15

Active Filter Circuits

Assessment Problems

AP 15.1

$$H(s) = \frac{-(R_2/R_1)s}{s + (1/R_1C)}$$

$$\frac{1}{R_1C} = 1\,\text{rad/s}; \qquad R_1 = 1\,\Omega, \quad \therefore \quad C = 1\,\text{F}$$

$$\frac{R_2}{R_1} = 1, \quad \therefore \quad R_2 = R_1 = 1\,\Omega$$

$$\therefore \qquad H_{\text{prototype}}(s) = \frac{-s}{s+1}$$

AP 15.2

$$H(s) = \frac{-(1/R_1C)}{s + (1/R_2C)} = \frac{-20{,}000}{s + 5000}$$

$$\frac{1}{R_1C} = 20{,}000; \quad C = 5\,\mu\text{F}$$

$$\therefore \quad R_1 = \frac{1}{(20{,}000)(5 \times 10^{-6})} = 10\,\Omega$$

$$\frac{1}{R_2C} = 5000$$

$$\therefore \quad R_2 = \frac{1}{(5000)(5 \times 10^{-6})} = 40\,\Omega$$

AP 15.3

$$\omega_c = 2\pi f_c = 2\pi \times 10^4 = 20{,}000\pi \text{ rad/s}$$

$$\therefore \quad k_f = 20{,}000\pi = 62{,}831.85$$

$$C' = \frac{C}{k_f k_m} \qquad \therefore \qquad 0.5 \times 10^{-6} = \frac{1}{k_f k_m}$$

$$\therefore \quad k_m = \frac{1}{(0.5 \times 10^{-6})(62{,}831.85)} = 31.83$$

AP 15.4 For a 2nd order Butterworth high pass filter

$$H(s) = \frac{s^2}{s^2 + \sqrt{2}s + 1}$$

For the circuit in Fig. 15.25

$$H(s) = \frac{s^2}{s^2 + \left(\frac{2}{R_2C}\right)s + \left(\frac{1}{R_1R_2C^2}\right)}$$

Equate the transfer functions. For $C = 1$F,

$$\frac{2}{R_2C} = \sqrt{2}, \quad \therefore \quad R_2 = \sqrt{2} = 1.414\,\Omega$$

$$\frac{1}{R_1R_2C^2} = 1, \quad \therefore \quad R_1 = \frac{1}{\sqrt{2}} = 0.707\,\Omega$$

AP 15.5

$$Q = 8, K = 5, \omega_o = 1000\,\text{rad/s}, C = 1\,\mu\text{F}$$

For the circuit in Fig 15.26

$$H(s) = \frac{-\left(\frac{1}{R_1C}\right)s}{s^2 + \left(\frac{2}{R_3C}\right)s + \left(\frac{R_1+R_2}{R_1R_2R_3C^2}\right)} = \frac{K\beta s}{s^2 + \beta s + \omega_o^2}$$

$$\beta = \frac{2}{R_3C}, \qquad \therefore \qquad R_3 = \frac{2}{\beta C}$$

$$\beta = \frac{\omega_o}{Q} = \frac{1000}{8} = 125\,\text{rad/s}$$

$$\therefore \quad R_3 = \frac{2\times 10^6}{(125)(1)} = 16\,\text{k}\Omega$$

$$K\beta = \frac{1}{R_1C}$$

$$\therefore \quad R_1 = \frac{1}{K\beta C} = \frac{1}{5(125)(1\times 10^{-6})} = 1.6\,\text{k}\Omega$$

$$\omega_o^2 = \frac{R_1+R_2}{R_1R_2R_3C^2}$$

$$10^6 = \frac{(1600+R_2)}{(1600)(R_2)(16{,}000)(10^{-6})^2}$$

Solving for R_2,

$$R_2 = \frac{(1600+R_2)10^6}{256\times 10^5}, \quad 246R_2 = 16{,}000, \quad R_2 = 65.04\,\Omega$$

AP 15.6

$$\omega_o = 1000\,\text{rad/s}; \qquad Q = 4;$$

$$C = 2\,\mu\text{F}$$

$$H(s) = \frac{s^2+(1/R^2C^2)}{s^2+\left[\dfrac{4(1-\sigma)}{RC}\right]s+\left(\dfrac{1}{R^2C^2}\right)}$$

$$= \frac{s^2+\omega_o^2}{s^2+\beta s+\omega_o^2}; \qquad \omega_o = \frac{1}{RC}; \qquad \beta = \frac{4(1-\sigma)}{RC}$$

$$R = \frac{1}{\omega_oC} = \frac{1}{(1000)(2\times 10^{-6})} = 500\,\Omega$$

$$\beta = \frac{\omega_o}{Q} = \frac{1000}{4} = 250$$

$$\therefore \qquad \frac{4(1-\sigma)}{RC} = 250$$

$$4(1-\sigma) = 250RC = 250(500)(2\times 10^{-6}) = 0.25$$

$$1-\sigma = \frac{0.25}{4} = 0.0625; \quad \therefore \qquad \sigma = 0.9375$$

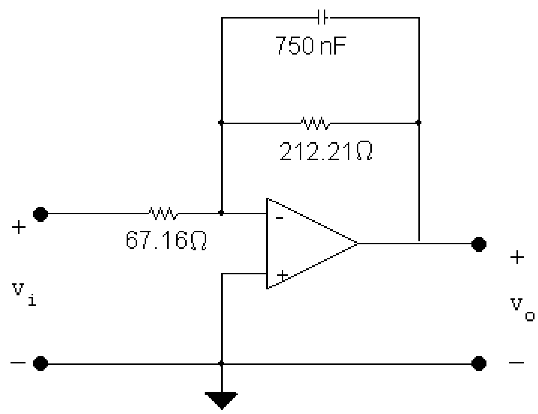
Problems

P 15.1 [a] $K = 10^{(10/20)} = 3.16 = \dfrac{R_2}{R_1}$

$$R_2 = \frac{1}{\omega_c C} = \frac{1}{(2\pi)(10^3)(750 \times 10^{-9})} = 212.21\,\Omega$$

$$R_1 = \frac{R_2}{K} = \frac{212.21}{3.16} = 67.16\,\Omega$$

[b]



P 15.2 [a] $\dfrac{1}{RC} = 2\pi(1000)$ so $RC = 1.5915 \times 10^{-4}$

There are several possible approaches. Here, choose $R_f = 150\,\Omega$. Then

$$C = \frac{1.5915 \times 10^{-4}}{150} = 1.06 \times 10^{-6}$$

Choose $C = 1\,\mu$F. This gives

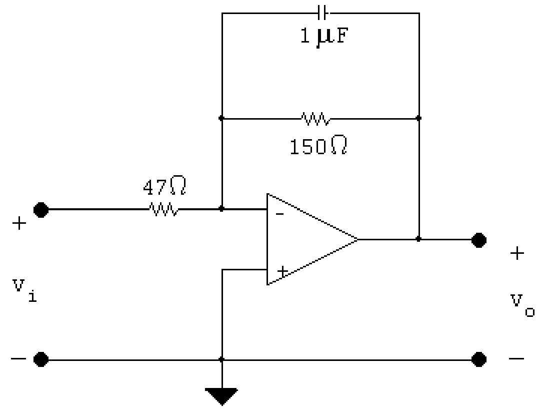
$$\omega_c = \frac{1}{(150)(10^{-6})} = 6.67 \times 10^3 \text{ rad/s} \quad \text{so} \quad f_c = 1061\,\text{Hz}$$

To get a passband gain of 10 dB, choose

$$R_i = \frac{R_f}{3.16} = \frac{150}{3.16} = 47.47\,\Omega$$

Choose $R_i = 47\,\Omega$ to give $K = 20\log_{10}(150/47) = 10.08$ dB.

The resulting circuit is

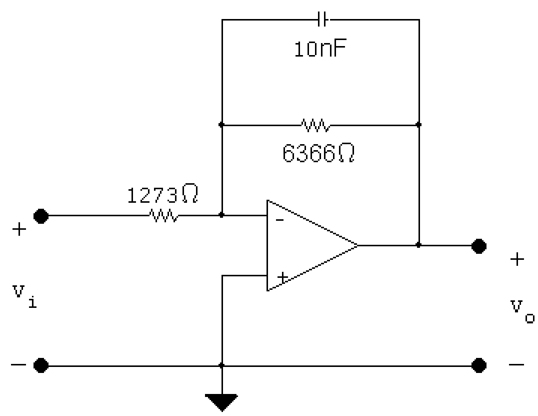


[b] $\%\text{ error in } f_c = \dfrac{1061 - 1000}{1000}(100) = 6.1\%$

$$\%\text{ error in passband gain} = \frac{10.08 - 10}{10}(100) = 0.8\%$$

P 15.3 **[a]** $\omega_c = \dfrac{1}{R_2C}$ so $R_2 = \dfrac{1}{\omega_c C} = \dfrac{1}{2\pi(2500)(10\times10^{-9})} = 6366\,\Omega$

$$K = \frac{R_2}{R_1} \quad \text{so} \quad R_1 = \frac{R_2}{K} = \frac{6366}{5} = 1273\,\Omega$$



[b] Both the cutoff frequency and the passband gain are changed.

P 15.4 **[a]** $5(3.5) = 17.5\,\text{V}$ so $V_{cc} \geq 17.5\,\text{V}$

[b] $H(j\omega) = \dfrac{-5(2\pi)(2500)}{j\omega + 2\pi(2500)}$

$$H(j5000\pi) = \frac{-5(5000\pi)}{5000\pi + j5000\pi} = -2.5 + j2.5 = \frac{5}{\sqrt{2}}\underline{/135^\circ}$$

$$V_o = \frac{17.5}{\sqrt{2}}\underline{/135^\circ}V_i \quad \text{so} \quad v_o(t) = 12.37\cos(5000\pi t + 135^\circ)\,\text{V}$$

[c] $H(j625\pi) = \dfrac{-5(5000\pi)}{5000\pi + j625\pi} = 4.96\underline{/172.9^\circ}$

$V_o = 17.36\underline{/172.9^\circ}V_i \quad \text{so} \quad v_o(t) = 17.36\cos(625\pi t + 172.9^\circ)\,\text{V}$

[d] $H(j40{,}000\pi) = \dfrac{-5(5000\pi)}{5000\pi + j40{,}000\pi} = 0.62\underline{/97.1^\circ}$

$V_o = 2.2\underline{/97.1^\circ}V_i \quad \text{so} \quad v_o(t) = 2.2\cos(40{,}000\pi t + 97.1^\circ)\,\text{V}$

P 15.5 Summing the currents at the inverting input node yields

$$\frac{0 - V_i}{Z_i} + \frac{0 - V_o}{Z_f} = 0$$

$$\therefore \quad \frac{V_o}{Z_f} = -\frac{V_i}{Z_i}$$

$$\therefore \quad H(s) = \frac{V_o}{V_i} = -\frac{Z_f}{Z_i}$$

P 15.6 **[a]** $Z_f = \dfrac{R_2(1/sC_2)}{[R_2 + (1/sC_2)]} = \dfrac{R_2}{R_2C_2s + 1}$

$$= \frac{(1/C_2)}{s + (1/R_2C_2)}$$

Likewise

$$Z_i = \frac{(1/C_1)}{s + (1/R_1C_1)}$$

$$\therefore \quad H(s) = \frac{-(1/C_2)[s + (1/R_1C_1)]}{[s + (1/R_2C_2)](1/C_1)}$$

$$= -\frac{C_1}{C_2}\frac{[s + (1/R_1C_1)]}{[s + (1/R_2C_2)]}$$

[b] $H(j\omega) = \dfrac{-C_1}{C_2}\left[\dfrac{j\omega + (1/R_1C_1)}{j\omega + (1/R_2C_2)}\right]$

$$H(j0) = \frac{-C_1}{C_2}\left(\frac{R_2C_2}{R_1C_1}\right) = \frac{-R_2}{R_1}$$

[c] $H(j\infty) = -\dfrac{C_1}{C_2}\left(\dfrac{j}{j}\right) = \dfrac{-C_1}{C_2}$

[d] As $\omega \to 0$ the two capacitor branches become open and the circuit reduces to a resistive inverting amplifier having a gain of $-R_2/R_1$.
As $\omega \to \infty$ the two capacitor branches approach a short circuit and in this case we encounter an indeterminate situation; namely $v_n \to v_i$ but $v_n = 0$ because of the ideal op amp. At the same time the gain of the ideal op amp is infinite so we have the indeterminate form $0 \cdot \infty$.
Although $\omega = \infty$ is indeterminate we can reason that for finite large values of ω $H(j\omega)$ will approach $-C_1/C_2$ in value. In other words, the circuit approaches a purely capacitive inverting amplifier with a gain of $(-1/j\omega C_2)/(1/j\omega C_1)$ or $-C_1/C_2$.

P 15.7 **[a]** $$Z_f = \frac{(1/C_2)}{s + (1/R_2C_2)}$$

$$Z_i = R_1 + \frac{1}{sC_1} = \frac{R_1}{s}[s + (1/R_1C_1)]$$

$$H(s) = -\frac{(1/C_2)}{[s + (1/R_2C_2)]} \cdot \frac{s}{R_1[s + (1/R_1C_1)]}$$

$$= -\frac{1}{R_1C_2}\frac{s}{[s + (1/R_1C_1)][s + (1/R_2C_2)]}$$

[b] $$H(j\omega) = -\frac{1}{R_1C_2}\frac{j\omega}{\left(j\omega + \frac{1}{R_1C_1}\right)\left(j\omega + \frac{1}{R_2C_2}\right)}$$

$$H(j0) = 0$$

[c] $H(j\infty) = 0$

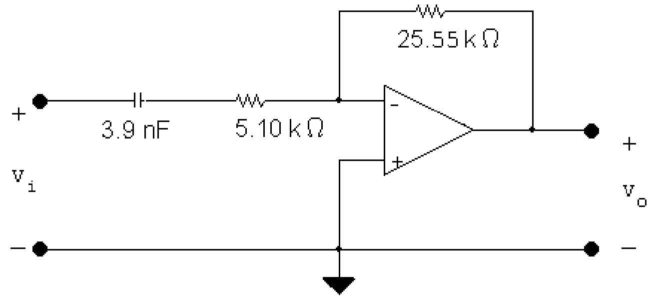
[d] As $\omega \to 0$ the capacitor C_1 disconnects v_i from the circuit. Therefore $v_o = v_n = 0$.
As $\omega \to \infty$ the capacitor short circuits the feedback network, thus $Z_F = 0$ and therefore $v_o = 0$.

P 15.8 **[a]** $$R_1 = \frac{1}{\omega_c C} = \frac{1}{(2\pi)(8 \times 10^3)(3.9 \times 10^{-9})} = 5.10\,\text{k}\Omega$$

$$K = 10^{(14/20)} = 5.01 = \frac{R_2}{R_1}$$

$$\therefore \quad R_2 = 5.01R_1 = 25.55\,\text{k}\Omega$$

[b]



P 15.9 **[a]** $\frac{1}{RC} = 2\pi(8000) \quad \text{so} \quad RC = 19.89 \times 10^{-6}$

There are several possible approaches. Here, choose $C = 0.047\,\mu$F. Then

$$R_i = \frac{19.89 \times 10^{-6}}{0.047 \times 10^{-6}} = 423$$

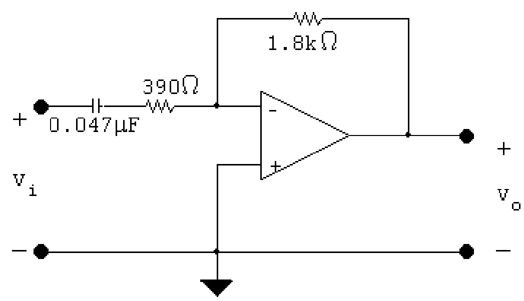
Choose $R_i = 390\,\Omega$. This gives

$$\omega_c = \frac{1}{(0.047 \times 10^{-6})(390)} = 54.56 \text{ krad/s} \quad \text{so} \quad f_c = 8.68\,\text{kHz}$$

To get a passband gain of 14 dB, choose

$$R_f = 5R_i = 5(390) = 1950\,\Omega$$

Choose $R_f = 1.8\,\text{k}\Omega$ to give a passband gain of $20\log_{10}(1800/390) = 13.3$ dB. The resulting circuit is

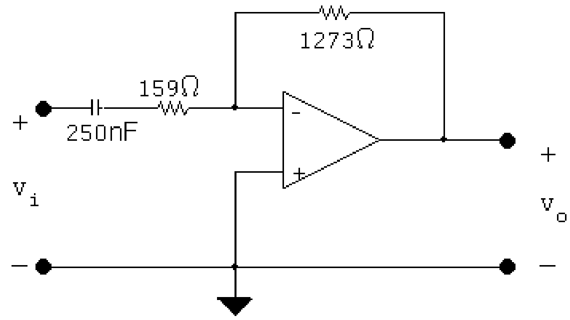


[b] $\%$ error in $f_c = \frac{8683.76 - 8000}{8000}(100) = 8.5\%$

$$\%\text{ error in passband gain} = \frac{13.3 - 14}{14}(100) = -5.1\%$$

P 15.10 **[a]** $\omega_c = \frac{1}{R_1 C} \quad \text{so} \quad R_1 = \frac{1}{\omega_c C} = \frac{1}{2\pi(4000)(250 \times 10^{-9})} = 159\,\Omega$

$$K = \frac{R_2}{R_1} \quad \text{so} \quad R_2 = KR_1 = (8)(159) = 1273\,\Omega$$



[**b**] The passband gain changes but the cutoff frequency is unchanged.

P 15.11 [**a**] $8(2.5) = 20\,\text{V} \quad \text{so} \quad V_{cc} \geq 20\,\text{V}$

[**b**] $H(j\omega) = \dfrac{-8j\omega}{j\omega + 8000\pi}$

$$H(j8000\pi) = \frac{-8(j8000\pi)}{8000\pi + j8000\pi} = \frac{8}{\sqrt{2}}\underline{/-135^\circ}$$

$$V_o = 14.14\underline{/-135^\circ}V_i \quad \text{so} \quad v_o(t) = 14.14\cos(8000\pi t - 135^\circ)\,\text{V}$$

[**c**] $H(j1000\pi) = \dfrac{-8(j1000\pi)}{8000\pi + j1000\pi} = 0.99\underline{/-97.1^\circ}$

$$V_o = 2.48\underline{/-97.1^\circ}V_i \quad \text{so} \quad v_o(t) = 2.48\cos(1000\pi t - 97.1^\circ)\,\text{mV}$$

[**d**] $H(j64{,}000\pi) = \dfrac{-8(j64{,}000\pi)}{8000\pi + j64{,}000\pi} = 7.94\underline{/-172.9^\circ}$

$$V_o = 19.85\underline{/-172.9^\circ}V_i \quad \text{so} \quad v_o(t) = 19.85\cos(64{,}000\pi t - 172.9^\circ)\,\text{V}$$

P 15.12 For the RC circuit

$$H(s) = \frac{V_o}{V_i} = \frac{s}{s + (1/RC)}$$

$$R' = k_m R; \qquad C' = \frac{C}{k_m k_f}$$

$$\therefore \quad R'C' = \frac{RC}{k_f} = \frac{1}{k_f}; \qquad \frac{1}{R'C'} = k_f$$

$$H'(s) = \frac{s}{s + (1/R'C')} = \frac{s}{s + k_f} = \frac{(s/k_f)}{(s/k_f) + 1}$$

For the RL circuit

$$H(s) = \frac{s}{s + (R/L)}$$

$$R' = k_m R; \qquad L' = \frac{k_m L}{k_f}$$

$$\frac{R'}{L'} = k_f \left(\frac{R}{L}\right) = k_f$$

$$H'(s) = \frac{s}{s + k_f} = \frac{(s/k_f)}{(s/k_f) + 1}$$

P 15.13 For the RC circuit

$$H(s) = \frac{V_o}{V_i} = \frac{(1/RC)}{s + (1/RC)}$$

$$R' = k_m R; \qquad C' = \frac{C}{k_m k_f}$$

$$\therefore \quad R'C' = k_m R \frac{C}{k_m k_f} = \frac{1}{k_f} RC = \frac{1}{k_f}$$

$$\frac{1}{R'C'} = k_f$$

$$H'(s) = \frac{(1/R'C')}{s + (1/R'C')} = \frac{k_f}{s + k_f}$$

$$H'(s) = \frac{1}{(s/k_f) + 1}$$

For the RL circuit $\qquad H(s) = \dfrac{R/L}{s + R/L} \qquad$ so

$$R' = k_m R; \qquad L' = \frac{k_m}{k_f} L$$

$$\frac{R'}{L'} = \frac{k_m R}{\frac{k_m}{k_f} L} = k_f \left(\frac{R}{L}\right) = k_f$$

$$H'(s) = \frac{(R'/L')}{s + (R'/L')} = \frac{k_f}{s + k_f}$$

$$H'(s) = \frac{1}{(s/k_f) + 1}$$

P 15.14 $H(s) = \dfrac{(R/L)s}{s^2 + (R/L)s + (1/LC)} = \dfrac{\beta s}{s^2 + \beta s + \omega_o^2}$

For the prototype circuit $\omega_o = 1$ and $\beta = \omega_o/Q = 1/Q$.
For the scaled circuit

$$H'(s) = \frac{(R'/L')s}{s^2 + (R'/L')s + (1/L'C')}$$

where $R' = k_m R$; $L' = \dfrac{k_m}{k_f}L$; and $C' = \dfrac{C}{k_f k_m}$

$$\therefore \quad \frac{R'}{L'} = \frac{k_m R}{\frac{k_m}{k_f}L} = k_f\left(\frac{R}{L}\right) = k_f\beta$$

$$\frac{1}{L'C'} = \frac{k_f k_m}{\frac{k_m}{k_f}LC} = \frac{k_f^2}{LC} = k_f^2$$

$$Q' = \frac{\omega_o'}{\beta'} = \frac{k_f\omega_o}{k_f\beta} = Q$$

therefore the Q of the scaled circuit is the same as the Q of the unscaled circuit. Also note $\beta' = k_f\beta$.

$$\therefore \quad H'(s) = \frac{\left(\frac{k_f}{Q}\right)s}{s^2 + \left(\frac{k_f}{Q}\right)s + k_f^2}$$

$$H'(s) = \frac{\left(\frac{1}{Q}\right)\left(\frac{s}{k_f}\right)}{\left[\left(\frac{s}{k_f}\right)^2 + \frac{1}{Q}\left(\frac{s}{k_f}\right) + 1\right]}$$

P 15.15 **[a]** $L = 1\,\text{H}; \qquad C = 1\,\text{F}$

$$R = \frac{1}{Q} = \frac{1}{20} = 0.05\,\Omega$$

[b] $k_f = \dfrac{\omega_o'}{\omega_o} = 40{,}000; \qquad k_m = \dfrac{R'}{R} = \dfrac{5000}{0.05} = 100{,}000$

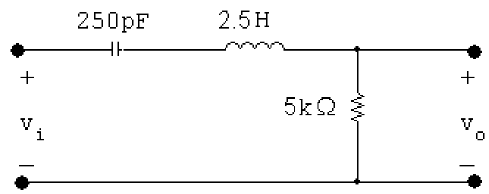
Thus,

$$R' = k_m R = (0.05)(100{,}000) = 5\,\text{k}\Omega$$

$$L' = \frac{k_m}{k_f}L = \frac{100{,}000}{40{,}000}(1) = 2.5\,\text{H}$$

$$C' = \frac{C}{k_m k_f} = \frac{1}{(40{,}000)(100{,}000)} = 250\,\text{pF}$$

[c]



P 15.16 [a] Since $\omega_o^2 = 1/LC$ and $\omega_o = 1$ rad/s,

$$C = \frac{1}{L} = \frac{1}{Q}\,\text{F}$$

[b] $H(s) = \dfrac{(R/L)s}{s^2 + (R/L)s + (1/LC)}$

$$H(s) = \frac{(1/Q)s}{s^2 + (1/Q)s + 1}$$

[c] In the prototype circuit

$$R = 1\,\Omega; \qquad L = 16\,\text{H}; \qquad C = \frac{1}{L} = 0.0625\,\text{F}$$

$$\therefore \quad k_m = \frac{R'}{R} = 10{,}000; \qquad k_f = \frac{\omega_o'}{\omega_o} = 25{,}000$$

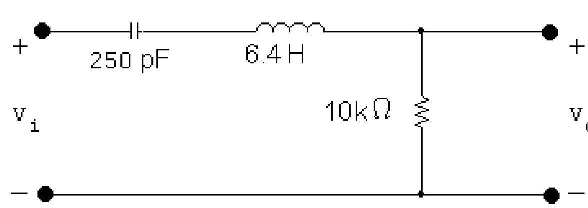
Thus

$$R' = k_m R = 10\,\text{k}\Omega$$

$$L' = \frac{k_m}{k_f}L = \frac{10{,}000}{25{,}000}(16) = 6.4\,\text{H}$$

$$C' = \frac{C}{k_m k_f} = \frac{0.0625}{(10{,}000)(25{,}000)} = 250\,\text{pF}$$

[d]



[e] $H'(s) = \dfrac{\frac{1}{16}\left(\frac{s}{25{,}000}\right)}{\left(\frac{s}{25{,}000}\right)^2 + \frac{1}{16}\left(\frac{s}{25{,}000}\right) + 1}$

$$H'(s) = \frac{1562.5s}{s^2 + 1562.5s + 625 \times 10^6}$$

P 15.17 **[a]** Using the first prototype

$$\omega_o = 1 \text{ rad/s}; \qquad C = 1\,\text{F}; \qquad L = 1\,\text{H}; \qquad R = 25\,\Omega$$

$$k_m = \frac{R'}{R} = \frac{40{,}000}{25} = 1600; \qquad k_f = \frac{\omega_o'}{\omega_o} = 50{,}000$$

Thus,

$$R' = k_m R = 40\,\text{k}\Omega; \qquad L' = \frac{k_m}{k_f}L = \frac{1600}{50{,}000}(1) = 32\,\text{mH};$$

$$C' = \frac{C}{k_m k_f} = \frac{1}{(1600)(50{,}000)} = 12.5\,\text{nF}$$

Using the second prototype

$$\omega_o = 1 \text{ rad/s}; \qquad C = 25\,\text{F}$$

$$L = \frac{1}{25} = 40\,\text{mH}; \qquad R = 1\,\Omega$$

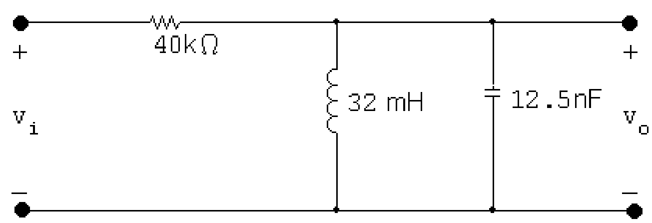
$$k_m = \frac{R'}{R} = 40{,}000; \qquad k_f = \frac{\omega_o'}{\omega_o} = 50{,}000$$

Thus,

$$R' = k_m R = 40\,\text{k}\Omega; \qquad L' = \frac{k_m}{k_f}L = \frac{40{,}000}{50{,}000}(0.04) = 32\,\text{mH};$$

$$C' = \frac{C}{k_m k_f} = \frac{25}{(40{,}000)(50{,}000)} = 12.5\,\text{nF}$$

[b]



P 15.18 **[a]** For the circuit in Fig. P15.18(a)

$$H(s) = \frac{V_o}{V_i} = \frac{s + \dfrac{1}{s}}{\dfrac{1}{Q} + s + \dfrac{1}{s}} = \frac{s^2 + 1}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

For the circuit in Fig. P15.18(b)

$$H(s) = \frac{V_o}{V_i} = \frac{Qs + \frac{Q}{s}}{1 + Qs + \frac{Q}{s}}$$

$$= \frac{Q(s^2+1)}{Qs^2 + s + Q}$$

$$H(s) = \frac{s^2+1}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

[b] $$H'(s) = \frac{\left(\frac{s}{8000}\right)^2 + 1}{\left(\frac{s}{8000}\right)^2 + \frac{1}{10}\left(\frac{s}{8000}\right) + 1}$$

$$= \frac{s^2 + 64\times 10^6}{s^2 + 800s + 64\times 10^6}$$

P 15.19 For the scaled circuit

$$H'(s) = \frac{s^2 + \left(\frac{1}{L'C'}\right)}{s^2 + \left(\frac{R'}{L'}\right)s + \left(\frac{1}{L'C'}\right)}$$

$$L' = \frac{k_m}{k_f}L; \qquad C' = \frac{C}{k_m k_f}$$

$$\therefore \quad \frac{1}{L'C'} = \frac{k_f^2}{LC}; \qquad R' = k_m R$$

$$\therefore \quad \frac{R'}{L'} = k_f\left(\frac{R}{L}\right)$$

It follows then that

$$H'(s) = \frac{s^2 + \left(\frac{k_f^2}{LC}\right)}{s^2 + \left(\frac{R}{L}\right)k_f s + \frac{k_f^2}{LC}}$$

$$= \frac{\left(\frac{s}{k_f}\right)^2 + \left(\frac{1}{LC}\right)}{\left[\left(\frac{s}{k_f}\right)^2 + \left(\frac{R}{L}\right)\left(\frac{s}{k_f}\right) + \left(\frac{1}{LC}\right)\right]}$$

$$= H(s)|_{s=s/k_f}$$

P 15.20 For the circuit in Fig. 15.31

$$H(s) = \frac{s^2 + \left(\frac{1}{LC}\right)}{s^2 + \frac{s}{RC} + \left(\frac{1}{LC}\right)}$$

It follows that

$$H'(s) = \frac{s^2 + \frac{1}{L'C'}}{s^2 + \frac{s}{R'C'} + \frac{1}{L'C'}}$$

where $R' = k_m R;\qquad L' = \frac{k_m}{k_f} L;$

$$C' = \frac{C}{k_m k_f}$$

$$\therefore \quad \frac{1}{L'C'} = \frac{k_f^2}{LC}$$

$$\frac{1}{R'C'} = \frac{k_f}{RC}$$

$$\begin{aligned} H'(s) &= \frac{s^2 + \left(\frac{k_f^2}{LC}\right)}{s^2 + \left(\frac{k_f}{RC}\right) s + \frac{k_f^2}{LC}} \\ &= \frac{\left(\frac{s}{k_f}\right)^2 + \frac{1}{LC}}{\left(\frac{s}{k_f}\right)^2 + \left(\frac{1}{RC}\right)\left(\frac{s}{k_f}\right) + \frac{1}{LC}} \\ &= H(s)\big|_{s=s/k_f} \end{aligned}$$

P 15.21 For prototype circuit (a):

$$H(s) = \frac{V_o}{V_i} = \frac{Q}{Q + \frac{1}{s+\frac{1}{s}}} = \frac{Q}{Q + \frac{s}{s^2+1}}$$

$$H(s) = \frac{Q(s^2+1)}{Q(s^2+1) + s} = \frac{s^2+1}{s^2 + \left(\frac{1}{Q}\right) s + 1}$$

For prototype circuit (b):

$$\begin{aligned} H(s) &= \frac{V_o}{V_i} = \frac{1}{1 + \frac{(s/Q)}{(s^2+1)}} \\ &= \frac{s^2+1}{s^2 + \left(\frac{1}{Q}\right) s + 1} \end{aligned}$$

P 15.22 **[a]** $k_m = \dfrac{80}{200} = 0.4$

$$\frac{k_m}{k_f}(0.4) = 0.02 \qquad \text{so} \qquad k_f = \frac{(0.4)(0.4)}{0.02} = 8$$

$$C' = \frac{20 \times 10^{-6}}{(0.4)(8)} = 6.25\,\mu\text{F}$$

[b] $Z_{\text{ab}} = \dfrac{1}{j\omega C} + R \| j\omega L = \dfrac{1}{j\omega C} + \dfrac{j\omega RL}{R + j\omega L}$

$$= \frac{(R + j\omega L - \omega^2 RLC)(-\omega^2 LC - j\omega RC)}{(-\omega^2 LC + j\omega RC)(-\omega^2 LC - j\omega RC)}$$

The denominator is purely real, so set the imaginary part of the numerator equal to 0 and solve for ω:

$$-\omega R^2 C - \omega^3 L^2 C^3_\omega R^2 LC^2 = 0$$

$$\therefore \qquad \omega^2 = \frac{R^2}{R^2 LC - L^2} = \frac{(80)^2}{(80)^2(0.02(6.25 \times 10^{-6}) - (0.02)^2} = 16 \times 10^6$$

Thus,

$\omega = 4000$ rad/s

[c] In the original, unscaled circuit, the frequency at which the impedance Z_{ab} is purely real is

$$\omega^2_{\text{us}} = \frac{(200)^2}{(200)^2(0.4)(20 \times 10^{-6}) - (0.4)^2} = 250{,}000$$

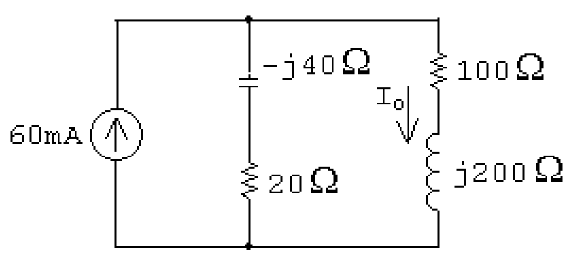
Thus, $\qquad \omega_{\text{us}} = 500$ rad/s

$$\frac{\omega_{\text{scaled}}}{\omega_{\text{us}}} = \frac{4000}{500} = 8 = k_f$$

P 15.23 **[a]** $k_f = \dfrac{10{,}000}{250} = 40$

$$C' = \frac{100 \times 10^{-6}}{40} = 2.5\,\mu\text{F}$$

$$L' = \frac{0.8}{40} = 20\,\text{mH}$$



[b] $I_o = \dfrac{20 - j40}{120 + j160}(0.06) = 13.42\underline{/-116.57^\circ}\,\text{mA}$

$i_o = 13.42\cos(10{,}000t - 166.57^\circ)\,\text{mA}$

The magnitude and phase angle of the output current are the same as in the unscaled circuit.

P 15.24 From the solution to Problem 14.18, $\omega_o = 100$ krad/s and $\beta = 12.5$ krad/s. Compute the two scale factors:

$$k_f = \frac{\omega_o'}{\omega_o} = \frac{2\pi(25 \times 10^3)}{100 \times 10^3} = \frac{\pi}{2}$$

$$L' = \frac{k_m L}{k_f} \qquad \text{so} \qquad k_m = \frac{L'}{L}k_f = \frac{25 \times 10^{-6}}{10 \times 10^{-3}}\left(\frac{\pi}{2}\right) = \frac{\pi}{800}$$

Thus,

$$R' = k_m R = \frac{8000\pi}{800} = 31.4\,\Omega \qquad\qquad C' = \frac{C}{k_m k_f} = \frac{10 \times 10^{-9}}{(\pi/800)(\pi/2)} = 1.62\,\mu\text{H}$$

Calculate the cutoff frequencies:

$$\omega_{c1}' = k_f\omega_{c1} = \frac{\pi}{2}(93.95 \times 10^3) = 147.576 \text{ krad/s}$$

$$\omega_{c2}' = k_f\omega_{c2} = \frac{\pi}{2}(106.45 \times 10^3) = 167.211 \text{ krad/s}$$

To check, calculate the bandwidth:

$$\beta' = \omega_{c2}' - \omega_{c1}' = 19.6 \text{ krad/s} = (\pi/2)\beta \text{ (checks!)}$$

P 15.25 From the solution to Problem 14.38, $\omega_o = 80$ krad/s and $\beta = 5.33$ krad/s. Calculate the scale factors:

$$k_f = \frac{\omega_o'}{\omega_o} = \frac{16 \times 10^3}{80 \times 10^3} = 0.2$$

$$k_m = \frac{C}{C'k_f} = \frac{62.5 \times 10^{-9}}{(50 \times 10^{-9})(0.2)} = 6.25$$

Thus,

$$R' = k_m R = (6.25)(3000) = 18.75\,\text{k}\Omega$$

$$L' = \frac{k_m L}{k_f} = \frac{(6.25)(2.5 \times 10^{-3})}{0.2} = 78.125\,\text{mH}$$

Calculate the bandwidth:

$$\beta' = k_f\beta = (0.2)[5333.33] = 1066.67 \text{ rad/s}$$

To check, calculate the quality factor:

$$Q = \frac{\omega_o}{\beta} = \frac{80{,}000}{5333.33} = 15$$

$$Q' = \frac{\omega_o'}{\beta'} = \frac{16{,}000}{1066.67} = 15 \text{ (checks)}$$

P 15.26 **[a]** From Eq 15.1 we have

$$H(s) = \frac{-K\omega_c}{s + \omega_c}$$

where $K = \dfrac{R_2}{R_1}, \quad \omega_c = \dfrac{1}{R_2 C}$

$$\therefore \quad H'(s) = \frac{-K'\omega_c'}{s + \omega_c'}$$

where $K' = \dfrac{R_2'}{R_1'} \qquad \omega_c' = \dfrac{1}{R_2' C'}$

By hypothesis $R_1' = k_m R_1; \qquad R_2' = k_m R_2,$

and $C' = \dfrac{C}{k_f k_m}$. It follows that

$K' = K$ and $\omega_c' = k_f\omega_c$, therefore

$$H'(s) = \frac{-Kk_f\omega_c}{s + k_f\omega_c} = \frac{-K\omega_c}{\left(\frac{s}{k_f}\right) + \omega_c}$$

[b] $H(s) = \dfrac{-K}{s+1}$

[c] $H'(s) = \dfrac{-K}{\left(\frac{s}{k_f}\right) + 1} = \dfrac{-Kk_f}{s + k_f}$

P 15.27 **[a]** From Eq. 15.4

$$H(s) = \frac{-Ks}{s+\omega_c} \text{ where } K = \frac{R_2}{R_1} \text{ and}$$

$$\omega_c = \frac{1}{R_1C}$$

$$\therefore \quad H'(s) = \frac{-K's}{s+\omega_c'} \text{ where } K' = \frac{R_2'}{R_1'}$$

$$\text{and } \omega_c' = \frac{1}{R_1'C'}$$

By hypothesis

$$R_1' = k_mR_1; \qquad R_2' = k_mR_2; \qquad C' = \frac{C}{k_mk_f}$$

It follows that

$$K' = K \text{ and } \omega_c' = k_f\omega_c$$

$$\therefore \quad H'(s) = \frac{-Ks}{s+k_f\omega_c} = \frac{-K(s/k_f)}{\left(\frac{s}{k_f}\right)+\omega_c}$$

[b] $H(s) = \dfrac{-Ks}{s+1}$

[c] $H'(s) = \dfrac{-K(s/k_f)}{\left(\frac{s}{k_f}+1\right)} = \dfrac{-Ks}{s+k_f}$

P 15.28 **[a]** $H_{\rm hp} = \dfrac{s}{s+1}; \qquad k_f = \dfrac{\omega_o'}{\omega} = \dfrac{200(2\pi)}{1} = 400\pi$

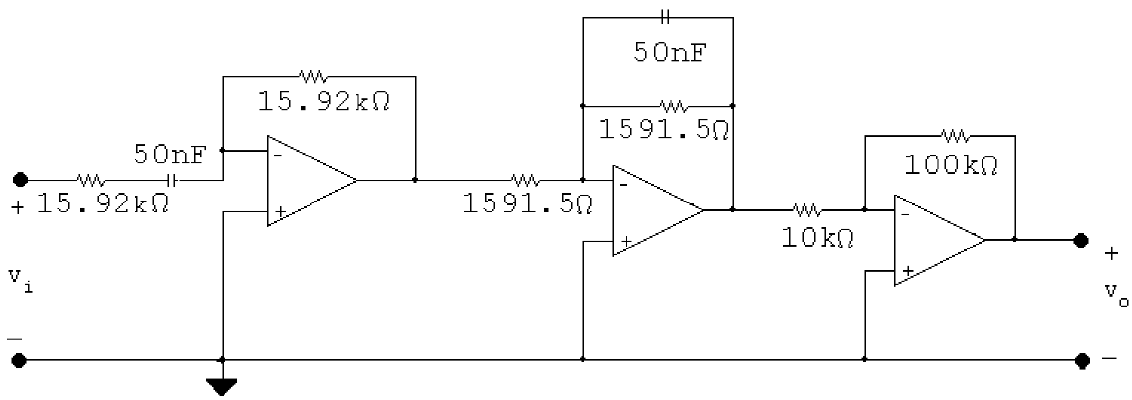
$$\therefore \quad H_{\rm hp}' = \frac{s}{s+400\pi}$$

$$\frac{1}{R_HC_H} = 400\pi; \qquad \therefore \quad R_H = \frac{1}{(400\pi)(50\times10^{-9})} = 15.915\,\text{k}\Omega$$

$$H_{\rm lp} = \frac{1}{s+1}; \qquad k_f = \frac{\omega_o'}{\omega} = \frac{2000(2\pi)}{1} = 4000\pi$$

$$\therefore \quad H_{\rm lp}' = \frac{4000\pi}{s+4000\pi}$$

$$\frac{1}{R_LC_L} = 4000\pi; \qquad \therefore \quad R_L = \frac{1}{(4000\pi)(50\times10^{-9})} = 1591.5\,\Omega$$



[b] $$H'(s) = \frac{s}{s+400\pi}\cdot\frac{4000\pi}{s+4000\pi}\cdot 10$$

$$= \frac{40{,}000\pi s}{(s+400\pi)(s+4000\pi)}$$

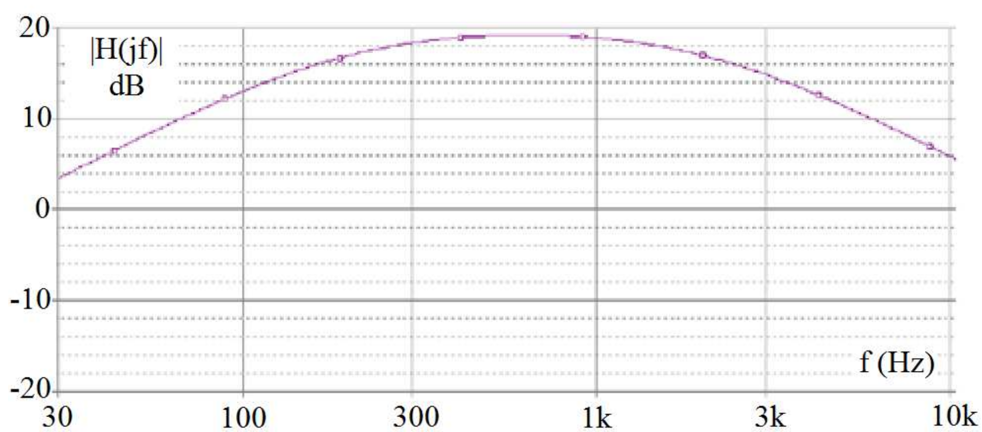
[c] $$\omega_o = \sqrt{\omega_{c1}\omega_{c1}} = \sqrt{(400\pi)(4000\pi)} = 3973.8 \text{ rad/s}$$

$$H'(j\omega_o) = \frac{(40{,}000\pi)(j3973.8)}{(400\pi + j3973.8)(4000\pi + j3973.8)}$$

$$= 9.1\underline{/0^\circ}$$

[d] $G = 20\log_{10}(9.1) = 19.2$ dB

[e]



P 15.29 **[a]** For the high-pass section:

$$k_f = \frac{\omega_o'}{\omega} = \frac{800(2\pi)}{1} = 1600\pi$$

$$H'(s) = \frac{s}{s+1600\pi}$$

$$\therefore \quad \frac{1}{R_1(2\times10^{-6})} = 1600\pi; \qquad R_1 = 99.5\,\Omega \quad \therefore \quad R_2 = 99.5\,\Omega$$

For the low-pass section:

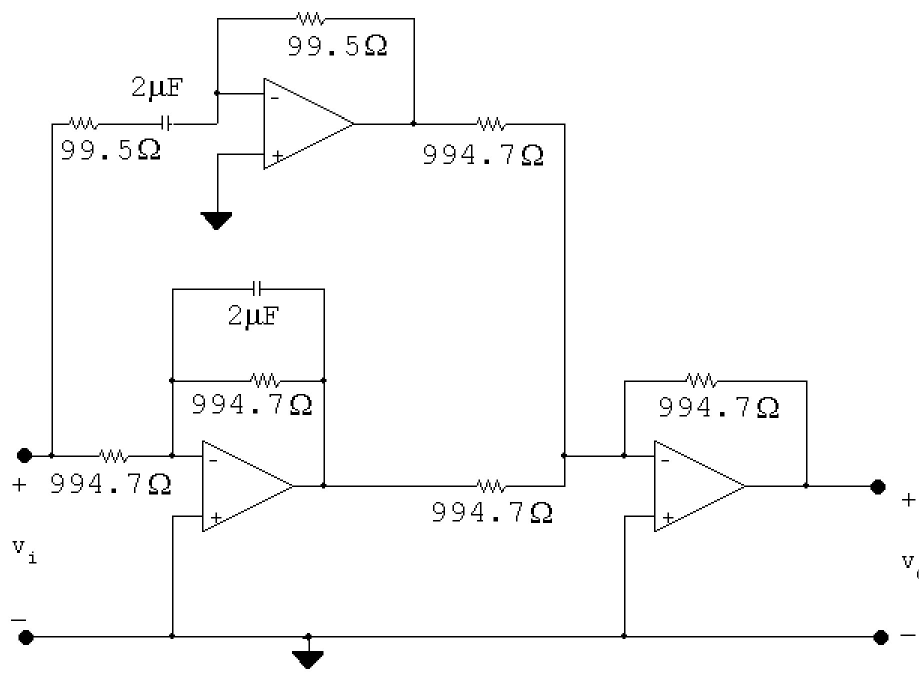
$$k_f = \frac{\omega_o'}{\omega} = \frac{80(2\pi)}{1} = 160\pi$$

$$H'(s) = \frac{160\pi}{s + 160\pi}$$

$$\therefore \qquad \frac{1}{R_2(2\times10^{-6})} = 160\pi; \qquad R_2 = 994.7\,\Omega \quad \therefore \qquad R_1 = 994.7\,\Omega$$

0 dB gain corresponds to $K = 1$. In the summing amplifier we are free to choose R_f and R_i so long as $R_f/R_i = 1$. To keep from having many different resistance values in the circuit we opt for $R_f = R_i = 994.7\,\Omega$.

[b]



[c] $H'(s) = \dfrac{s}{s + 1600\pi} + \dfrac{160\pi}{s + 160\pi}$

$$= \frac{s^2 + 320\pi s + 256\times10^3\pi^2}{(s + 160\pi)(s + 1600\pi)}$$

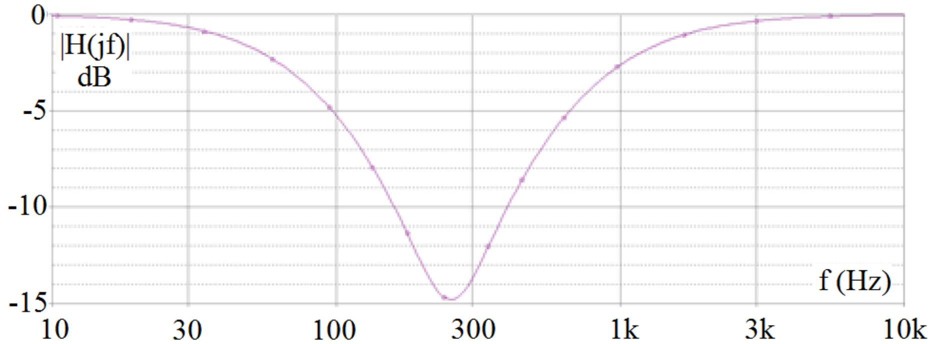
[d] $\omega_o = \sqrt{(1600\pi)(160\pi)} = 1589.5$

$$H'(j1589.5) = \frac{-(1589.5)^2 + 320\pi(j1589.5) + 256\times10^3\pi^2}{(160\pi + j1589.5)(1600\pi + j1589.5)}$$

$$= 0.1818\underline{/0^\circ}$$

[e] $G = 20\log_{10} 0.1818 = -14.81$ dB

[f]



P 15.30 $\omega_o = 2\pi f_o = 400\pi\,\text{rad/s}$

$\beta = 2\pi(1000) = 2000\pi\,\text{rad/s}$

$\therefore\quad \omega_{c_2} - \omega_{c_1} = 2000\pi$

$\sqrt{\omega_{c_1}\omega_{c_2}} = \omega_o = 400\pi$

Solve for the cutoff frequencies:

$\omega_{c_1}\omega_{c_2} = 16 \times 10^4\pi^2$

$$\omega_{c_2} = \frac{16 \times 10^4\pi^2}{\omega_{c_1}}$$

$$\therefore\quad \frac{16 \times 10^4\pi^2}{\omega_{c_1}} - \omega_{c_1} = 2000\pi$$

or $\omega_{c_1}^2 + 2000\pi\omega_{c_1} - 16 \times 10^4\pi^2 = 0$

$\omega_{c_1} = -1000\pi \pm \sqrt{10^6\pi^2 + 0.16 \times 10^6\pi^2}$

$\omega_{c_1} = 1000\pi(-1 \pm \sqrt{1.16}) = 242.01\,\text{rad/s}$

$\therefore\quad \omega_{c_2} = 2000\pi + 242.01 = 6525.19\,\text{rad/s}$

Thus, $f_{c1} = 38.52$ Hz and $f_{c2} = 1038.52$ Hz

Check: $\beta = f_{c2} - f_{c1} = 1000\text{Hz}$

$$\omega_{c2} = \frac{1}{R_L C_L} = 6525.19$$

$$R_L = \frac{1}{(6525.19)(5 \times 10^{-6})} = 30.65\,\Omega$$

$$\omega_{c1} = \frac{1}{R_H C_H} = 242.01$$

$$R_H = \frac{1}{(242.01)(5 \times 10^{-6})} = 826.43\,\Omega$$

P 15.31 $\omega_o = 1000\,\text{rad/s}; \qquad \text{GAIN} = 6$

$$\beta = 4000\ \text{rad/s}; \qquad C = 0.2\,\mu\text{F}$$

$$\beta = \omega_{c_2} - \omega_{c_1} = 4000$$

$$\omega_o = \sqrt{\omega_{c_1}\omega_{c_2}} = 1000$$

Solve for the cutoff frequencies:

$$\therefore \quad \omega_{c_1}^2 + 4000\omega_{c_1} - 10^6 = 0$$

$$\omega_{c_1} = -2000 \pm 1000\sqrt{5} = 236.07\,\text{rad/s}$$

$$\omega_{c_2} = 4000 + \omega_{c_1} = 4236.07\,\text{rad/s}$$

Check: $\quad \beta = \omega_{c2} - \omega_{c1} = 4000\ \text{rad/s}$

$$\omega_{c_1} = \frac{1}{R_L C_L}$$

$$\therefore \quad R_L = \frac{1}{(0.2 \times 10^{-6})(236.07)} = 21.18\,\text{k}\Omega$$

$$\frac{1}{R_H C_H} = 4236.07$$

$$R_H = \frac{1}{(0.2 \times 10^{-6})(4236.07)} = 1.18\,\text{k}\Omega$$

$$\frac{R_f}{R_i} = 6$$

If $R_i = 1\,\text{k}\Omega \qquad R_f = 6R_i = 6\,\text{k}\Omega$

P 15.32 $H(s) = \dfrac{V_o}{V_i} = \dfrac{-Z_f}{Z_i}$

$$Z_f = \frac{1}{sC_2}\|R_2 = \frac{(1/C_2)}{s + (1/R_2C_2)}; \qquad Z_i = R_1 + \frac{1}{sC_1} = \frac{sR_1C_1 + 1}{sC_1}$$

$$\therefore \quad H(s) = \frac{\dfrac{-1/C_2}{s + (1/R_2C_2)}}{\dfrac{s + (1/R_1C_1)}{s/R_1}} = \frac{-(1/R_1C_2)s}{[s + (1/R_1C_1)][s + (1/R_2C_2)]}$$

$$= \frac{-K\beta s}{s^2 + \beta s + \omega_o^2}$$

[a] $H(s) = \dfrac{-250s}{(s+50)(s+20)} = \dfrac{-250s}{s^2 + 70s + 1000} = \dfrac{-3.57(70s)}{s^2 + 70s + (\sqrt{1000})^2}$

$\omega_o = \sqrt{1000} = 31.6$ rad/s

$\beta = 70$ rad/s

$K = -3.57$

[b] $Q = \dfrac{\omega_o}{\beta} = 0.45$

$$\omega_{c1,2} = \pm\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2} = \pm 35 + \sqrt{35^2 + 1000} = \pm 35 + 47.17$$

$\omega_{c1} = 12.17$ rad/s $\qquad \omega_{c2} = 82.17$ rad/s

P 15.33 **[a]** $H(s) = \dfrac{(1/sC)}{R + (1/sC)} = \dfrac{(1/RC)}{s + (1/RC)}$

$$H(j\omega) = \frac{(1/RC)}{j\omega + (1/RC)}$$

$$|H(j\omega)| = \frac{(1/RC)}{\sqrt{\omega^2 + (1/RC)^2}}$$

$$|H(j\omega)|^2 = \frac{(1/RC)^2}{\omega^2 + (1/RC)^2}$$

[b] Let V_a be the voltage across the capacitor, positive at the upper terminal. Then

$$\frac{V_a - V_{in}}{R_1} + sCV_a + \frac{V_a}{R_2 + sL} = 0$$

Solving for V_a yields

$$V_a = \frac{(R_2 + sL)V_{in}}{R_1LCs^2 + (R_1R_2C + L)s + (R_1 + R_2)}$$

But

$$v_o = \frac{sLV_a}{R_2 + sL}$$

Therefore

$$V_o = \frac{sLV_{in}}{R_1LCs^2 + (L + R_1R_2C)s + (R_1 + R_2)}$$

$$H(s) = \frac{sL}{R_1LCs^2 + (L + R_1R_2C)s + (R_1 + R_2)}$$

$$H(j\omega) = \frac{j\omega L}{[(R_1 + R_2) - R_1LC\omega^2] + j\omega(L + R_1R_2C)}$$

$$|H(j\omega)| = \frac{\omega L}{\sqrt{[R_1 + R_2 - R_1LC\omega^2]^2 + \omega^2(L + R_1R_2C)^2}}$$

$$|H(j\omega)|^2 = \frac{\omega^2L^2}{(R_1 + R_2 - R_1LC\omega^2)^2 + \omega^2(L + R_1R_2C)^2}$$

$$= \frac{\omega^2L^2}{R_1^2L^2C^2\omega^4 + (L^2 + R_1^2R_2^2C^2 - 2R_1^2LC)\omega^2 + (R_1 + R_2)^2}$$

[c] Let V_a be the voltage across R_2 positive at the upper terminal. Then

$$\frac{V_a - V_{in}}{R_1} + \frac{V_a}{R_2} + V_asC + V_asC = 0$$

$$(0 - V_a)sC + (0 - V_a)sC + \frac{0 - V_o}{R_3} = 0$$

$$\therefore \quad V_a = \frac{R_2V_{in}}{2R_1R_2Cs + R_1 + R_2}$$

and $V_a = -\dfrac{V_o}{2R_3Cs}$

It follows directly that

$$H(s) = \frac{V_o}{V_{in}} = \frac{-2R_2R_3Cs}{2R_1R_2Cs + (R_1 + R_2)}$$

$$H(j\omega) = \frac{-2R_2R_3C(j\omega)}{(R_1+R_2)+j\omega(2R_1R_2C)}$$

$$|H(j\omega)| = \frac{2R_2R_3C\omega}{\sqrt{(R_1+R_2)^2+\omega^2 4R_1^2R_2^2C^2}}$$

$$|H(j\omega)|^2 = \frac{4R_2^2R_3^2C^2\omega^2}{(R_1+R_2)^2+4R_1^2R_2^2C^2\omega^2}$$

P 15.34 **[a]** $n \cong \dfrac{(-0.05)(-30)}{\log_{10}(7000/2000)} \cong 2.76$

$$\therefore \quad n = 3$$

[b] Gain $= 20\log_{10}\dfrac{1}{\sqrt{1+(7000/2000)^6}} = -32.65$ dB

P 15.35 For the scaled circuit

$$H'(s) = \frac{1/(R')^2C_1'C_2'}{s^2+\frac{2}{R'C_1'}s+\frac{1}{(R')^2C_1'C_2'}}$$

where

$$R' = k_mR; \qquad C_1' = C_1/k_fk_m; \qquad C_2' = C_2/k_fk_m$$

It follows that

$$\frac{1}{(R')^2C_1'C_2'} = \frac{k_f^2}{R^2C_1C_2}$$

$$\frac{2}{R'C_1'} = \frac{2k_f}{RC_1}$$

$$\therefore \quad H'(s) = \frac{k_f^2/RC_1C_2}{s^2+\frac{2k_f}{RC_1}s+\frac{k_f^2}{R^2C_1C_2}}$$

$$= \frac{1/RC_1C_2}{\left(\frac{s}{k_f}\right)^2+\frac{2}{RC_1}\left(\frac{s}{k_f}\right)+\frac{1}{R^2C_1C_2}}$$

P 15.36 **[a]** $H(s) = \dfrac{1}{(s+1)(s^2+s+1)}$

[b] $f_c = 2000\,\text{Hz};\qquad \omega_c = 4000\pi\ \text{rad/s};\qquad k_f = 4000\pi$

$$H'(s) = \frac{1}{(\frac{s}{k_f}+1)[(\frac{s}{k_f})^2 + \frac{s}{k_f} + 1]}$$

$$= \frac{k_f^3}{(s+k_f)(s^2 + k_f s + k_f^2)}$$

$$= \frac{(4000\pi)^3}{(s+4000\pi)[s^2 + 4000\pi s + (4000\pi)^2]}$$

[c] $$H'(j14{,}000\pi) = \frac{64}{(4+j14)(-180+j52)}$$

$$= 0.02332\underline{/-236.77^\circ}$$

$$\text{Gain } = 20\log_{10}(0.02332) = -32.65\text{ dB}$$

P 15.37 [a] In the first-order circuit $R = 1\,\Omega$ and $C = 1$ F.

$$k_m = \frac{R'}{R} = \frac{1000}{1} = 1000;\qquad k_f = \frac{\omega_o'}{\omega_o} = \frac{2\pi(2000)}{1} = 4000\pi$$

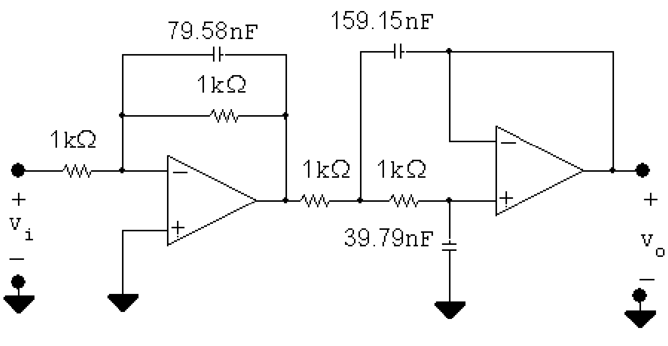
$$R' = k_m R = 1000\,\Omega;\qquad C' = \frac{C}{k_m k_f} = \frac{1}{(1000)(4000\pi)} = 79.58\,\text{nF}$$

In the second-order circuit $R = 1\,\Omega$, $2/C_1 = 1$ so $C_1 = 2$ F, and $C_2 = 1/C_1 = 0.5$ F. Therefore in the scaled second-order circuit

$$R' = k_m R = 1000\,\Omega;\qquad C_1' = \frac{C_1}{k_m k_f} = \frac{2}{(1000)(4000\pi)} = 159.15\,\text{nF}$$

$$C_2' = \frac{C_2}{k_m k_f} = \frac{0.5}{(1000)(4000\pi)} = 39.79\,\text{nF}$$

[b]



P 15.38 [a] $$y = 20\log_{10}\frac{1}{\sqrt{1+\omega^{2n}}} = -10\log_{10}(1+\omega^{2n})$$

From the laws of logarithms we have

$$y = \left(\frac{-10}{\ln 10}\right)\ln(1+\omega^{2n})$$

Thus

$$\frac{dy}{d\omega} = \left(\frac{-10}{\ln 10}\right)\frac{2n\omega^{2n-1}}{(1+\omega^{2n})}$$

$$x = \log_{10}\omega = \frac{\ln\omega}{\ln 10}$$

$$\therefore \quad \ln\omega = x\ln 10$$

$$\frac{1}{\omega}\frac{d\omega}{dx} = \ln 10, \qquad \frac{d\omega}{dx} = \omega\ln 10$$

$$\frac{dy}{dx} = \left(\frac{dy}{d\omega}\right)\left(\frac{d\omega}{dx}\right) = \frac{-20n\omega^{2n}}{1+\omega^{2n}}\,\text{dB/decade}$$

at $\omega = \omega_c = 1\,\text{rad/s}$

$$\frac{dy}{dx} = -10n\,\text{dB/decade.}$$

[b] $y = 20\log_{10}\dfrac{1}{[\sqrt{1+\omega^2}]^n} = -10n\log_{10}(1+\omega^2)$

$$= \frac{-10n}{\ln 10}\ln(1+\omega^2)$$

$$\frac{dy}{d\omega} = \frac{-10n}{\ln 10}\left(\frac{1}{1+\omega^2}\right)2\omega = \frac{-20n\omega}{(\ln 10)(1+\omega^2)}$$

As before

$$\frac{d\omega}{dx} = \omega(\ln 10); \qquad \therefore \quad \frac{dy}{dx} = \frac{-20n\omega^2}{(1+\omega^2)}$$

At the corner $\omega_c = \sqrt{2^{1/n}-1} \qquad \therefore \quad \omega_c^2 = 2^{1/n}-1$

$$\frac{dy}{dx} = \frac{-20n[2^{1/n}-1]}{2^{1/n}}\,\text{dB/decade.}$$

[c] For the Butterworth Filter

n	dy/dx (dB/decade)
1	-10
2	-20
3	-30
4	-40
∞	$-\infty$

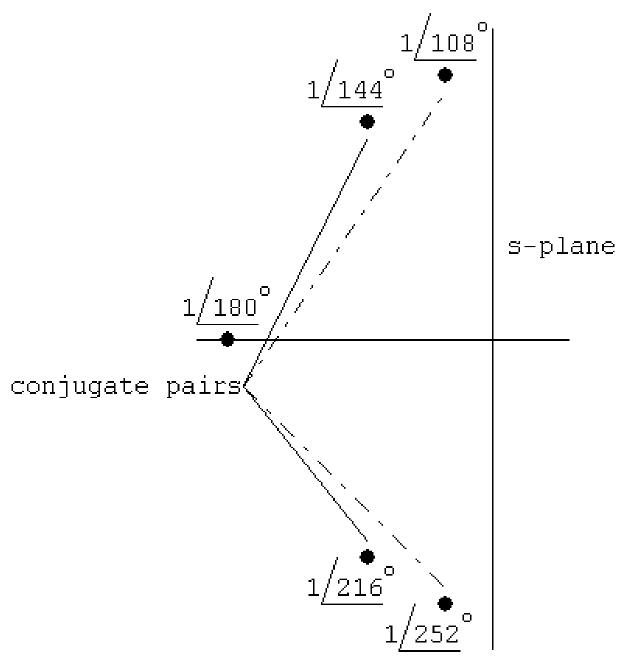
For the cascade of identical sections

n	dy/dx (dB/decade)
1	-10
2	-11.72
3	-12.38
4	-12.73
∞	-13.86

[d] It is apparent from the calculations in part (c) that as n increases the amplitude characteristic at the cutoff frequency decreases at a much faster rate for the Butterworth filter.
Hence the transition region of the Butterworth filter will be much narrower than that of the cascaded sections.

P 15.39 $n = 5$: $1 + (-1)^5 s^{10} = 0; \qquad s^{10} = 1$

$$s^{10} = 1\underline{/(0 + 360k)^\circ} \qquad \text{so} \qquad s = 1\underline{/36k^\circ}$$



k	s_{k+1}	k	s_{k+1}
0	$1\underline{/0^\circ}$	5	$1\underline{/180^\circ}$
1	$1\underline{/36^\circ}$	6	$1\underline{/216^\circ}$
2	$1\underline{/72^\circ}$	7	$1\underline{/252^\circ}$
3	$1\underline{/108^\circ}$	8	$1\underline{/288^\circ}$
4	$1\underline{/144^\circ}$	9	$1\underline{/324^\circ}$

Group by conjugate pairs to form denominator polynomial.

$$(s + 1)[s - (\cos 108^\circ + j \sin 108^\circ)][(s - (\cos 252^\circ + j \sin 252^\circ)]$$

$$\cdot\, [(s - (\cos 144^\circ + j \sin 144^\circ)][(s - (\cos 216^\circ + j \sin 216^\circ)]$$

$$= (s + 1)(s + 0.309 - j0.951)(s + 0.309 + j0.951)\cdot$$

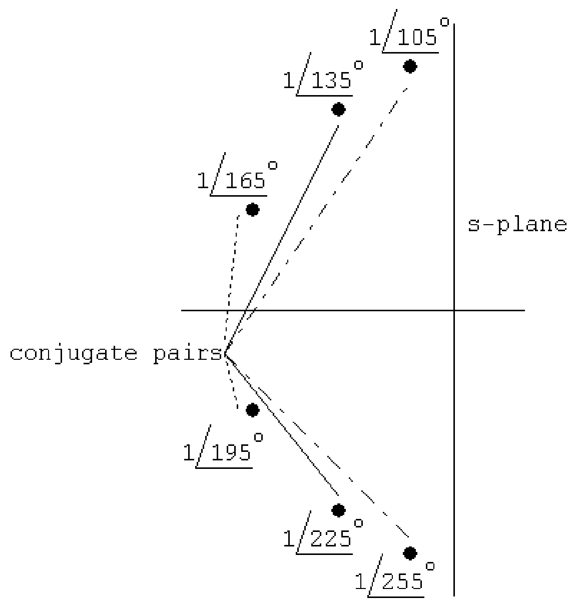
$$(s + 0.809 - j0.588)(s + 0.809 + j0.588)$$

which reduces to

$$(s + 1)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)$$

$n = 6$: $1 + (-1)^6 s^{12} = 0 \qquad s^{12} = -1$

$s^{12} = 1\angle 180^\circ + 360k$



k	s_{k+1}	k	s_{k+1}
0	$1\angle 15^\circ$	6	$1\angle 195^\circ$
1	$1\angle 45^\circ$	7	$1\angle 225^\circ$
2	$1\angle 75^\circ$	8	$1\angle 255^\circ$
3	$1\angle 105^\circ$	9	$1\angle 285^\circ$
4	$1\angle 135^\circ$	10	$1\angle 315^\circ$
5	$1\angle 165^\circ$	11	$1\angle 345^\circ$

Grouping by conjugate pairs yields

$$(s + 0.2588 - j0.9659)(s + 0.2588 + j0.9659)\times$$

$$(s + 0.7071 - j0.7071)(s + 0.7071 + j0.7071)\times$$

$$(s + 0.9659 - j0.2588)(s + 0.9659 + j0.2588)$$

or $(s^2 + 0.5176s + 1)(s^2 + 1.4142s + 1)(s^2 + 1.9318s + 1)$

P 15.40 $H'(s) = \dfrac{s^2}{s^2 + \dfrac{2}{k_m R_2 (C/k_m k_f)} s + \dfrac{1}{k_m R_1 k_m R_2 (C^2/k_m^2 k_f^2)}}$

$$H'(s) = \frac{s^2}{s^2 + \dfrac{2k_f}{R_2 C} s + \dfrac{k_f^2}{R_1 R_2 C^2}}$$

$$= \frac{(s/k_f)^2}{(s/k_f)^2 + \dfrac{2}{R_2 C}\left(\dfrac{s}{k_f}\right) + \dfrac{1}{R_1 R_2 C^2}}$$

P 15.41 **[a]** $n = \dfrac{(-0.05)(-55)}{\log_{10}(2500/500)} = 3.93 \quad \therefore \qquad n = 4$

From Table 15.1 the transfer function is

$$H(s) = \frac{s^2}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

The capacitor values for the first stage prototype circuit are

$$\frac{2}{C_1} = 0.765 \quad \therefore \qquad C_1 = 2.61\,\text{F}$$

$$C_2 = \frac{1}{C_1} = 0.38\,\text{F}$$

The values for the second stage prototype circuit are

$$\frac{2}{C_1} = 1.848 \quad \therefore \qquad C_1 = 1.08\,\text{F}$$

$$C_2 = \frac{1}{C_1} = 0.92\,\text{F}$$

The scaling factors are

$$k_m = \frac{R'}{R} = 8000; \qquad k_f = \frac{\omega_o'}{\omega_o} = 5000\pi$$

Therefore the scaled values for the components in the first stage are

$$R_1 = R_2 = R = 8000\,\Omega$$

$$C_1 = \frac{2.61}{(5000\pi)(8000)} = 20.8\,\text{nF}$$

$$C_2 = \frac{0.38}{(5000\pi)(8000)} = 3.04\,\text{nF}$$

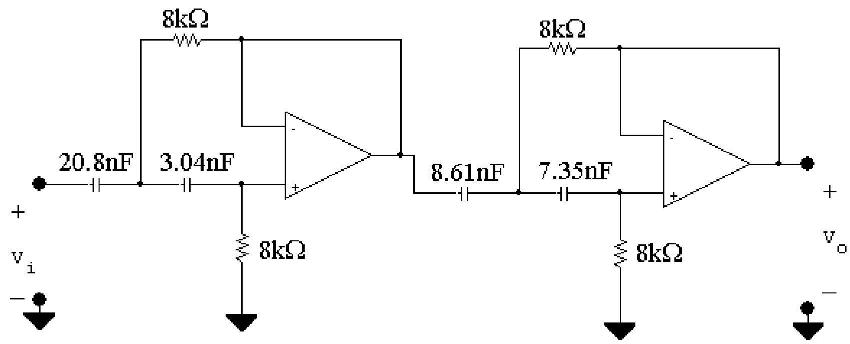
The scaled values for the second stage are

$$R_1 = R_2 = R = 8000\,\Omega$$

$$C_1 = \frac{1.08}{(5000\pi)(8000)} = 8.61\,\text{nF}$$

$$C_2 = \frac{0.92}{(5000\pi)(8000)} = 7.35\,\text{nF}$$

[b]



P 15.42 [a] $n = \dfrac{(-0.05)(-55)}{\log_{10}(200/40)} = 3.93 \quad \therefore \quad n = 4$

From Table 15.1 the transfer function of the first section is

$$H_1(s) = \frac{1}{s^2 + 0.765s + 1}$$

For the prototype circuit

$$\frac{2}{R_2} = 0.765; \qquad R_2 = 2.61\,\Omega; \qquad R_1 = \frac{1}{R_2} = 0.383\,\Omega$$

The transfer function of the second section is

$$H_2(s) = \frac{1}{s^2 + 1.848s + 1}$$

For the prototype circuit

$$\frac{2}{R_2} = 1.848; \qquad R_2 = 1.082\,\Omega; \qquad R_1 = \frac{1}{R_2} = 0.9240\,\Omega$$

The scaling factors are:

$$k_f = \frac{\omega_o'}{\omega_o} = \frac{2\pi(40{,}000)}{1} = 80{,}000\pi$$

$$C' = \frac{C}{k_m k_f} \qquad \therefore \qquad 250 \times 10^{-9} = \frac{1}{80{,}000\pi k_m}$$

$$\therefore \qquad k_m = \frac{1}{80{,}000\pi(250 \times 10^{-9})} = 15.9$$

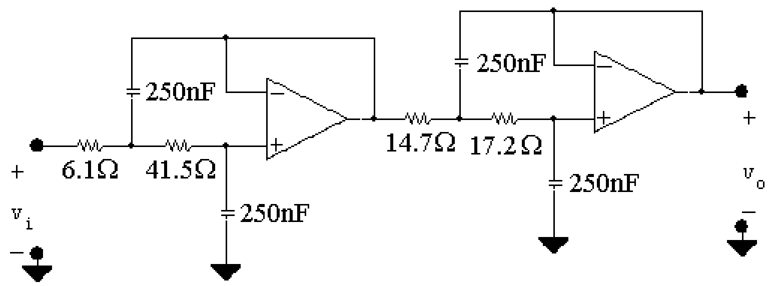
Therefore in the first section

$$R_1' = k_m R_1 = 6.1\,\Omega; \qquad R_2' = k_m R_2 = 41.5\,\Omega$$

In the second section

$R_1' = k_m R_1 = 14.7\,\Omega; \qquad R_2' = k_m R_2 = 17.2\,\Omega$

[b]



P 15.43 [a] The cascade connection is a bandpass filter.

[b] The cutoff frequencies are 2.5 kHz and 40 kHz.
The center frequency is $\sqrt{(2.5)(40)} = 10$ kHz.
The Q is $10/(40 - 2.5) = 0.267$

[c] For the high pass section $k_f = 5000\pi$. The prototype transfer function is

$$H_{\mathrm{hp}}(s) = \frac{s^4}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

$$\therefore \qquad H'_{\mathrm{hp}}(s) = \frac{(s/5000\pi)^4}{[(s/5000\pi)^2 + 0.765(s/5000\pi) + 1]} \cdot \frac{1}{[(s/5000\pi)^2 + 1.848(s/5000\pi) + 1]}$$

$$= \frac{s^4}{(s^2 + 3825\pi s + 25 \times 10^6\pi^2)(s^2 + 9240\pi s + 25 \times 10^6\pi^2)}$$

For the low pass section $k_f = 80{,}000\pi$

$$H_{\mathrm{lp}}(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

$$\therefore \qquad H'_{\mathrm{lp}}(s) = \frac{1}{[(s/80{,}000\pi)^2 + 0.765(s/80{,}000\pi) + 1]} \cdot \frac{1}{[(s/80{,}000\pi)^2 + 1.848(s/80{,}000\pi) + 1]}$$

$$= \frac{(80{,}000\pi)^4}{([s^2 + 61{,}200\pi s + (80{,}000\pi)^2)][s^2 + 147{,}840\pi s + (80{,}000\pi)^2]}$$

The cascaded transfer function is

$$H'(s) = H'_{\mathrm{hp}}(s)H'_{\mathrm{lp}}(s)$$

For convenience let

$$D_1 = s^2 + 3825\pi s + 25 \times 10^6\pi^2$$

$$D_2 = s^2 + 9240\pi s + 25 \times 10^6\pi^2$$

$$D_3 = s^2 + 61{,}200\pi s + 6400 \times 10^6\pi^2$$

$$D_4 = s^2 + 147{,}840\pi s + 6400 \times 10^6\pi^2$$

Then

$$H'(s) = \frac{4096 \times 10^{16}\pi^4 s^4}{D_1 D_2 D_3 D_4}$$

[d] $\omega_o = 2\pi(10{,}000) = 20{,}000\pi$ rad/s

$$s = j20{,}000\pi$$

$$s^4 = 16 \times 10^{16}\pi^4$$

$$D_1 = (25 \times 10^6\pi^2 - 400 \times 10^6\pi^2) + j(20{,}000\pi)(3825\pi)$$

$$= 10^6\pi^2(-375 - j76.5) = 10^6\pi^2(382.72\underline{/-168.47^\circ})$$

$$D_2 = (25 \times 10^6\pi^2 - 400 \times 10^6\pi^2) + j(20{,}000\pi)(9240\pi)$$

$$= 10^6\pi^2(-375 + j184.8) = 10^6\pi^2(418.06\underline{/153.77^\circ})$$

$$D_3 = (6400 \times 10^6\pi^2 - 400 \times 10^6\pi^2) + j(20{,}000\pi)(61{,}200\pi)$$

$$= 10^6\pi^2(6000 + j1224) = 10^6\pi^2(6123.58\underline{/11.53^\circ})$$

$$D_4 = (6400 \times 10^6\pi^2 - 400 \times 10^6\pi^2) + j(20{,}000\pi)(147{,}840\pi)$$

$$= 10^6\pi^2(6000 + j2956.8) = 10^6\pi^2(6689\underline{/26.23^\circ})$$

$$|H'(j\omega_o)| = \frac{(4096)(16)\pi^8 \times 10^{32}}{(\pi^8 \times 10^{24})[(382.72)(418.06)(6123.58)(6689)]}$$

$$= 0.999$$

P 15.44 **[a]** First we will design a unity gain filter and then provide the passband gain with an inverting amplifier. For the high pass section the cut-off frequency is 200 Hz. The order of the Butterworth is

$$n = \frac{(-0.05)(-40)}{\log_{10}(200/40)} = 2.86$$

$$\therefore \quad n = 3$$

$$H_{hp}(s) = \frac{s^3}{(s+1)(s^2+s+1)}$$

For the prototype first-order section

$R_1 = R_2 = 1\,\Omega, \quad C = 1\,\text{F}$

For the prototype second-order section

$R_1 = 0.5\,\Omega, \quad R_2 = 2\,\Omega, \quad C = 1\,\text{F}$

The scaling factors are

$$k_f = \frac{\omega_o'}{\omega_o} = 2\pi(200) = 400\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(10^{-6})(400\pi)} = 795.77$$

In the scaled first-order section

$R_1' = R_2' = k_m R_1 = 795.77\,\Omega$

$C' = 1\,\mu\text{F}$

In the scaled second-order section

$R_1' = 0.5k_m = 397.9\,\Omega$

$R_2' = 2k_m = 1591.5\,\Omega$

$C' = 1\,\mu\text{F}$

For the low-pass section the cut-off frequency is 2500 Hz. The order of the Butterworth filter is

$$n = \frac{(-0.05)(-40)}{\log_{10}(12{,}500/2500)} = 2.86; \qquad \therefore \quad n = 3$$

$$H_{\text{lp}}(s) = \frac{1}{(s+1)(s^2+s+1)}$$

For the prototype first-order section

$R_1 = R_2 = 1\,\Omega, \quad C = 1\,\text{F}$

For the prototype second-order section

$R_1 = R_2 = 1\,\Omega; \qquad C_1 = 2\,\text{F}; \qquad C_2 = 0.5\,\text{F}$

The low-pass scaling factors are

$$k_m = \frac{R'}{R} = 2500; \qquad k_f = \frac{\omega_o'}{\omega_o} = (2500)(2\pi) = 5000\pi$$

For the scaled first-order section

$$R_1' = R_2' = 2.5\,\text{k}\Omega; \qquad C' = \frac{C}{k_f k_m} = \frac{1}{(5000\pi)(2500)} = 25.46\,\text{nF}$$

For the scaled second-order section

$$R_1' = R_2' = 2.5\,\text{k}\Omega$$

$$C_1' = \frac{C_1}{k_f k_m} = \frac{2}{(5000\pi)(2500)} = 50.93\,\text{nF}$$

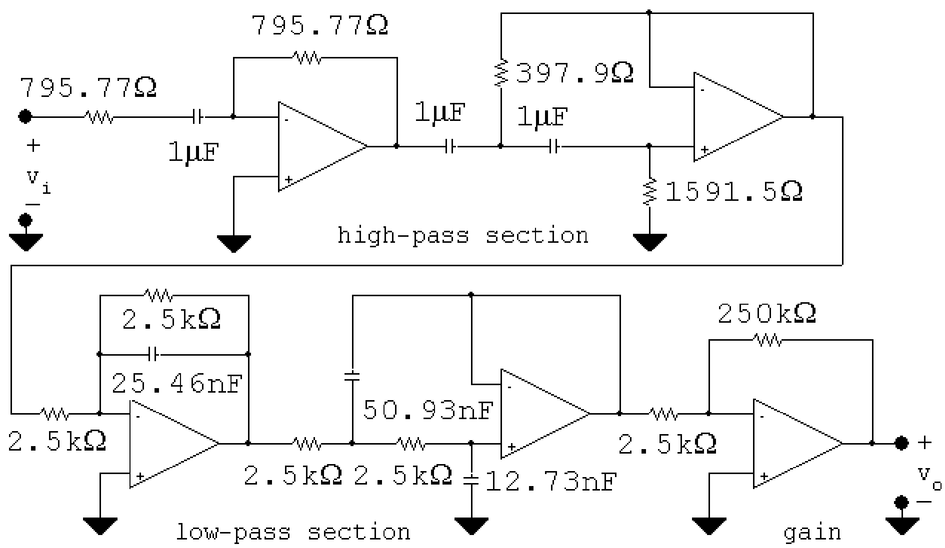
$$C_2' = \frac{C_2}{k_f k_m} = \frac{0.5}{(5000\pi)(2500)} = 12.73\,\text{nF}$$

GAIN AMPLIFIER

$20\log_{10} K = 40$ dB, $\quad\therefore\quad K = 100$

Since we are using $2.5\,\text{k}\Omega$ resistors in the low-pass stage, we will use $R_f = 250\,\text{k}\Omega$ and $R_i = 2.5\,\text{k}\Omega$ in the inverting amplifier stage.

[**b**]



P 15.45 [**a**] Unscaled high-pass stage

$$H_{hp}(s) = \frac{s^3}{(s+1)(s^2+s+1)}$$

The frequency scaling factor is $k_f = (\omega_o'/\omega_o) = 400\pi$. Therefore the scaled transfer function is

$$H_{hp}'(s) = \frac{(s/400\pi)^3}{\left(\frac{s}{400\pi}+1\right)\left[\left(\frac{s}{400\pi}\right)^2 + \frac{s}{400\pi} + 1\right]}$$

$$= \frac{s^3}{(s+400\pi)[s^2 + 400\pi s + 160{,}000\pi^2]}$$

Unscaled low-pass stage

$$H_{lp}(s) = \frac{1}{(s+1)(s^2+s+1)}$$

The frequency scaling factor is $k_f = (\omega_o'/\omega_o) = 5000\pi$. Therefore the scaled transfer function is

$$H_{lp}'(s) = \frac{1}{\left(\frac{s}{5000\pi}+1\right)\left[\left(\frac{s}{5000\pi}\right)^2+\left(\frac{s}{5000\pi}\right)+1\right]}$$
$$= \frac{(5000\pi)^3}{(s+5000\pi)(s^2+5000\pi s+25\times10^6\pi^2)}$$

Thus the transfer function for the filter is

$$H'(s) = 100H_{hp}'(s)H_{lp}'(s) = \frac{125\times10^{11}\pi^3 s^3}{D_1D_2D_3D_4}$$

where

$$D_1 = s + 400\pi$$

$$D_2 = s + 5000\pi$$

$$D_3 = s^2 + 400\pi s + 160{,}000\pi^2$$

$$D_4 = s^2 + 5000\pi s + 25\times10^6\pi^2$$

[b] At 40 Hz $\quad \omega = 80\pi$ rad/s

$$D_1(j80\pi) = \pi(400 + j80)$$

$$D_2(j80\pi) = \pi(5000 + j80)$$

$$D_3(j80\pi) = \pi^2(153{,}600 + j32{,}000)$$

$$D_4(j80\pi) = \pi^2(24{,}993{,}600 + j400{,}000)$$

Therefore

$$D_1D_2D_3D_4(j80\pi) = \pi^6 10^{18}(8\underline{/24.9^\circ})$$

$$H'(j80\pi) = \frac{(125\pi^3\times10^{11})(512\times10^3\pi^3)\underline{/-90^\circ}}{\pi^6\times10^{18}(8\underline{/24.9^\circ})}$$

$$= 0.8\underline{/-114.9^\circ}$$

$$\therefore \quad 20\log_{10}|H'(j80\pi)| = 20\log_{10}(0.8) = -1.94 \text{ dB}$$

At $f = 1000$ Hz, $\quad \omega = 2000\pi$ rad/s

Then

$$D_1(j2000\pi) = \pi(400 + j2000)$$

$$D_2(j2000\pi) = \pi(5000 + j2000)$$

$$D_3(j2000\pi) = \pi^2(-3{,}840{,}000 + j800{,}000)$$

$$D_4(j2000\pi) = \pi^2(21{,}000{,}000 + j10^7)$$

$$H'(j2000\pi) = \frac{(125\times\pi^3\times 10^{11})(8\times 10^9\pi^3)\underline{/-90^\circ}}{10.02\times 10^{20}\pi^6\underline{/-65.8^\circ}}$$

$$= 99.8\underline{/-24.2^\circ}$$

$$\therefore\quad 20\log_{10}|H'(j2000\pi)| = 39.98\ \text{dB}$$

[c] From the transfer function the gain is down $39.98 + 1.94$ or 41.9 dB at 40 Hz. Because the upper cut-off frequency is 25 times the lower cut-off frequency we would expect the high-pass stage of the filter to predict the loss in gain at 40 Hz. For a 3nd order Butterworth,

$$\text{GAIN} = 20\log_{10}\frac{1}{\sqrt{1+(200/40)^6}} = -41.9\ \text{dB}.$$

1000 Hz is in the passband for this bandpass filter. Hence we expect the gain at 1000 Hz to nearly equal 40 dB as specified in Problem 15.44. Thus our scaled transfer function confirms that the filter meets the specifications.

P 15.46 [a] From Table 15.1

$$H_{lp}(s) = \frac{1}{(s+1)(s^2+0.618s+1)(s^2+1.618s+1)}$$

$$H_{hp}(s) = \frac{1}{\left(\frac{1}{s}+1\right)\left(\frac{1}{s^2}+0.618\left(\frac{1}{s}\right)+1\right)\left(\frac{1}{s^2}+1.618\left(\frac{1}{s}\right)+1\right)}$$

$$H_{hp}(s) = \frac{s^5}{(s+1)(s^2+0.618s+1)(s^2+1.618s+1)}$$

P 15.47 [a] $k_f = 800$

$$H'_{\text{hp}}(s) = \frac{(s/800)^5}{[(s/800)+1]}$$

$$\cdot\frac{1}{[(s/800)^2+0.618s/800+1][(s/800)^2+1.618s/800+1]}$$

$$= \frac{s^5}{(s+800)(s^2+494.4s+64\times 10^4)}$$

$$\cdot\frac{1}{(s^2+1294.4s+64\times 10^4)}$$

[b] $H'(j800) = \dfrac{(j800)^5}{[800 + j800][494.4(j800)][1294.4(j800)]}$

$$= \frac{j(800)^2}{-(1 + j1)(494.4)(1294.4)}$$

$$= 0.707\underline{/-135^\circ}$$

$$20\log_{10}|H'(j800)| = -3.01 \text{ dB}$$

P 15.48 From Eq 15.56 we can write

$$H(s) = \frac{-\left(\dfrac{2}{R_3C}\right)\left(\dfrac{R_3C}{2}\right)\left(\dfrac{1}{R_1C}\right)s}{s^2 + \dfrac{2}{R_3C}s + \dfrac{R_1 + R_2}{R_1R_2R_3C^2}}$$

or

$$H(s) = \frac{-\left(\dfrac{R_3}{2R_1}\right)\left(\dfrac{2}{R_3C}s\right)}{s^2 + \dfrac{2}{R_3C}s + \dfrac{R_1 + R_2}{R_1R_2R_3C^2}}$$

Therefore

$$\frac{2}{R_3C} = \beta = \frac{\omega_o}{Q}; \qquad \frac{R_1 + R_2}{R_1R_2R_3C^2} = \omega_o^2;$$

and $K = \dfrac{R_3}{2R_1}$

By hypothesis $C = 1\,\text{F}$ and $\omega_o = 1\,\text{rad/s}$

$$\therefore \quad \frac{2}{R_3} = \frac{1}{Q} \text{ or } R_3 = 2Q$$

$$R_1 = \frac{R_3}{2K} = \frac{Q}{K}$$

$$\frac{R_1 + R_2}{R_1R_2R_3} = 1$$

$$\frac{Q}{K} + R_2 = \left(\frac{Q}{K}\right)(2Q)R_2$$

$$\therefore \quad R_2 = \frac{Q}{2Q^2 - K}$$

P 15.49 [a] From the statement of the problem, $K = 10$ ($= 20$ dB). Therefore for the prototype bandpass circuit

$$R_1 = \frac{Q}{K} = \frac{16}{10} = 1.6\,\Omega$$

$$R_2 = \frac{Q}{2Q^2 - K} = \frac{16}{502}\,\Omega$$

$$R_3 = 2Q = 32\,\Omega$$

The scaling factors are

$$k_f = \frac{\omega_o'}{\omega_o} = 2\pi(6400) = 12{,}800\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(20 \times 10^{-9})(12{,}800\pi)} = 1243.40$$

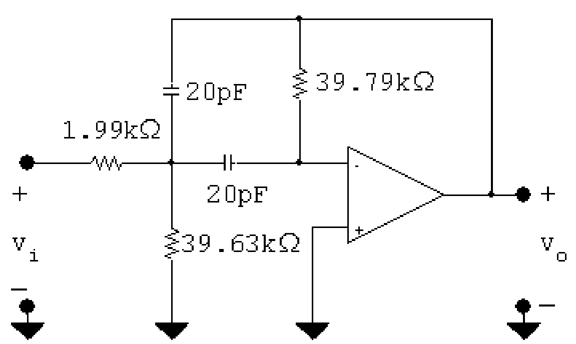
Therefore,

$$R_1' = k_m R_1 = (1.6)(1243.30) = 1.99\,\text{k}\Omega$$

$$R_2' = k_m R_2 = (16/502)(1243.40) = 39.63\,\Omega$$

$$R_3' = k_m R_3 = 32(1243.40) = 39.79\,\text{k}\Omega$$

[b]



P 15.50 [a] At very low frequencies the two capacitor branches are open and because the op amp is ideal the current in R_3 is zero. Therefore at low frequencies the circuit behaves as an inverting amplifier with a gain of R_2/R_1. At very high frequencies the capacitor branches are short circuits and hence the output voltage is zero.

[b] Let the node where R_1, R_2, R_3, and C_2 join be denoted as a, then

$$(V_a - V_i)G_1 + V_a sC_2 + (V_a - V_o)G_2 + V_a G_3 = 0$$

$$-V_a G_3 - V_o sC_1 = 0$$

or

$$(G_1 + G_2 + G_3 + sC_2)V_a - G_2 V_o = G_1 V_i$$

$$V_a = \frac{-sC_1}{G_3}V_o$$

Solving for V_o/V_i yields

$$H(s) = \frac{-G_1G_3}{(G_1 + G_2 + G_3 + sC_2)sC_1 + G_2G_3}$$

$$= \frac{-G_1G_3}{s^2C_1C_2 + (G_1 + G_2 + G_3)C_1s + G_2G_3}$$

$$= \frac{-G_1G_3/C_1C_2}{s^2 + \left[\frac{(G_1+G_2+G_3)}{C_2}\right]s + \frac{G_2G_3}{C_1C_2}}$$

$$= \frac{-\frac{G_1G_2G_3}{G_2C_1C_2}}{s^2 + \left[\frac{(G_1+G_2+G_3)}{C_2}\right]s + \frac{G_2G_3}{C_1C_2}}$$

$$= \frac{-Kb_o}{s^2 + b_1s + b_o}$$

where $K = \frac{G_1}{G_2}$; $\qquad b_o = \frac{G_2G_3}{C_1C_2}$

and $b_1 = \frac{G_1 + G_2 + G_3}{C_2}$

[c] Rearranging we see that

$$G_1 = KG_2$$

$$G_3 = \frac{b_oC_1C_2}{G_2} = \frac{b_oC_1}{G_2}$$

since by hypothesis $C_2 = 1\,\text{F}$

$$b_1 = \frac{G_1 + G_2 + G_3}{C_2} = G_1 + G_2 + G_3$$

$$\therefore\ b_1 = KG_2 + G_2 + \frac{b_oC_1}{G_2}$$

$$b_1 = G_2(1 + K) + \frac{b_oC_1}{G_2}$$

Solving this quadratic equation for G_2 we get

$$G_2 = \frac{b_1}{2(1+K)} \pm \sqrt{\frac{b_1^2 - b_oC_14(1+K)}{4(1+K)^2}}$$

$$= \frac{b_1 \pm \sqrt{b_1^2 - 4b_o(1+K)C_1}}{2(1+K)}$$

For G_2 to be realizable

$$C_1 < \frac{b_1^2}{4b_o(1+K)}$$

[d] 1. Select $C_2 = 1\,\text{F}$

2. Select C_1 such that $C_1 < \dfrac{b_1^2}{4b_o(1+K)}$

3. Calculate G_2 (R_2)

4. Calculate G_1 (R_1); $G_1 = KG_2$

5. Calculate G_3 (R_3); $G_3 = b_oC_1/G_2$

P 15.51 **[a]** In the second order section of a third order Butterworth filter $b_o = b_1 = 1$
Therefore,

$$C_1 \leq \frac{b_1^2}{4b_o(1+K)} = \frac{1}{(4)(1)(5)} = 0.05\,\text{F}$$

$$\therefore \qquad C_1 = 0.05\,\text{F} \quad \text{(limiting value)}$$

[b] $G_2 = \dfrac{1}{2(1+4)} = 0.1\,\text{S}$

$$G_3 = \frac{1}{0.1}(0.05) = 0.5\,\text{S}$$

$$G_1 = 4(0.1) = 0.4\,\text{S}$$

Therefore,

$$R_1 = \frac{1}{G_1} = 2.5\,\Omega; \qquad R_2 = \frac{1}{G_2} = 10\,\Omega; \qquad R_3 = \frac{1}{G_3} = 2\,\Omega$$

[c] $k_f = \dfrac{\omega_o'}{\omega_o} = 2\pi(2500) = 5000\pi$

$$k_m = \frac{C_2}{C_2'k_f} = \frac{1}{(10\times10^{-9})k_f} = 6366.2$$

$$C_1' = \frac{0.05}{k_fk_m} = 0.5\times10^{-9} = 500\,\text{pF}$$

$$R_1' = (2.5)(6366.2) = 15.92\,\text{k}\Omega$$

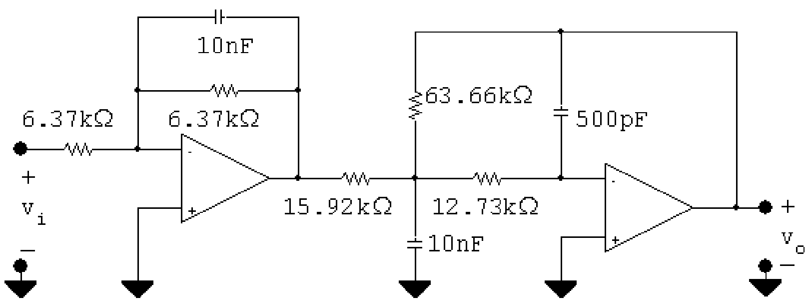
$$R_2' = (10)(6366.2) = 63.66\,\text{k}\Omega$$

$$R_3' = (2)(6366.2) = 12.73\,\text{k}\Omega$$

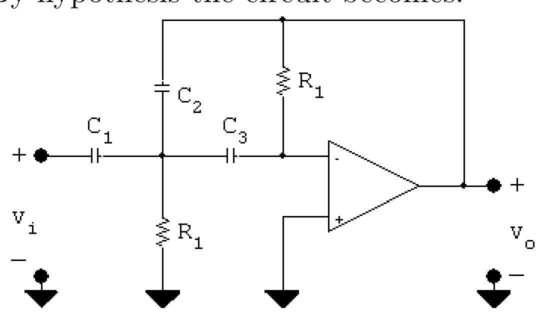
[d] $R_1' = R_2' = (6366.2)(1) = 6.37\,\text{k}\Omega$

$$C' = \frac{C}{k_fk_m} = \frac{1}{10^8} = 10\,\text{nF}$$

[e]



P 15.52 [a] By hypothesis the circuit becomes:



For very small frequencies the capacitors behave as open circuits and therefore v_o is zero. As the frequency increases, the capacitive branch impedances become small compared to the resistive branches. When this happens the circuit becomes an inverting amplifier with the capacitor C_2 dominating the feedback path. Hence the gain of the amplifier approaches $(1/j\omega C_2)/(1/j\omega C_1)$ or C_1/C_2. Therefore the circuit behaves like a high-pass filter with a passband gain of C_1/C_2.

[b] Summing the currents away from the upper terminal of R_2 yields

$$V_aG_2 + (V_a - V_i)sC_1 + (V_a - V_o)sC_2 + V_asC_3 = 0$$

or

$$V_a[G_2 + s(C_1 + C_2 + C_3)] - V_osC_2 = sC_1V_i$$

Summing the currents away from the inverting input terminal gives

$$(0 - V_a)sC_3 + (0 - V_o)G_1 = 0$$

or

$$sC_3V_a = -G_1V_o; \qquad V_a = \frac{-G_1V_o}{sC_3}$$

Therefore we can write

$$\frac{-G_1V_o}{sC_3}[G_2 + s(C_1 + C_2 + C_3)] - sC_2V_o = sC_1V_i$$

Solving for V_o/V_i gives

$$H(s) = \frac{V_o}{V_i} = \frac{-C_1C_3s^2}{C_2C_3s^2 + G_1(C_1 + C_2 + C_3)s + G_1G_2]}$$

$$= \frac{\frac{-C_1}{C_2}s^2}{\left[s^2 + \frac{G_1}{C_2C_3}(C_1 + C_2 + C_3)s + \frac{G_1G_2}{C_2C_3}\right]}$$

$$= \frac{-Ks^2}{s^2 + b_1s + b_o}$$

Therefore the circuit implements a second-order high-pass filter with a passband gain of C_1/C_2.

[c] $C_1 = K$:

$$b_1 = \frac{G_1}{(1)(1)}(K+2) = G_1(K+2)$$

$$\therefore \quad G_1 = \frac{b_1}{K+2}; \qquad R_1 = \left(\frac{K+2}{b_1}\right)$$

$$b_o = \frac{G_1G_2}{(1)(1)} = G_1G_2$$

$$\therefore \quad G_2 = \frac{b_o}{G_1} = \frac{b_o}{b_1}(K+2)$$

$$\therefore \quad R_2 = \frac{b_1}{b_o(K+2)}$$

[d] From Table 15.1 the transfer function of the second-order section of a third-order high-pass Butterworth filter is

$$H(s) = \frac{Ks^2}{s^2 + s + 1}$$

Therefore $b_1 = b_o = 1$

Thus

$$C_1 = K = 8\,\text{F}$$

$$R_1 = \frac{8+2}{1} = 10\,\Omega$$

$$R_2 = \frac{1}{1(8+2)} = 0.1\,\Omega$$

P 15.53 **[a]** Low-pass filter:

$$n = \frac{(-0.05)(-24)}{\log_{10}(2000/1000)} = 3.99; \qquad \therefore \quad n = 4$$

In the first prototype second-order section: $b_1 = 0.765,\ b_o = 1,\ C_2 = 1\,\text{F}$

$$C_1 \le \frac{b_1^2}{4b_o(1+K)} \le \frac{(0.765)^2}{(4)(2)} \le 0.0732$$

choose $C_1 = 0.03\,\text{F}$

$$G_2 = \frac{0.765 \pm \sqrt{(0.765)^2 - 4(2)(0.03)}}{4} = \frac{0.765 \pm 0.588}{4}$$

Arbitrarily select the larger value for G_2, then

$$G_2 = 0.338\ \text{S}; \quad \therefore \quad R_2 = \frac{1}{G_2} = 2.96\,\Omega$$

$$G_1 = KG_2 = 0.338\ \text{S}; \quad \therefore \quad R_1 = \frac{1}{G_1} = 2.96\,\Omega$$

$$G_3 = \frac{b_oC_1}{G_2} = \frac{(1)(0.03)}{0.338} = 0.089 \quad \therefore \quad R_3 = 1/G_3 = 11.3\,\Omega$$

Therefore in the first second-order prototype circuit

$$R_1 = R_2 = 2.96\,\Omega; \qquad R_3 = 11.3\,\Omega$$

$$C_1 = 0.03\,\text{F}; \qquad C_2 = 1\,\text{F}$$

In the second second-order prototype circuit:
$b_1 = 1.848,\ b_0 = 1,\ C_2 = 1\,\text{F}$

$$\therefore \quad C_1 \le \frac{(1.848)^2}{8} \le 0.427$$

choose $C_1 = 0.30\,\text{F}$

$$G_2 = \frac{1.848 \pm \sqrt{(1.848)^2 - 8(0.3)}}{4} = \frac{1.848 \pm 1.008}{4}$$

Arbitrarily select the larger value, then

$$G_2 = 0.7139\ \text{S}; \quad \therefore \quad R_2 = \frac{1}{G_2} = 1.4008\,\Omega$$

$$G_1 = KG_2 = 0.7139\ \text{S}; \quad \therefore \quad R_1 = \frac{1}{G_1} = 1.4008\,\Omega$$

$$G_3 = \frac{b_oC_1}{G_2} = \frac{(1)(0.30)}{0.7139} = 0.4202\ \text{S} \quad \therefore \quad R_3 = 1/G_3 = 2.3796\,\Omega$$

In the low-pass section of the filter

$$k_f = \frac{\omega_o'}{\omega_o} = 2\pi(1000) = 2000\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(25\times 10^{-9})k_f} = \frac{20{,}000}{\pi}$$

Therefore in the first scaled second-order section

$$R_1' = R_2' = 2.96k_m = 18.84\,\text{k}\Omega$$

$$R_3' = 11.3k_m = 71.94\,\text{k}\Omega$$

$$C_1' = \frac{0.03}{k_f k_m} = 750\,\text{pF}$$

$$C_2' = 25\,\text{nF}$$

In the second scaled second-order section

$$R_1' = R_2' = 1.4008k_m = 8.92\,\text{k}\Omega$$

$$R_3' = 2.38k_m = 15.15\,\text{k}\Omega$$

$$C_1' = \frac{0.3}{k_f k_m} = 7.5\,\text{nF}$$

$$C_2' = 25\,\text{nF}$$

High-pass filter section

$$n = \frac{(-0.05)(-24)}{\log_{10}(8000/4000)} = 3.99; \qquad n = 4$$

In the first prototype second-order section:
$b_1 = 0.765;\ b_o = 1;\ C_2 = C_3 = 1\,\text{F}$

$$C_1 = K = 1\,\text{F}$$

$$R_1 = \frac{K+2}{b_1} = \frac{3}{0.765} = 3.92\,\Omega$$

$$R_2 = \frac{b_1}{b_o(K+2)} = \frac{0.765}{3} = 0.255\,\Omega$$

In the second prototype second-order section: $b_1 = 1.848$; $b_o = 1$;
$C_2 = C_3 = 1\,\text{F}$

$$C_1 = K = 1\,\text{F}$$

$$R_1 = \frac{K+2}{b_1} = \frac{3}{1.848} = 1.623\,\Omega$$

$$R_2 = \frac{b_1}{b_o(K+2)} = \frac{1.848}{3} = 0.616\,\Omega$$

In the high-pass section of the filter

$$k_f = \frac{\omega_o'}{\omega_o} = 2\pi(8000) = 16{,}000\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(25 \times 10^{-9})(16{,}000\pi)} = \frac{2500}{\pi}$$

In the first scaled second-order section

$R'_1 = 3.92k_m = 3119.4\,\Omega$

$R'_2 = 0.255k_m = 202.9\,\Omega$

$C'_1 = C'_2 = C'_3 = 25\,\text{nF}$

In the second scaled second-order section

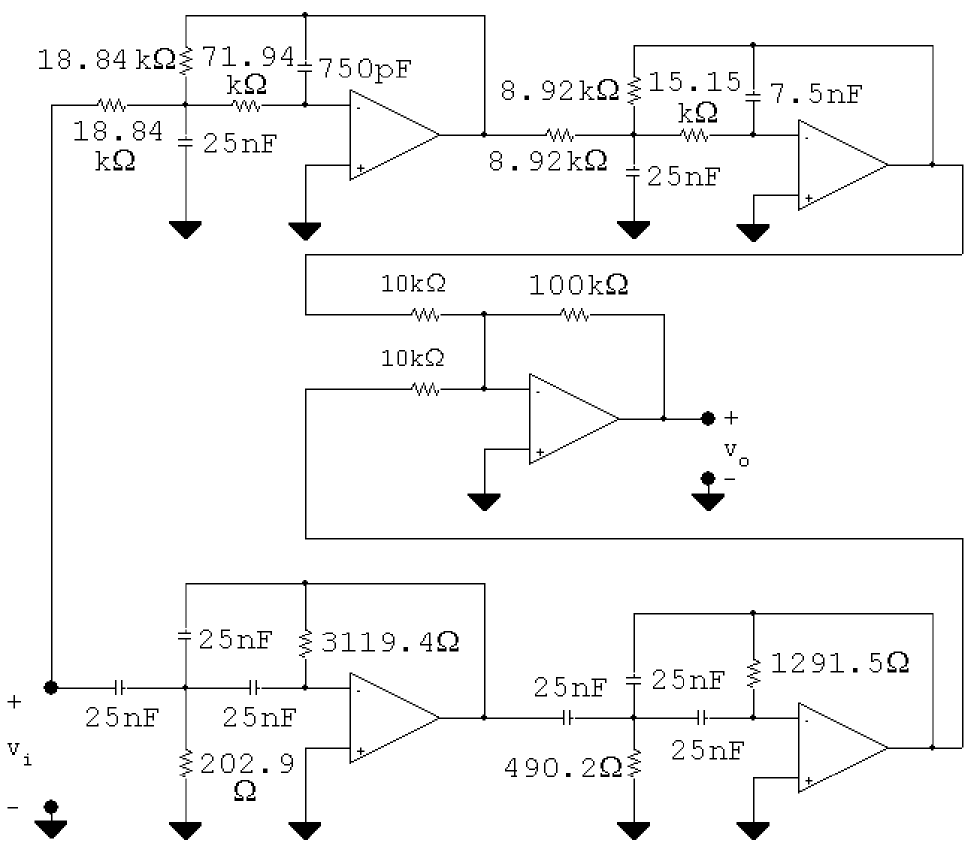
$R'_1 = 1.623k_m = 1291.5\,\Omega$

$R'_2 = 0.616k_m = 490.2\,\Omega$

$C'_1 = C'_2 = C'_3 = 25\,\text{nF}$

In the gain section, let $R_i = 10\,\text{k}\Omega$ and $R_f = 100\,\text{k}\Omega$.

[**b**]



P 15.54 [**a**] The prototype low-pass transfer function is

$$H_{lp}(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

The low-pass frequency scaling factor is

$k_{f_{lp}} = 2\pi(1000) = 2000\pi$

The scaled transfer function for the low-pass filter is

$$H'_{lp}(s) = \frac{1}{\left[\left(\frac{s}{2000\pi}\right)^2 + \frac{0.765s}{2000\pi} + 1\right]\left[\left(\frac{s}{2000\pi}\right)^2 + \frac{1.848s}{2000\pi} + 1\right]}$$

$$= \frac{16 \times 10^{12}\pi^4}{[s^2 + 1530\pi s + (2000\pi)^2]\,[s^2 + 3696\pi s + (2000\pi)^2]}$$

The prototype high-pass transfer function is

$$H_{hp}(s) = \frac{s^4}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

The high-pass frequency scaling factor is

$k_{f_{hp}} = 2\pi(8000) = 16{,}000\pi$

The scaled transfer function for the high-pass filter is

$$H'_{hp}(s) = \frac{(s/16{,}000\pi)^4}{\left[\left(\frac{s}{16{,}000\pi}\right)^2 + \frac{0.765s}{16{,}000\pi} + 1\right]\left[\left(\frac{s}{16{,}000\pi}\right)^2 + \frac{1.848s}{16{,}000\pi} + 1\right]}$$

$$= \frac{s^4}{[s^2 + 12{,}240\pi s + (16{,}000\pi)^2][s^2 + 29{,}568\pi s + (16{,}000\pi)^2]}$$

The transfer function for the filter is

$H'(s) = \left[H'_{lp}(s) + H'_{hp}(s)\right]$

[b] $f_o = \sqrt{f_{c1} f_{c2}} = \sqrt{(1000)(8000)} = 2000\sqrt{2}\,\text{Hz}$

$\omega_o = 2\pi f_o = 4000\sqrt{2}\pi\ \text{rad/s}$

$(j\omega_o)^2 = -32 \times 10^6\pi^2$

$(j\omega_o)^4 = 1024 \times 10^{12}\pi^4$

$$H'_{lp}(j\omega_o) = \frac{16 \times 10^{12}\pi^4}{[-28 \times 10^6\pi^2 + j1530(4000\sqrt{2}\pi^2)]} \times \frac{1}{[-28 \times 10^6\pi^2 + j3696(4000\sqrt{2}\pi^2)]}$$

$$= 0.0156\underline{/53.9^\circ}$$

$$H'_{hp}(j\omega_o) = \frac{1024 \times 10^{12}\pi^4}{[224 \times 10^6\pi^2 + j12{,}240(4000\sqrt{2}\pi^2)]} \cdot \frac{1}{[224 \times 10^6\pi^2 + j29{,}568(4000\sqrt{2}\pi^2)]}$$

$$= 0.0156\underline{/-53.9^\circ}$$

$$\therefore \quad H'(j\omega_o) = 0.0156(1\underline{/53.9^\circ} + 1\underline{/-53.9^\circ})$$

$$= 0.0156(1.18\underline{/0^\circ}) = 0.0184\underline{/0^\circ}$$

$$G = 20\log_{10}|H'(j\omega_o)| = 20\log_{10}(0.0184) = -34.7 \text{ dB}$$

P 15.55 **[a]** At low frequencies the capacitor branches are open; $v_o = v_i$. At high frequencies the capacitor branches are short circuits and the output voltage is zero. Hence the circuit behaves like a unity-gain low-pass filter.

[b] Let v_a represent the voltage-to-ground at the right-hand terminal of R_1. Observe this will also be the voltage at the left-hand terminal of R_2. The s-domain equations are

$$(V_a - V_i)G_1 + (V_a - V_o)sC_1 = 0$$

$$(V_o - V_a)G_2 + sC_2V_o = 0$$

or

$$(G_1 + sC_1)V_a - sC_1V_o = G_1V_i$$

$$-G_2V_a + (G_2 + sC_2)V_o = 0$$

$$\therefore \quad V_a = \frac{G_2 + sC_2V_o}{G_2}$$

$$\therefore \quad \left[(G_1 + sC_1)\frac{(G_2 + sC_2)}{G_2} - sC_1\right]V_o = G_1V_i$$

$$\therefore \quad \frac{V_o}{V_i} = \frac{G_1G_2}{(G_1 + sC_1)(G_2 + sC_2) - C_1G_2s}$$

which reduces to

$$\frac{V_o}{V_i} = \frac{G_1G_2/C_1C_2}{s^2 + \frac{G_1}{C_1}s + \frac{G_1G_2}{C_1C_2}} = \frac{b_o}{s^2 + b_1s + b_o}$$

[c] There are four circuit components and two restraints imposed by $H(s)$; therefore there are two free choices.

[d] $b_1 = \dfrac{G_1}{C_1} \quad \therefore \quad G_1 = b_1C_1$

$$b_o = \frac{G_1G_2}{C_1C_2} \quad \therefore \quad G_2 = \frac{b_o}{b_1}C_2$$

[e] No, all physically realizeable capacitors will yield physically realizeable resistors.

[f] From Table 15.1 we know the transfer function of the prototype 4th order Butterworth filter is

$$H(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

In the first section $b_o = 1, \quad b_1 = 0.765$

$$\therefore \quad G_1 = (0.765)(1) = 0.765 \text{ S}$$

$$R_1 = 1/G_1 = 1.307\,\Omega$$

$$G_2 = \frac{1}{0.765}(1) = 1.307 \text{ S}$$

$$R_2 = 1/G_2 = 0.765\,\Omega$$

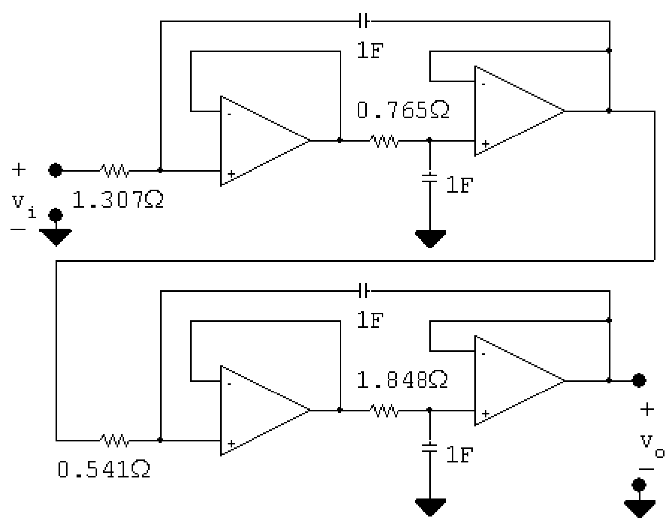
In the second section $b_o = 1, \quad b_1 = 1.848$

$$\therefore \quad G_1 = 1.848\,\text{S}$$

$$R_1 = 1/G_1 = 0.541\,\Omega$$

$$G_2 = \left(\frac{1}{1.848}\right)(1) = 0.541\,\text{S}$$

$$R_2 = 1/G_2 = 1.848\,\Omega$$



P 15.56 **[a]** $k_f = \frac{\omega_o'}{\omega_o} = 2\pi(3000) = 6000\pi$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(4.7 \times 10^{-9})(6000\pi)} = \frac{10^6}{28.2\pi}$$

In the first section

$$R_1' = 1.307k_m = 14.75\,\text{k}\Omega$$

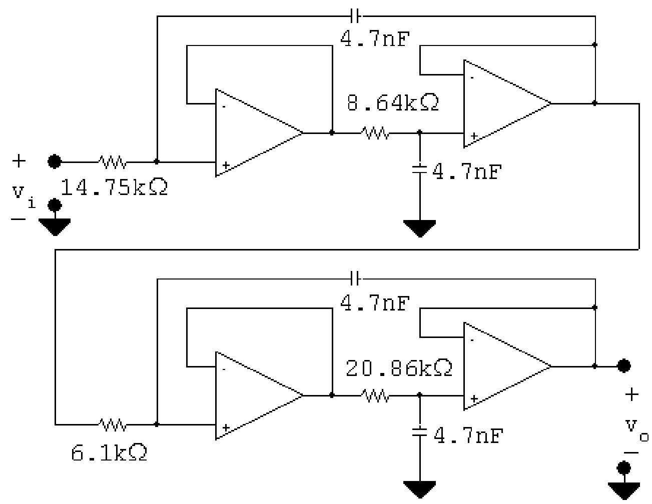
$R_2' = 0.765k_m = 8.64\,\text{k}\Omega$

In the second section

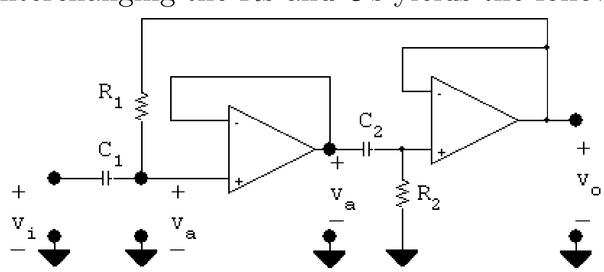
$R_1' = 0.541k_m = 6.1\,\text{k}\Omega$

$R_2' = 1.848k_m = 20.86\,\text{k}\Omega$

[b]



P 15.57 **[a]** Interchanging the Rs and Cs yields the following circuit.



At low frequencies the capacitors appear as open circuits and hence the output voltage is zero. As the frequency increases the capacitor branches approach short circuits and $v_a = v_i = v_o$. Thus the circuit is a unity-gain, high-pass filter.

[b] The s-domain equations are

$$(V_a - V_i)sC_1 + (V_a - V_o)G_1 = 0$$

$$(V_o - V_a)sC_2 + V_oG_2 = 0$$

It follows that

$$V_a(G_1 + sC_1) - G_1V_o = sC_1V_i$$

and $V_a = \dfrac{(G_2 + sC_2)V_o}{sC_2}$

Thus

$$\left\{\left[\frac{(G_2 + sC_2)}{sC_2}\right](G_1 + sC_1) - G_1\right\} V_o = sC_1V_i$$

$$V_o\{s^2C_1C_2 + sC_1G_2 + G_1G_2\} = s^2C_1C_2V_i$$

$$H(s) = \frac{V_o}{V_i} = \frac{s^2}{\left(s^2 + \frac{G_2}{C_2}s + \frac{G_1G_2}{C_1C_2}\right)}$$

$$= \frac{V_o}{V_i} = \frac{s^2}{s^2 + b_1s + b_o}$$

[c] There are 4 circuit components: R_1, R_2, C_1 and C_2.
There are two transfer function constraints: b_1 and b_o.
Therefore there are two free choices.

[d] $b_o = \dfrac{G_1G_2}{C_1C_2}$; $\qquad b_1 = \dfrac{G_2}{C_2}$

$$\therefore \quad G_2 = b_1C_2; \qquad R_2 = \frac{1}{b_1C_2}$$

$$G_1 = \frac{b_o}{b_1}C_1 \therefore \quad R_1 = \frac{b_1}{b_oC_1}$$

[e] No, all realizeable capacitors will produce realizeable resistors.

[f] The second-order section in a 3rd-order Butterworth high-pass filter is $s^2/(s^2 + s + 1)$. Therefore $b_o = b_1 = 1$ and

$$R_1 = \frac{1}{(1)(1)} = 1\,\Omega.$$

$$R_2 = \frac{1}{(1)(1)} = 1\,\Omega.$$

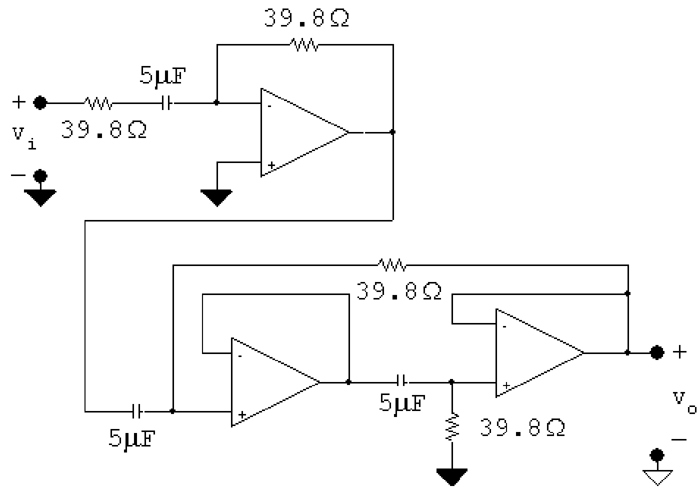
P 15.58 **[a]** $k_f = \dfrac{\omega_o'}{\omega_o} = 1600\pi$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(5 \times 10^{-6})(1600\pi)} = 39.8$$

$$C_1' = C_2' = 5\,\mu\text{F}; \qquad R_1' = R_2' = k_mR = 39.8\,\Omega$$

[b] $R = 39.8\,\Omega$; $\qquad C = 5\,\mu\text{F}$

[**c**]



[**d**] $H_{\rm hp}(s) = \dfrac{s^3}{(s+1)(s^2+s+1)}$

$$H'_{\rm hp}(s) = \frac{(s/1600\pi)^3}{[(s/1600\pi)+1][(s/1600\pi)^2+(s/1600\pi)+1]}$$

$$= \frac{s^3}{(s+1600\pi)(s^2+1600\pi s+256\times 10^4\pi^2)}$$

[**e**] $H'_{\rm hp}(j1600\pi) = \dfrac{(j1600\pi)^3}{(j1600\pi+1600\pi)[(j1600\pi)^2+1600\pi(j1600\pi)+256\times 10^4\pi^2]}$

$$= 0.7071\underline{/135^\circ}$$

$$\therefore \qquad |H'_{\rm hp}| = 0.7071 = -3.01\text{ dB}$$

P 15.59 [**a**] It follows directly from Eqs 15.64 and 15.65 that

$$H(s) = \frac{s^2+1}{s^2+4(1-\sigma)s+1}$$

Now note from Eq 15.69 that $(1-\sigma)$ equals $1/4Q$, hence

$$H(s) = \frac{s^2+1}{s^2+\frac{1}{Q}s+1}$$

[**b**] For Example 15.13 $\omega_o = 5000\,\text{rad/s}$ and $Q = 5$. Therefore $k_f = 5000$ and

$$H'(s) = \frac{(s/5000)^2+1}{(s/5000)^2+\frac{1}{5}\left(\frac{s}{5000}\right)+1}$$

$$= \frac{s^2+25\times 10^6}{s^2+1000s+25\times 10^6}$$

P 15.60 **[a]** $\omega_o = 8000\pi$ rad/s

$$\therefore \quad k_f = \frac{\omega_o'}{\omega_o} = 8000\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(0.5 \times 10^{-6})(8000\pi)} = \frac{250}{\pi}$$

$$R' = k_m R = 79.6\,\Omega \qquad \text{so} \qquad R'/2 = 39.8\,\Omega$$

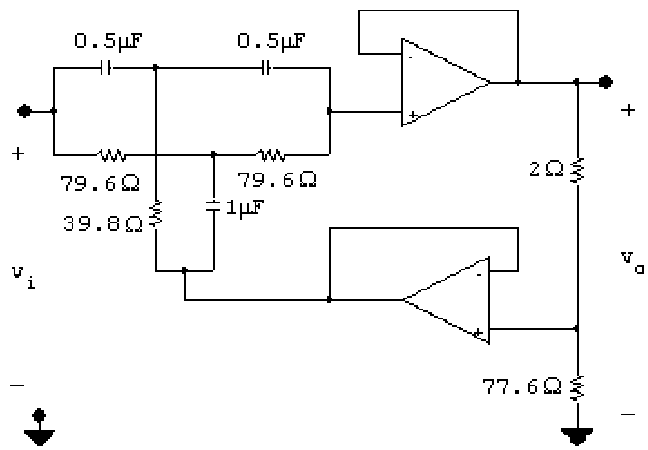
$$\sigma = 1 - \frac{1}{4Q} = 1 - \frac{1}{4(10)} = 0.975$$

$$\sigma R' = 77.6\,\Omega; \qquad (1-\sigma)R' = 2\,\Omega$$

$$C' = 0.5\,\mu\text{F}$$

$$2C' = 1\,\mu\text{F}$$

[b]



[c] $k_f = 8000\pi$

$$H(s) = \frac{(s/8000\pi)^2 + 1}{(s/8000\pi)^2 + \frac{1}{10}(s/8000\pi) + 1}$$

$$= \frac{s^2 + 64 \times 10^6\pi^2}{s^2 + 800\pi s + 64 \times 10^6\pi^2}$$

P 15.61 To satisfy the gain specification of 14 dB at $\omega = 0$ and $\alpha = 1$ requires

$$\frac{R_1 + R_2}{R_1} = 5 \qquad \text{or} \qquad R_2 = 4R_1$$

Use the specified resistor of $10\,\text{k}\Omega$ for R_1 and a $50\,\text{k}\Omega$ potentiometer for R_2. Since $(R_1 + R_2)/R_1 \gg 1$ the value of C_1 is

$$C_1 = \frac{1}{2\pi(50)(50{,}000)} = 63.66 \text{ nF}$$

Choose a capacitor value of 64 nF. Using the selected values of R_1 and R_2 the maximum gain for $\alpha = 1$ is

$$20\log_{10}\left(\frac{50}{10}\right)_{\alpha=1} = 13.98 \text{ dB}$$

When $C_1 = 64$ nF the frequency $1/R_2C_1$ is

$$\frac{1}{R_2C_1} = \frac{10^9}{50{,}000(64)} = 312.5 \text{ rad/s} = 49.7 \text{ Hz}$$

The magnitude of the transfer function at 312.5 rad/s is

$$|H(j312.5)|_{\alpha=1} = \left|\frac{50\times10^3 + j312.5(10)(50)(64)10^{-3}}{10\times10^3 + j312.5(10)(50)(64)10^{-3}}\right| = 3.61$$

Therefore the gain at 49.7 Hz is

$$20\log_{10}(3.61)_{\alpha=1} = 11.1 \text{ dB}$$

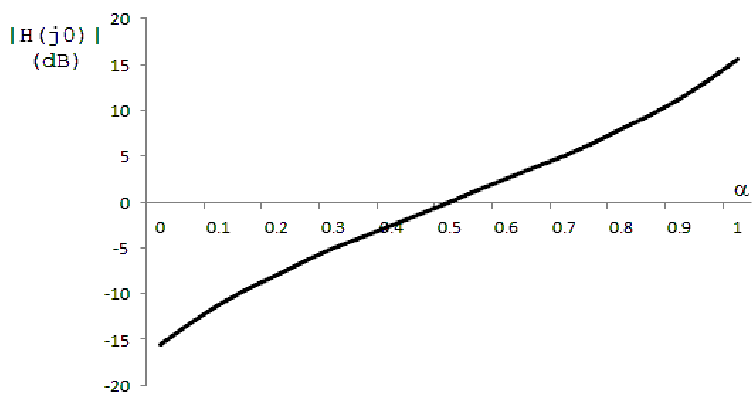
P 15.62 $20\log_{10}\left(\frac{R_1+R_2}{R_1}\right) = 20$

$$\therefore \quad \frac{R_1+R_2}{R_1} = 10; \qquad \therefore \quad R_2 = 9R_1$$

Choose $R_1 = 100\,\text{k}\Omega$. Then $R_2 = 900\,\text{k}\Omega$

$$\frac{1}{R_2C_1} = 150\pi \text{ rad/s}; \qquad \therefore \quad C_1 = \frac{1}{(150\pi)(900\times10^3)} = 2.36\,\text{nF}$$

P 15.63 $|H(j0)| = \frac{R_1+\alpha R_2}{R_1+(1-\alpha)R_2} = \frac{10+\alpha(50)}{10+(1-\alpha)50}$

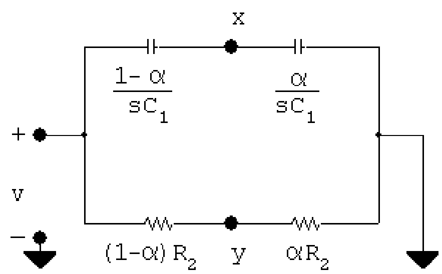


P 15.64 **[a]** Combine the impedances of the capacitors in series in Fig. P15.64(b) to get

$$\frac{1}{sC_{\text{eq}}} = \frac{1-\alpha}{sC_1} + \frac{\alpha}{sC_1} = \frac{1}{sC_1}$$

which is identical to the impedance of the capacitor in Fig. P15.60(a).

[b]



$$V_x = \frac{\alpha/sC_1}{(1-\alpha)/sC_1 + \alpha/sC_1}V = \alpha V$$

$$V_y = \frac{\alpha R_2}{(1-\alpha)R_2 + \alpha R_2} = \alpha V = V_x$$

[c] Since x and y are both at the same potential, they can be shorted together, and the circuit in Fig. 15.34 can thus be drawn as shown in Fig. 15.53(c).

[d] The feedback path between V_o and V_s containing the resistance $R_4 + 2R_3$ has no effect on the ratio V_o/V_s, as this feedback path is not involved in the nodal equation that defines the voltage ratio. Thus, the circuit in Fig. P15.64(c) can be simplified into the form of Fig. 15.2, where the input impedance is the equivalent impedance of R_1 in series with the parallel combination of $(1-\alpha)/sC_1$ and $(1-\alpha)R_2$, and the feedback impedance is the equivalent impedance of R_1 in series with the parallel combination of α/sC_1 and αR_2:

$$Z_i = R_1 + \frac{\frac{(1-\alpha)}{sC_1}\cdot(1-\alpha)R_2}{(1-\alpha)R_2 + \frac{(1-\alpha)}{sC_1}}$$

$$= \frac{R_1 + (1-\alpha)R_2 + R_1R_2C_1s}{1 + R_2C_1s}$$

$$Z_f = R_1 + \frac{\frac{\alpha}{sC_1}\cdot \alpha R_2}{\alpha R_2 + \frac{\alpha}{sC_1}}$$

$$= \frac{R_1 + \alpha R_2 + R_1R_2C_1s}{1 + R_2C_1s}$$

P 15.65 As $\omega \to 0$

$$|H(j\omega)| \to \frac{2R_3 + R_4}{2R_3 + R_4} = 1$$

Therefore the circuit would have no effect on low frequency signals. As $\omega \to \infty$

$$|H(j\omega)| \to \frac{[(1-\beta)R_4 + R_o](\beta R_4 + R_3)}{[(1-\beta)R_4 + R_3](\beta R_4 + R_o)}$$

When $\beta = 1$

$$|H(j\infty)|_{\beta=1} = \frac{R_o(R_4 + R_3)}{R_3(R_4 + R_o)}$$

If $R_4 \gg R_o$

$$|H(j\infty)|_{\beta=1} \cong \frac{R_o}{R_3} > 1$$

Thus, when $\beta = 1$ we have amplification or "boost". When $\beta = 0$

$$|H(j\infty)|_{\beta=0} = \frac{R_3(R_4 + R_o)}{R_o(R_4 + R_3)}$$

If $R_4 \gg R_o$

$$|H(j\infty)|_{\beta=0} \cong \frac{R_3}{R_0} < 1$$

Thus, when $\beta = 0$ we have attenuation or "cut".
Also note that when $\beta = 0.5$

$$|H(j\omega)|_{\beta=0.5} = \frac{(0.5R_4 + R_o)(0.5R_4 + R_3)}{(0.5R_4 + R_3)(0.5R_4 + R_o)} = 1$$

Thus, the transition from amplification to attenuation occurs at $\beta = 0.5$. If $\beta > 0.5$ we have amplification, and if $\beta < 0.5$ we have attenuation.
Also note the amplification an attenuation are symmetric about $\beta = 0.5$. i.e.

$$|H(j\omega)|_{\beta=0.6} = \frac{1}{|H(j\omega)|_{\beta=0.4}}$$

Yes, the circuit can be used as a treble volume control because

- The circuit has no effect on low frequency signals
- Depending on β the circuit can either amplify ($\beta > 0.5$) or attenuate ($\beta < 0.5$) signals in the treble range
- The amplification (boost) and attenuation (cut) are symmetric around $\beta = 0.5$. When $\beta = 0.5$ the circuit has no effect on signals in the treble frequency range.

P 15.66 **[a]** $|H(j\infty)|_{\beta=1} = \dfrac{R_o(R_4+R_3)}{R_3(R_4+R_o)} = \dfrac{(65.9)(505.9)}{(5.9)(565.9)} = 9.99$

$\therefore$ maximum boost $= 20\log_{10} 9.99 = 19.99$ dB

[b] $|H(j\infty)|_{\beta=0} = \dfrac{R_3(R_4+R_3)}{R_o(R_4+R_o)}$

$\therefore$ maximum cut $= -19.99$ dB

[c] $R_4 = 500\,\text{k}\Omega; \qquad R_o = R_1 + R_3 + 2R_2 = 65.9\,\text{k}\Omega$

$\therefore \quad R_4 = 7.59R_o$

Yes, R_4 is significantly greater than R_o.

[d] $$|H(j/R_3C_2)|_{\beta=1} = \left|\frac{(2R_3+R_4)+j\frac{R_o}{R_3}(R_4+R_3)}{(2R_3+R_4)+j(R_4+R_o)}\right|$$

$$= \left|\frac{511.8 + j\frac{65.9}{5.9}(505.9)}{511.8 + j565.9}\right|$$

$$= 7.44$$

$20\log_{10}|H(j/R_3C_2)|_{\beta=1} = 20\log_{10} 7.44 = 17.43$ dB

[e] When $\beta = 0$

$$|H(j/R_3C_2)|_{\beta=0} = \frac{(2R_3+R_4)+j(R_4+R_o)}{(2R_3+R_4)+j\dfrac{R_o}{R_3}(R_4+R_3)}$$

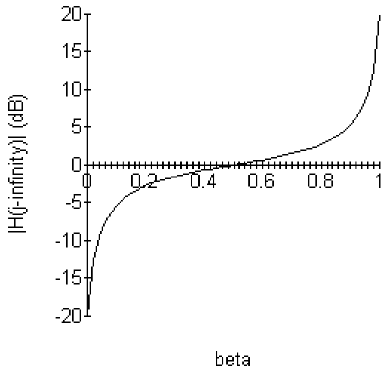
Note this is the reciprocal of $|H(j/R_3C_2)|_{\beta=1}$.

$\therefore \quad 20\log_{10}|H(j/R_3C_2)|_{\beta+0} = -17.43$ dB

[f] The frequency $1/R_3C_2$ is very nearly where the gain is 3 dB off from its maximum boost or cut. Therefore for frequencies higher than $1/R_3C_2$ the circuit designer knows that gain or cut will be within 3 dB of the maximum.

P 15.67 $$|H(j\infty)| = \frac{[(1-\beta)R_4+R_o][\beta R_4+R_3]}{[(1-\beta R_4+R_3][\beta R_4+R_o]}$$

$$= \frac{[(1-\beta)500+65.9][\beta 500+5.9]}{[(1-\beta)500+5.9][\beta 500+65.9]}$$


20
15
10
5
0
-5
-10
-15
-20
0
0.2
0.4
0.6
0.8
1
|H(j-infinity)| (dB)
beta