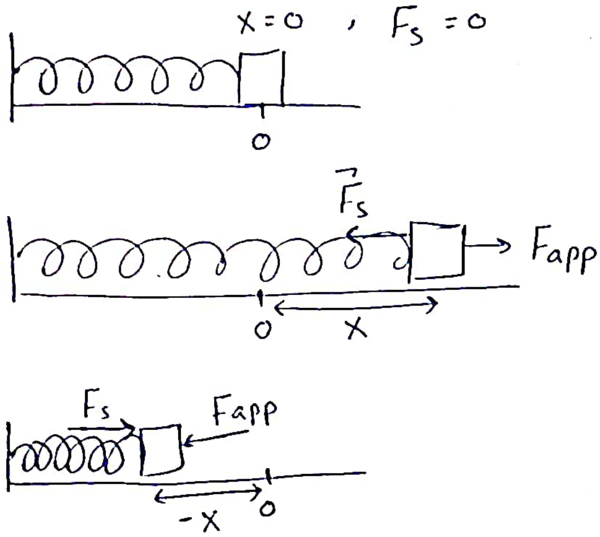
$$= [3^3 - 2^3] + \{4(0) - 4(3)\}$$

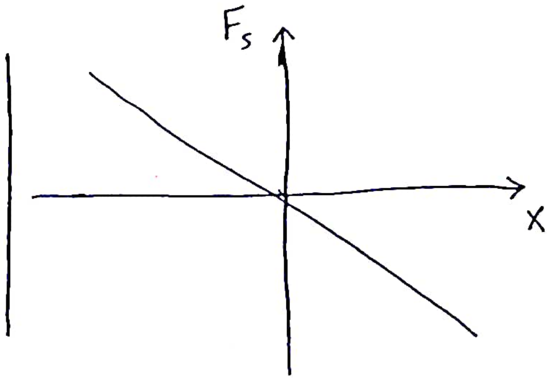
$= 7$ J $\rightarrow$ W is positive $\Rightarrow K_f > K_i$

if $\vec{r}_i = \underset{x_i}{2\hat{i}} + \underset{y_i}{3\hat{j}}$ m , $\vec{r}_f = \underset{x_f}{3\hat{i}}$, $y_f = 0$

* Work done by a spring force:

$F_s = -kx$ (Hook's Law) , k: sping constant (N/m)





$$W_s = \int_{x_i}^{x_f} F_x \, dx = \int_{x_i}^{x_f} -kx \, dx$$

$$= -k \frac{x^2}{2} \Big|_{x_i}^{x_f}$$

$$= -\frac{1}{2} k x_f^2 + \frac{1}{2} k x_i^2$$

$$W_s = \frac{1}{2} k x_i^2 - \frac{1}{2} k x_f^2$$

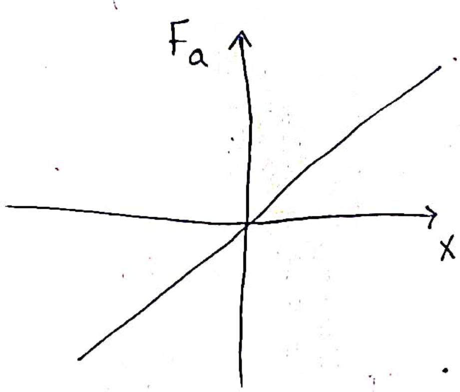
→ work done by the applied force:

if $V_i = V_f$

$$W_{net} = \Delta K$$

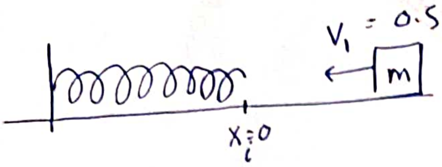
$$W_s + W_a = \cancel{K_f - K_i} \; 0$$

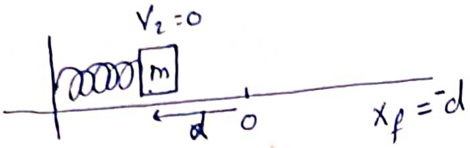
$$\boxed{W_a = -W_s}$$



Sample problem 7.06:

$m = 0.4\ kg \quad , K = 750\ N/m$





Find x_f? ~~($x_f = -d$)~~

$$W_s = \int_0^{-d} -kx\,dx = -k\frac{x^2}{2}\Big|_0^{-d}$$

$$= -\frac{kd^2}{2}$$

$$W_{net} = \Delta K$$

$$W_s = K_f - K_i$$

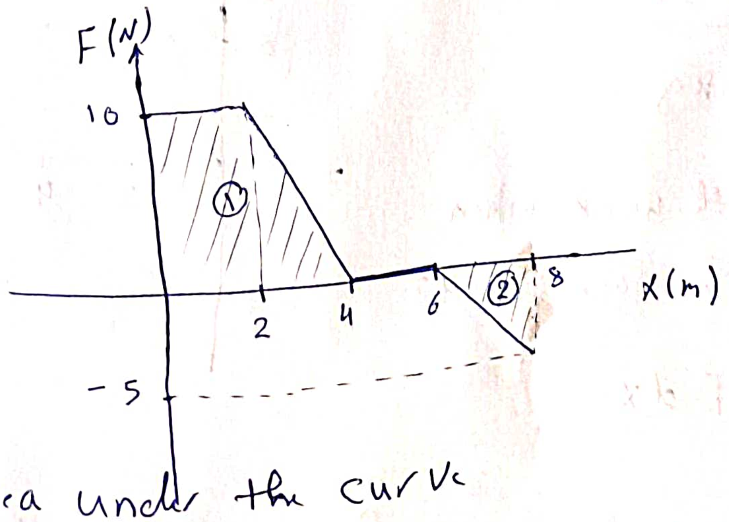
$$-\frac{kd^2}{2} = \cancel{\frac{1}{2} m v_f^2}^{\,0} - \frac{1}{2} m v_i^2$$

$$-\frac{750\,d^2}{2} = -\frac{1}{2}(0.4)(0.5)^2$$

$$d = 1.2 \times 10^{-2}\ m = 1.2\ cm$$

problem 7-36

Find the work done by this force from $x = 0 \to x = 8\ m$



$$W = \int_0^8 F\,dx = \text{Area under the curve}$$

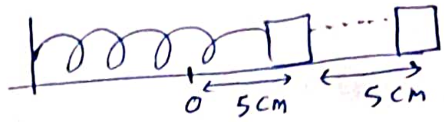
$$W = Area_{(1)} + Area_{(2)}$$

$$= \frac{1}{2}(2+4)\times 10 + \frac{1}{2}(2)(-5)$$

$$= 25\ \text{J}$$

problem 26

$K = 5\times 10^3$ N/m



Find W required to stretch it further by another 5 cm?

$x_i = 5$ cm , $x_f = 10$ cm

$$W_s = \int_{x_i}^{x_f} -Kx\,dx = -\frac{1}{2}Kx^2 \Big|_{x_i}^{x_f}$$

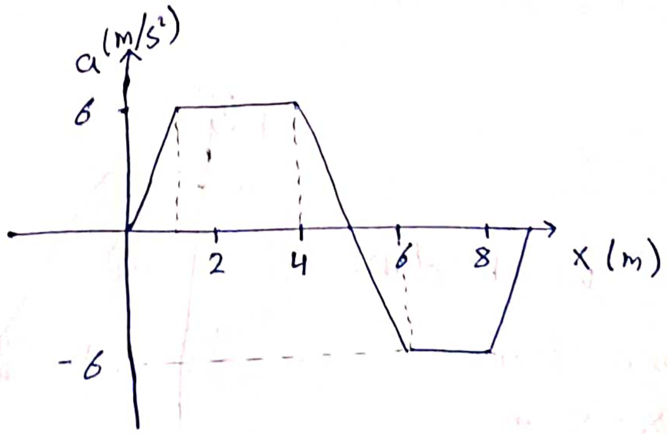
$$= -\frac{1}{2}Kx^2 \Big|_{0.05}^{0.1}$$

$$= -\frac{1}{2}(5\times 10^3)(0.1)^2 + \frac{1}{2}(5\times 10^3)(0.05)^2$$

$$= -18.75\ \text{J}$$

problem 37

$m = 2$ kg



a) Find the work when the particle reaches $x = 4$?

$$W = \int_0^4 F\,dx$$

$$= \int_0^4 ma\,dx = m\int_0^4 a\,dx = m(\text{area under the curve } (0\to 4))$$

$$W = 2\left[\frac{1}{2}(4+3)6\right]$$

$$= 42\ J$$

d) what is the particle's speed & direction when it reaches $x = 4\ m$?

$$W = \Delta K$$

$$= K_f - \cancel{K_i}^{\,0}$$, the particle start from rest. $V_i = 0$

$$W = \frac{1}{2} m V_f^2$$

$$42 = \frac{1}{2}(2)\, V_f^2 \Rightarrow V_f = 6.4\ m/s\ \ (+x)$$

* Average power = the time rate of doing work

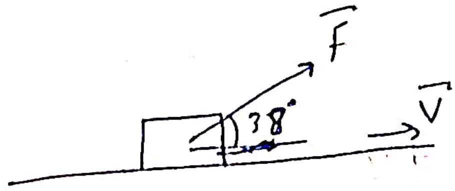
$$P_{avg} = \frac{\text{Work}}{\text{time}} = \frac{W}{\Delta t} \quad (J/s = \text{watt})$$

$$P_{inst.} = \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt}$$

$$= \frac{d}{dt}(\vec{F}\cdot d)$$

$$= \vec{F}\cdot\vec{v}$$

problem 45



$F = 125\ N$

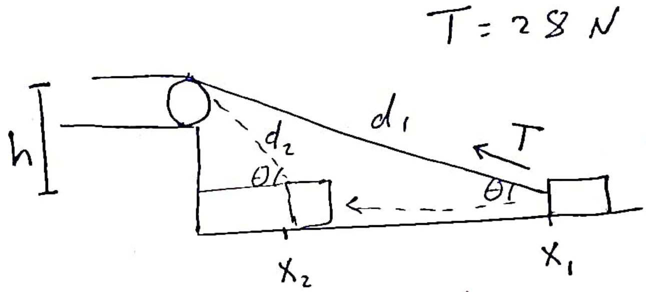
Find power when $v = 5.5\ m/s$?

$$P_{inst} = \vec{F} \cdot \vec{v} = 125 \times 5.5 \cos 38$$

$$= \cancel{487}\ \text{watt}$$

$$541.7$$

problem 42



$x_1 = 3\ m,\quad x_2 = 1\ m$

Find ΔK ? $\quad h = 1.25\ m$

$W = \Delta K$

$W = \vec{F} \cdot \vec{x} = F\, x \cos\theta$, θ is variable

$= F\, d$, d : is the length of the cord pulled as the cart slides from x_1 to x_2.

$$d = d_1 - d_2$$

$$= \sqrt{x_1^2 + h^2} - \sqrt{x_2^2 + h^2}$$

$$= \sqrt{3^2 + (1.25)^2} - \sqrt{1^2 + (1.25)^2}$$

$$= 1.65\ m$$

$$W = 28 \times 1.65 = 46\ J = \Delta K$$