

8.4 Integration Using Partial Fraction

Wednesday, February 16, 2022 3:06 PM

Exp 2 $\int \frac{dx}{x^2 - x - 2} = \int \frac{dx}{(x-2)(x+1)}$ Partial Fraction

كسر جزئية بيده كسر يعقد

$$= \int \left(\frac{A}{x-2} + \frac{B}{x+1} \right) dx$$

كسرينا جزئيتين (جزء)

$$\frac{1}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$$

المقام خطي linear Poly. of degree 1

Cover Method $\Rightarrow A = \frac{1}{2+1} = \frac{1}{3}$

$$B = \frac{1}{-3-2} = -\frac{1}{3}$$

$$\frac{1}{(x-2)(x+1)} = \frac{\frac{1}{3}}{x-2} + \frac{-\frac{1}{3}}{x+1}$$

$$= \frac{\frac{1}{3}(x+1) - \frac{1}{3}(x-2)}{(x-2)(x+1)}$$

$$= \frac{\frac{1}{3}x + \frac{1}{3} - \frac{1}{3}x + \frac{2}{3}}{(x-2)(x+1)}$$

$$= \frac{1}{(x-2)(x+1)}$$

$$\int \frac{dx}{(x-2)(x+1)} = \int \left(\frac{\frac{1}{3}}{x-2} + \frac{-\frac{1}{3}}{x+1} \right) dx$$

$$= \frac{1}{3} \int \frac{dx}{x-2} - \frac{1}{3} \int \frac{dx}{x+1}$$

$$= \frac{1}{3} \ln|x-2| - \frac{1}{3} \ln|x+1| + C$$

$$\int_{-x_0} \frac{x+4}{x+4} dx = \int \frac{x+4}{x+4} dx$$

$$\begin{aligned} \text{Exp } \int \frac{x+4}{x^2+5x-6} dx &= \int \frac{x+4}{(x+6)(x-1)} dx \\ &= \int \left(\frac{A}{x+6} + \frac{B}{x-1} \right) dx \\ A = \frac{[-6]+4}{[-6]-1} &= \frac{-2}{-7} = \frac{2}{7} \\ B = \frac{[-6]+4}{[-6]+6} &= \frac{-2}{0} \quad \checkmark \\ &= \int \left(\frac{\frac{2}{7}}{x+6} + \frac{\frac{5}{7}}{x-1} \right) dx \\ &= \frac{2}{7} \ln|x+6| + \frac{5}{7} \ln|x-1| + c \end{aligned}$$

$$\begin{aligned} \frac{x+4}{(x+6)(x-1)} &= \frac{A}{x+6} + \frac{B}{x-1} \\ \frac{x+4}{(x+6)(x-1)} &= \frac{A(x-1) + B(x+6)}{(x+6)(x-1)} \end{aligned}$$

$$\Leftrightarrow \boxed{x+4 = A(x-1) + B(x+6)}$$

↑

$$x=1 \Rightarrow 1+4 = 0 + B(1+6)$$

$$5 = 7B$$

$$\boxed{B = \frac{5}{7}}$$

$$x=-6 \Rightarrow -6+4 = A(-6-1) + 0$$

$$-2 = -7A$$

$$\boxed{A = \frac{2}{7}}$$

Cover Method

لازم يكون المقام عبارة عن حاصل ضرب عوامل مختلفة

Cover Method

عوامل مختلفة ومبطل في عملية

$$\checkmark \frac{x+7}{x(x-1)(x+3)(x+5)}$$



x

$$\frac{\quad}{(x^2-1)(x-1)(x-1)}$$

x

Ex P

$$\int \frac{x^2+4x+1}{(x-1)(x+1)(x+3)} dx$$

$$\int \left(\frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+3} \right) dx$$

$$\begin{aligned} c &= \frac{(-3)^2 + 4(-3) + 1}{(-3-1)(-3+1)} \\ &= \frac{9-12+1}{(-4)(-2)} \\ &= \frac{-2}{8} = -\frac{1}{4} \end{aligned}$$

Cover Method

$$A = \frac{0^2 + 4(0) + 1}{(0+1)(0+3)} = \frac{1+4+1}{(2)(4)} = \frac{6}{8} = \frac{3}{4}$$

$$B = \frac{(-1)^2 + 4(-1) + 1}{(-1-1)(-1+3)} = \frac{1-4+1}{(-2)(2)} = \frac{-2}{-4} = \frac{1}{2}$$

$$= \frac{3}{4} \ln|x-1| + \frac{1}{2} \ln|x+1| - \frac{1}{4} \ln|x+3| + C$$

