

Ch.1 | Digital Systems and binary Numbers

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Popular Number Systems

1. Binary = Radix 2

- each digit called bit
- only two digit 0s and 1s.

2. Octal = Radix 8

- only eight digit : 0 to 7
- Digits 8 and 9 not used.

3. Decimal = Radix 10

- 10 digits, 0 → 9

4. HexaDecimal = Radix 16

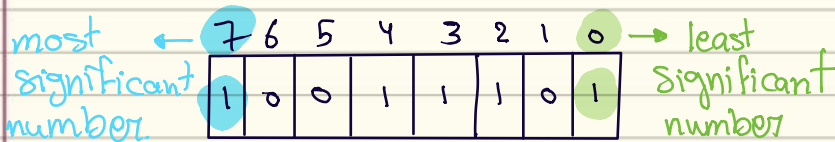
- 16 digits, 0 → 9 and A → F
- A=10, B=11, ..., F=15.

memorise table.

Decimal Radix 10	Binary Radix 2	Octal Radix 8	Hex Radix 16
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Bit numbering:

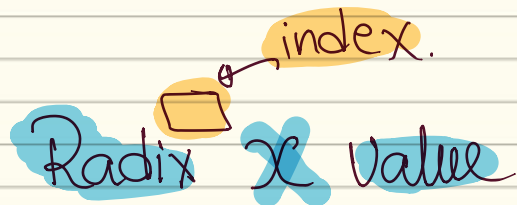
- least Significant Bit [LSB] is Right most. (bit 0)
- Most Significant Bit [MSB] is left most.



Decimal Value of Binary Number

- each bit represent a power of 2.
- every binary number is represent a power of 2.

to convert from any system to decimal



Binary to Decimal :-

ex:- $(10011101)_2$

index = 7 6 5 4 3 2 1 0

radix = 2

$$= 2^0 \times 1 + 2^1 \times 0 + 2^2 \times 1 + 2^3 \times 1 + 2^4 \times 1 + 2^5 \times 0 + 2^6 \times 0 + 2^7 \times 1$$

$$= (157)$$



Octal to Decimal

$$\text{ex: } (2107)_8 = (\quad)_{10}$$

$$8^0 \times 7 + 8^1 \times 0 + 8^2 \times 1 + 8^3 \times 2 = (1095)_{10}$$

ex. $(2051)_4$ to decimal

invalid.
Radix = 4, So possible numbers are between $0 \rightarrow 3$

ex. $(3BAY)_{16}$ to decimal $16^0 \times 4 + 16^1 \times 10 + 16^2 \times 11 + 16^3 \times 3$
 $= 15268$

NOTE:-

possible number for radix r

is $0 \rightarrow r-1$

ex, $r = 8, 0 \rightarrow 7$

$r = 2, (0, 1)$

$r = 4, 0 \rightarrow 3$

⋮

Convert from decimal to any system

- Repeatedly divide the decimal integer by the radix of the Sys
- each remainder is a digit in the translated value.
- Stop when quotient is Zero.

ex. Convert $(37)_{10}$ to Binary

radix = 2

37	2	
18	1	→ LSB
9	0	
4	1	
2	0	
1	0	
0	1	→ MSB

$$(37)_{10} = (100101)_2$$

ex. Convert $(422)_{10} \rightarrow (\quad)_{16}$

422	16	
26	6	→ LSB
1	10	
0	1	→ MSB

$$(422)_{10} = (1A6)_{16}$$



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* Converting octal and hexadecimal to Binary is easy.

→ $8 = 2^3$ → so each 3 digits equals 1 bit in Binary and vice versa.

→ $16 = 2^4$ → so each 4 digits in hexa equals 1 bit in Binary and vice versa.

7	5	3	0	5	5	2	3	6	2	4	Octal																					
1	1	1	0	1	0	1	1	0	0	0	1	0	1	1	0	1	0	1	0	0	1	1	1	1	0	0	1	0	1	0	0	32-bit binary
E	B	1	6	A	7	9	4	Hexadecimal																								

* Starting from [LSB] group each 4 Bits into a Hex digit or each 3 bits into one octal digit.

To convert from octal to hexa or vice versa

1) Convert from octal to binary.

2) Convert from binary to hexa.



important properties

* How many possible digits can we have in radix r ?

* What's the result of adding (1) to the largest digit in Radix r ?
ex. $(1)_2 + 1 = (10)_2$ $(7)_8 + 1 = (10)_8$

$$(9)_{10} + 1 = (10)_{10} \quad (F)_{16} + 1 = (10)_{16}$$

* What's the largest value using 3 digits in Radix r ?

$$r^3 - 1 \quad , \quad \text{ex:- In Binary } 2^3 - 1 = 7 \rightarrow (111)_2$$
$$\text{ex. In Octal } 8^3 - 1 \quad (777)_8$$

In general :- the largest value using n digit in Radix r is ..

n number of digit
radix - 1

* How many possible values can be represented

• using n binary digits 2^n values , $0 \rightarrow 2^n - 1$

• using n octal digits 8^n values , $0 \rightarrow 8^n - 1$

• using n decimal digits 10^n values , $0 \rightarrow 10^n - 1$

• using n Hexa decimal digits 16^n values , $0 \rightarrow 16^n - 1$

In general :- using n digits in Radix

r^n values , 0 to $r^n - 1$.



representing fractions

- * Integer values start with index 0.
- * Fraction values start with index -1.

Convert Decimal Fraction to Binary.

- by multiplying the fraction by 2 Repeatedly.
- Collect integer bits.
- stop when the number on the most right of the point equals 0.

ex. $(0.6875)_{10} = (0.1011)_2$

0.6875×2	1.3750
$0.3750 \times 2 =$	0.750
$0.75 \times 2 =$	1.50
$0.5 \times 2 =$	1.0

MSB

integers cuts *

stop when fraction equals zero.

LSB

Decimal إلى الثنائي *

أي نظام

integers

Fractions

Radix من

Radix من



ex. $(139.6875)_{10} = (\quad)_{8}$

5	8
139	•
17	3
2	1
0	2

$(213.54)_{8}$

- * ينقسم السؤال إلى جزأين
- integer جزء ✓
- fractions جزء ✓
- up to expression من ✓

$0.6875 \times 8 = 5.5000$
 $0.5 \times 8 = 4.0$ stop.

Converting Binary and Hexa to Binary 80

← integer: right to left fraction: left to right →

7	2	6	1	3	.	2	4	7	4	5	2	Octal
1111010110001011	.	010100111110010101	Binary									
7	5	8	B	.	5	3	C	A	8	Hexadecima		

important properties of fractions

* How many fractional values exist with m fractional Bits?
 2^m In general. r^m value
 radix

* whats the largest fraction value if m fraction digits are used in radix r?
 $1 - r^{-m}$ → number of digits
 Radix

ex. Convert $(299.8195)_{10} \rightarrow (20B.9A)_{16}$ at most two fractional digits, if necessary.



5	16
299	•
24	(11) = B
2	0
0	2

$0.8195 \times 16 = 9.834$
 $0.834 \times 16 = 10.008$

arithmetic operation

Binary Addition :-

$$\begin{array}{r}
 00110110 \\
 00011101 \\
 \hline
 01010011
 \end{array}
 + \begin{array}{r}
 (54) \\
 + (29) \\
 \hline
 (83)
 \end{array}$$

Binary Subtraction :-

$$\begin{array}{r}
 11011 \\
 10110 \\
 \hline
 00101
 \end{array}$$

* لما بنستاف
. ناخذ 2

Octal addition :-

$$\begin{array}{r}
 567 \\
 243 \\
 \hline
 1032
 \end{array}
 +$$

$$\begin{array}{l}
 7+3=10 > 8 \\
 10 = 1 \times 8 + 2 \\
 \text{Carry} \downarrow \text{Carry} \downarrow
 \end{array}$$

$$C_0 = A_0 \times 8 + B_0$$

Carry \downarrow \rightarrow sum

Octal Subtraction :-

$$\begin{array}{r}
 5624 \\
 265 \\
 \hline
 337
 \end{array}$$

* بنستاف 8

ex.

$$\begin{array}{r}
 56000 \\
 777 \\
 \hline
 5001
 \end{array}$$

Hexa Addition :-

ex.

$$\begin{array}{r}
 ADD \\
 DAD \\
 \hline
 188A
 \end{array}
 +$$

A B C D E F
10 11 12 13 14 15

$$D+D = 13+13 = 26 = 16 \times 1 + 10$$

Carry \downarrow \rightarrow sum

$$A+D+1 = 24 = 16 \times 1 + 8$$

Carry \downarrow \rightarrow sum

Hexa Subtraction :-

$$\begin{array}{r}
 8974B \\
 587C \\
 \hline
 3EAF
 \end{array}$$



complement of numbers

"for unsigned numbers"

1) Radix Complement 8-

$$A = r^n - N$$

$$B = (r-1)'s \text{ Comp} + 1$$

2) Diminished Radix Complement 8

$(r-1)'s \text{ Comp.}$

$$(r^n - 1) - N$$

$r \rightarrow$ radix

$n \rightarrow$ digits

$N \rightarrow$ the number

EX. Find the 9's Comp of (134795).

8	6	5	2	0	4
1	3	4	7	9	5
8	6	5	2	0	4

* طريقة التاكيد ← Comp
 قلة يقة م م م م م م
 م م م م م م م م م م م م

أو حسب القانون

$$r = 10, n = 6, N = 134795$$

1 2 3 4 5 6

$$= r^n - 1 - N$$

$n \rightarrow 6$ digits

$$= 10^6 - 1 - 134795$$

$N \rightarrow$ الرقم نفسه

$$= 999999 - 134795$$

$r =$ radix

$$= 865204$$

⇒ to find 10's Comp. 9's Comp + 1.

Find the 10's Comp of (134795).

* طريقة سريعة 8

← يترك كل الأرقام، وأول رقم بعد الألفا يضاف له (10)، والباقي بالمتكافؤ له (9)

$$10's \text{ Comp} = 9's \text{ Comp} + 1$$

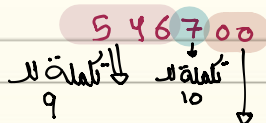
$$= 865204 + 1$$

$$= 865205$$

EX. Find 9's and 10's Comp of (546700).

$$9's \text{ Comp} = 453299$$

$$10's \text{ Comp} = 463300$$



$$453300$$

* to find the 1's Comp 8- Convert 0's to 1's and 1's to 0's

$$2's \text{ Comp} = 1's \text{ Comp} + 1.$$

Remarks-

The Comp of Comp restores the original value ⇒ $\text{Comp}(\text{Comp}(N)) = N$



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examples

1. Consider a binary number $3 = 1011001$
 1 's Complement: $- 0100110$
 2 's Complement: $- 0100111 \rightarrow +1$

2. Consider a binary number $8 = 00100100$
 1 's Complement: $- 11011011$
 2 's Complement: $- 11011100 \rightarrow +1$

00100100
 11011000
 flip same

طريقة ثانية -
 الألفا وأول رقم يسوم بـ 1
 والي يسوم بـ 0 بقلبهم [0 ← 1
 [1 ← 0]

Complement are used to simplify Subtraction operation in digital computers.

Advantages of simplification :-

1. it results in simpler circuits.
2. it results in low cost.

which means (fewer and simpler hardware components).

* Assume that we need to subtract x from y

then we have two cases & 1) $x \geq y$, 2) $x < y$

1) $x \geq y$

$(r-1)$'s Comp
 $x + (r-1)$'s Comp

if their exist carry
 add(1) to the result

* the final answer will be positive

r 's Comp
 $x + r$'s Comp.

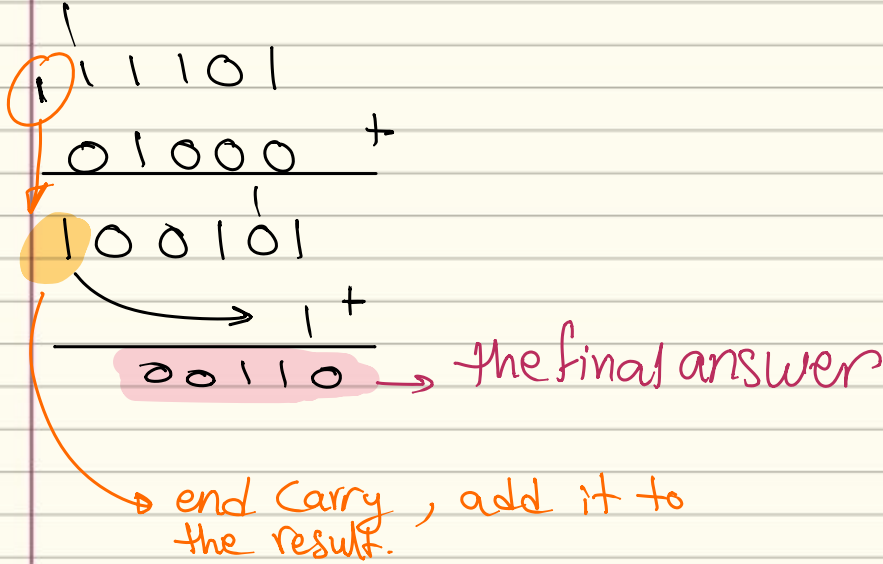
if end carry occurs
 will be discarded.



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ex. $X = 11101$, $Y = 10111$
find $X - Y$, using 1's complement.

$$\begin{aligned} X > Y, & \quad = X - Y \\ & \quad = X + 1's(Y) \\ & \quad = 11101 + (01000) \\ & \quad = 100110 \end{aligned}$$



ex. $X = 11101$, $Y = 10111$
find $X - Y$ by 2's comp

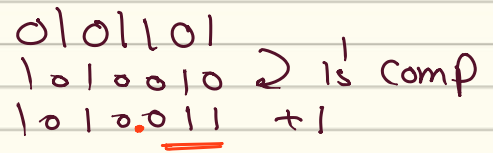
$$\begin{aligned} & \quad = X + 2's(Y) \\ & \quad = \begin{array}{r} \overset{1}{\circ} 11101 \\ + 01001 \\ \hline 100110 \end{array} + \\ & \quad \text{the final answer} \\ & \quad \text{will be discarded} \end{aligned}$$

NOTE:-

fraction subtraction
تفاضل كسور على رقم
صحيح ونحوها يرجع
إلى point على نفس
الخط

ex:- 10110.00
1's Comp = 01001.11

ex:- 0101.101
2's Comp = 101.011



- ② $X < Y$ →
- ① the final answer will be negative
 - ② won't produce end carry.
 - ③ after taking the comp of the final carry, put the negative sign.



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ex. using 10's Comp. $\rightarrow 3250 - 72532$

$$= 03250 + (27468)$$

$$= \begin{array}{r} 103250 \\ 3250 \end{array}$$

$$\underline{27468} +$$

$$30718 \rightarrow \text{no end carry}$$

$$10's(30718) = -69282$$

ex. using 9's Comp. do $285.31 - 3459.20$

$$= 285.31 + (6540.79)$$

$$\begin{array}{r} 1 \quad 1 \quad 1 \\ 285.31 \end{array}$$

$$6540.79 +$$

$$\underline{6826.10} \rightarrow \text{no end carry.}$$

$$9's(6826.10) = -3173.89$$



signed binary numbers

3 major techniques are used to represent signed numbers :-

1. Signed magnitude.
2. 1's Complement.
3. 2's Complement.

1 Signed magnitude :-

- left most bit is the sign bit :- 0 for \oplus^{ve}
1 for \ominus^{ve}
- two representation for Zero (NOT GOOD)!
- Range $-2^{(n-1)} \rightarrow + (2^{n-1})$.
- examples :-

-1	4 bits	→	1001	
+6	4 bit	→	0110	positive
+15	8 bit	→	00001111	
-79	8 bit	→	11001111	negative

- the only way, that represent \oplus^{ve} and \ominus^{ve} .

2 1's complement.

- 2 representation for 0.
- Rang $-2^{n-1} \rightarrow +2^{n-1}$

کُل ۱-۱ نکتہ الی رقم الی جدولین
 2) ناخن کا لہ
 Comp



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 الی أفند

ex. -16

+16 ①

3] 2's Complement.

- only one representation for 0.
- Range $-2^{n-1} \rightarrow +2^{n-1} - 1$

- 2's Comp = 1's Comp + 1
- نتيجة نأخذ 2's Comp عشوائياً نجيب القيمة الأصلية.

1] يكتب الرقم الموجب (أخذ bit صفر)
 2] 1's comp
 3] + 1

- only for negative number.

• $2's\text{Comp}(N) + N = 0$

ex. -16
 $+16 = 010000$
 $\hookrightarrow 1's\text{comp} = 101111$
 $+1 = 110000$

* negative weight to the sign bit (MSB)

ex. 10110100
 $-128 \quad 32 \quad 8 \quad 4 \quad 2 \quad 1$
 $64 \quad 16$

$4 + 16 + 32 + -128 = -76$

* Range values for unsigned number $0 \rightarrow +(2^n - 1)$

* Range values for signed number $-2^{n-1} \rightarrow +2^{n-1} - 1$

* positive integers $0 \rightarrow 2^{n-1} - 1$

* negative integers $-2^{n-1} \rightarrow -1$

8 bits $-128 \rightarrow 127$

} according to 2's Comp.

* Signed magnitude

$-2^{n-1} - 1 \rightarrow +2^{n-1} - 1$

ex. $A = (00101100) = +44$

$\hookrightarrow 2's (11010100) = -44$



* Subtraction :-

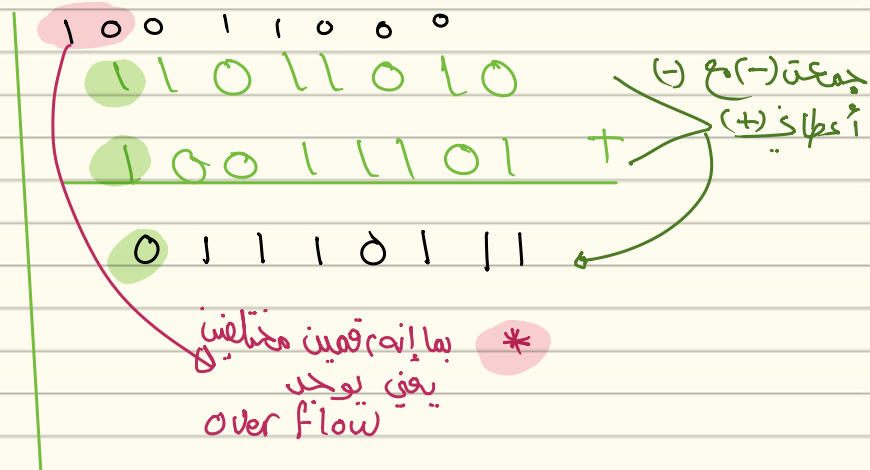
↳ Convert sub. into addition and get the 2's Comp of the second # with ignoring the carry.

* overflow occurs :-

1. adding two positive numbers and the sum is neg.
2. adding two negative numbers and the sum is pos.

ex. $\begin{array}{r} 1111000 \\ 10000111 \\ \hline 00000111 \end{array} +$

1 Carry



* إذا طلعنا متساويين يعني لا يوجد overflow

* Carry is important when

- Adding or Sub, unsigned numbers.
- indicates that the unsigned sum is out of range.

* إذا عني رقم متساويين ولي أي قيمة

ex: $1101 \rightarrow 2's \text{ Comp.}$

$\begin{array}{r} 0010 \\ 0010 \\ \hline 0011 \end{array} \xrightarrow{+1}$

= -3

ex. $\begin{array}{r} 00100100 \\ 11011011 \\ \hline 11011100 \end{array} \xrightarrow{+1}$

= -36

ex: $1001 \rightarrow 2's \text{ Comp}$

$\begin{array}{r} 0110 \\ 0110 \\ \hline 0111 \end{array} \xrightarrow{+1}$

= -7

ex. (+5) \rightarrow in 8bits

$\begin{array}{r} 0000101 \\ 11111010 \\ \hline 11111011 \end{array} \xrightarrow{+1}$



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Positive numbers in signed magnitude and 2's Comp. are the same.

* the 2's Comp of N is the negative of N

* the sum of N and the 2's Comp of N is Zero.

BINARY CODES

• do representation

• $\lceil \log_2 \square \rceil$

* لعيني 20 لون ودي اسيكل
لون عن الآخر بيعد البتة. زك
bit بحتيج ؟

$$\lceil \log_2 20 \rceil = 5$$

$$16 < 20 < 32$$

$$2^4 < 20 < 2^5$$

the minimum number of bits.

ex. if you have 3 bits
how many numbers can you
do?

$$2^3 = 8$$

2^n → n = number of bits.

Decimal/Codes 8-

$$\lceil \log_2 10 \rceil = 4$$

10 char
4 bit
بحتاج

Decimal	BCD
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
	1010
Unused	...
	1111



Conversion And Coding

$(13)_{10} = (1101)_2$, this is Conversion. , 4 bit needed

$13 = (0001\ 0011)_{BCD}$, this is coding , 8 bit needed

* In general, Coding requires more bits than Conversion

* كل رقم في BCD يعبر عنه بحاله

* أكبر عدد في BCD هو 9 (1001)

لأن

إذا طرقت الاتجاه أكبر من 9
يضيف عليه 6

	BCD
8	1000
5	0101 +
13	11101
	0110 +
	<u>0001 0011</u>
	1 3

ex. 1897

2905 +

0001	1000	1001	0111	+
0010	1001	0000	0101	+
0100	10010	1010	1100	+
0000	0110	0110	0110	+
0100	1000	10000	10010	
<div style="border: 1px solid black; padding: 5px; display: inline-block;"> 4 8 0 2 </div>				



Gray Code

↳ successive values differ in only one bit

used :- sequence of binary numbers generated by the hardware that may cause an error or ambiguity during the transition from one number to the next.

from binary to gray

1 0 0 1 1 0 Binary Code
↓ ⊕ ⊕ ⊕ ⊕ ⊕
1 1 0 1 0 1 gray Code

from gray to binary

1 1 0 1 0 1 gray
↓ ⊕ ⊕ ⊕ ⊕ ⊕
1 0 0 1 1 0 Binary

1. The Gray code for the binary value (100110) is Answer 1, and this code and the Gray code (110111) Answer 2 (are/are not) successive codes. (Fill in the blank)

Answer 1

Type your answer 110101

Answer 2

Type your answer yes



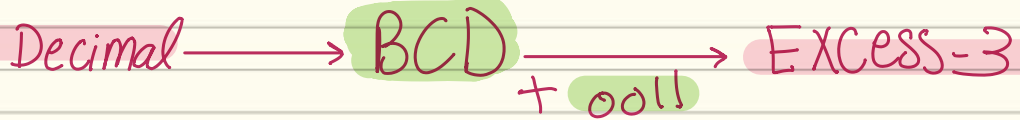
other decimal codes

Decimal	BCD 8421	5421 code	2421 code	8 4 -2 -1 code	Excess-3 code
0	0000	0000	0000	0000	0011
1	0001	0001	0001	0111	0100
2	0010	0010	0010	0110	0101
3	0011	0011	0011	0101	0110
4	0100	0100	0100	0100	0111
5	0101	1000	1011	1011	1000
6	0110	1001	1100	1010	1001
7	0111	1010	1101	1001	1010
8	1000	1011	1110	1000	1011
9	1001	1100	1111	1111	1100
Unused		

* BCD 5421, 2421
8 4 . 2 . 1
are weighted codes

* Excess-3 is not
weighted code.

EXcess-3



1. The decimal number 17 can be represented in binary as Answer 1 and in Excess-3 as Answer 2. (Fill in the blank)

Answer 1
Type your answer

Answer 2
Type your answer

17 BCD \rightarrow

$$\begin{array}{r} 0001\ 0111 \\ + 0011 \\ \hline 0001\ 1010 \\ + 0110 \\ \hline 0010\ 0000 \\ \hline 20 \end{array}$$

ex. $(27)_{10} + (39)_{10}$

$$\begin{array}{r|l} 0010\ 0111 & 0011\ 1001 \\ + 0011 & + 0011 \\ \hline 0010\ 1010 & 0011\ 1100 \\ + 0110 & + 0110 \\ \hline 0011\ 0000 & 0100\ 0010 \\ & 42 \end{array}$$

$30 + 12 = 42$



ربنا تقبل منا إنك أنت السميع العليم

روان فارس

