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Chapter 7
Kinetic Energy work.

* Kinetic energy (K) → mass, → speed

$$\boxed{K = \frac{1}{2} m v^2}$$

K ≡ scalar quantity (قياسية)

P5
$$\begin{bmatrix} K_F = \frac{1}{2} K_s \\ m_s = \frac{1}{2} m_F \end{bmatrix}$$ initial
f : father
s : son

$(K_f)_i = \frac{1}{2} (K_f)_f$

$\frac{1}{2} \cancel{m} (V_f)_i^2 = \frac{1}{2} \left[\frac{1}{2} \cancel{m} (V_{if} + 1)^2 \right]$

$V_{if} = ??$ ⟶ $(V_f)_f = V_{if} + 1 \cdot m/s$

$K_f = K_s$?

سبب العمل هو القوة المؤثرة
* work
- ① force ≡ constant (ثابتة) [gravitional force] قوة الجاذبية الأرضية
- ② force [variable] (متغيرة) [spring]

$\vec{f}$ ≡ constant ⟹ $\boxed{W = \vec{f} \cdot \Delta \vec{X}}$ ⟹ $\vec{f}$ = force, ΔX = displasement الإزاحة, W = work. } work scalar quantity

$[W] = N \cdot m$: Joule

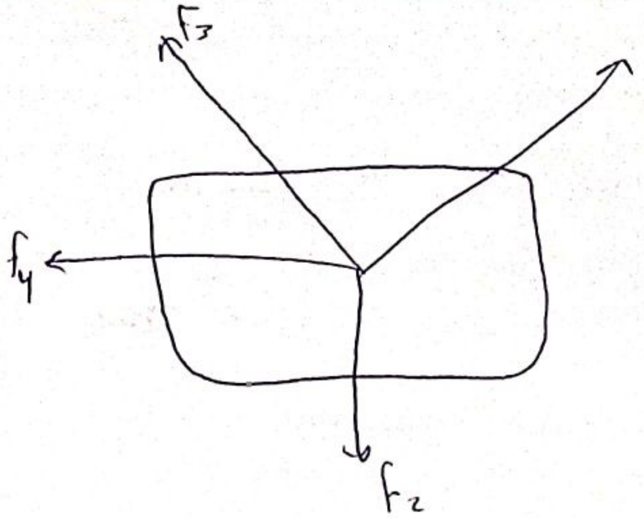
$W = |\vec{f}| \, |\Delta X| \cos \theta$ (θ بين DX و f)

$W = 0$ → $\vec{f} = 0$, → $\vec{\Delta X} = 0$, → $\vec{f} \perp \vec{\Delta X}$

$f = \frac{m v^2}{r}$

① $W_{net} = \vec{f}_{net} \cdot \vec{\Delta r}$

② $W_{net} = W_1 + W_2 + W_3 + \cdots$

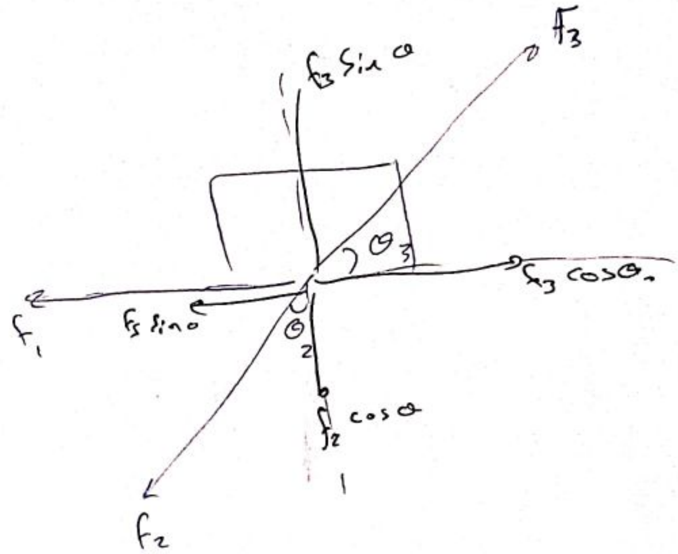


P [14] $f_1 = 3N$ $\theta_2 = 50°$

$f_2 = 4$ $\theta_3 = 35$

$f_3 = 9N$

$W = ??$



$W_{net} = \vec{f}_{net} \cdot \vec{\Delta x}$

$\sum f_x = f_3 \cos\theta_3 - f_2 \sin\theta_2 - f_1$

$= 9 \cos 35 - 4 \sin 50 - 3$

$= \boxed{1{,}24\,N}$

$\sum f_y = f_3 \sin\theta_3 - f_2 \cos\theta_2$

$= 9 \sin 35 - 4 \cos 50$

$= \boxed{2{,}6\,N}$

$f_{net} = \sqrt{(1{,}24)^2 + (2{,}6)^2} = \boxed{2{,}88}$

$W = f \cdot \Delta r$

$= 2{,}88 \cdot 4 = \boxed{11{,}5}$ Joul

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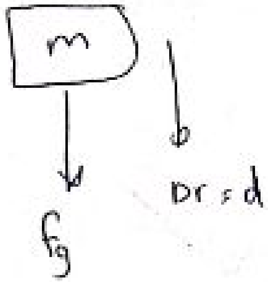
Exp: gravitional force

① $\vec{f_g} = mg$

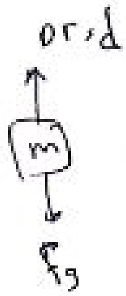
① $W_{f_g} = \vec{f_g} \cdot \vec{\Delta r}$

$= f_g \, d \cos\theta$

$\boxed{W_{f_g} = f_g \, d}$



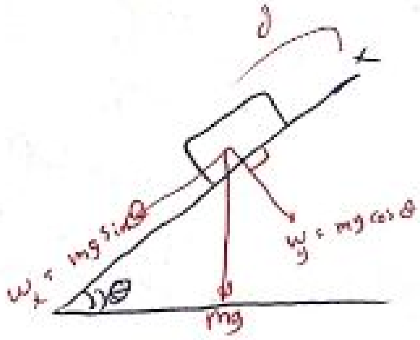
② $\boxed{W_{f_g} = -f_g \, d}$



$f_g \perp \Delta r$

$W_{f_g} = 0$

Case 3



① downword

$W_{w_y} = 0$

$W_{w_x} = w_x \cdot d$

constant speed (un) (+)

= —— (a = zero)

تناقص (-)

② upword $W_{w_x} = -w_x \cdot d$

Kientic energy - work theorem :-

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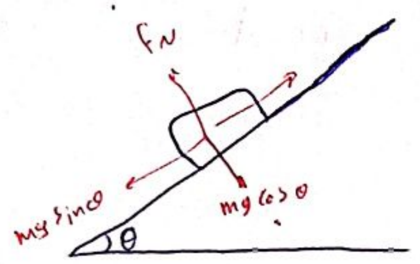
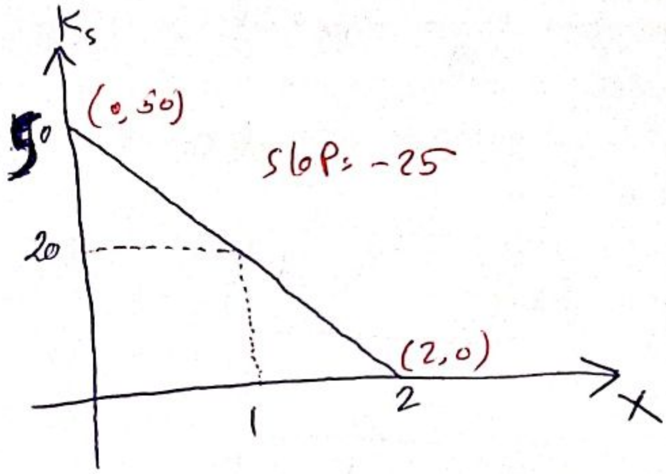
$$W_{net} = \Delta K$$

P_{20}





$v_i = 5\ m/s$

$f_N = ??$

$f_N = mg\cos\theta$

$= 4(9,8)\cos 39,6°$

$W_{net} = f_{net} \cdot \Delta X = \Delta K$

slope $= \frac{\Delta K}{\Delta X} = f_{net\ x} = -mg\sin\theta$

$25 = 4(9,8)\sin\theta$

$\boxed{\theta = 39,6}$

$K_i = 50\ J$

$50 = \frac{1}{2}m(5)^2$

$\boxed{m = 4\ kg}$

Part ②

work done by Variable

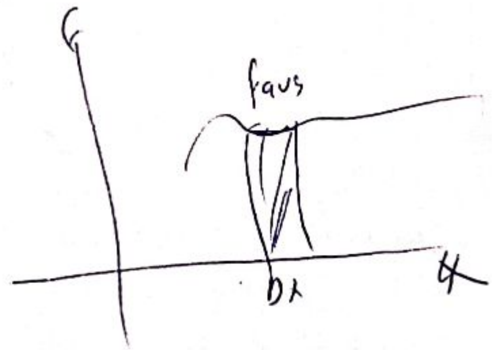
$\Delta X \longrightarrow f_{avg}$

$W_i = f_{avg} \cdot \Delta X$

$W = \sum_i f_{avg} \cdot \Delta X$

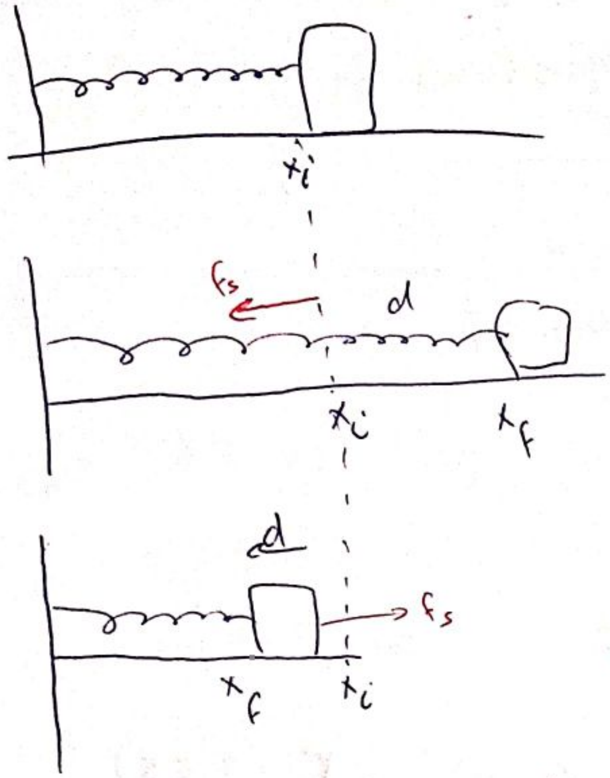
$W = \lim_{\Delta X \to 0} \sum f_{avg}\, \Delta X$

$$\boxed{W = \int_{x_i}^{x_f} F\, dx}$$



* Spring force قوة النابض

F_s عكس الازاحة



Hook's law.

$\vec{F} \propto \vec{d}$

$$\boxed{F_s = -K\vec{d}}$$

$K \equiv$ spring constant ثابت المرونة

$$W_{spring} = \int_{x_i}^{x_f} F_s \cdot dx$$

$$= \int_{x_i}^{x_f} -Kx \, dx$$

$$= \frac{1}{2}Kx \Big|_{x_f}^{x_i}$$

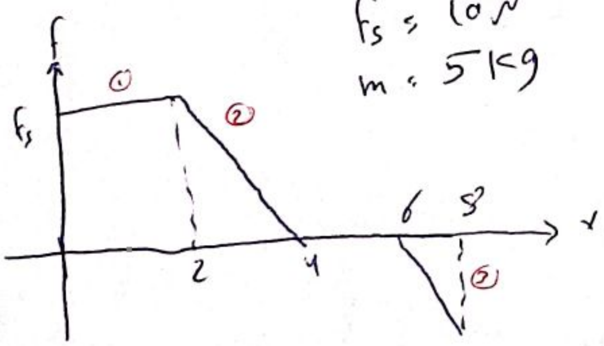
$$= \frac{1}{2}Kx_i^2 - \frac{1}{2}Kx_f^2$$

$$= \frac{1}{2}K\left(x_i^2 - x_f^2\right)$$

$$W_{applied} = -W_{sprin}$$

$$= \frac{1}{2}K\left(x_f^2 - x_i^2\right)$$

P36



$F_s = 10N$

$m = 5Kg$

$$W = \int_{x_i}^{x_f} F\, dx = Area(F\text{-}x)$$

$$F \xrightarrow{\int} W$$
$$W \xrightarrow{\frac{dW}{dt}} F$$

$$x_f = \sqrt{\frac{mv_i^2}{K}}$$

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عنان

force
- Constant $w = f \cdot d$ (gravitional force)
- Variable $W = \int_{x_i}^{x_f} F dx$ (spring force)
- $W_{net} = \Delta K$

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* $\vec{F}$ in 2D , 3D

$$\vec{F} = f_x \hat{i} + f_y \hat{j} + f_z \hat{k}$$

$$W = \int_{x_i}^{x_f} f_x\, dx + \int_{y_0}^{y_f} f_y\, dy + \int_{z_i}^{z_f} f_z\, dz$$

Exp1 $\vec{F} = 2x\,\hat{i} - 3y\,\hat{j} + \hat{k}$ $\vec{r}_1 = (0,2,4) \longrightarrow \vec{r}_2 = (2,3,5)$

$$W = \int_0^2 2x\, dx - \int_2^3 3y\, dy + \int_4^5 dz$$

$$= x^2 \Big|_0^2 - \frac{3y^2}{2}\Big|_2^3 + z\Big|_4^5$$

* work - Kentic theorem

$W_{net} = \int F_{net}\, dx$, $F_{net} = ma$, $a = \frac{dv}{dt}$

$$= \int ma\, dx = \int m \frac{dv}{dt} dx$$

$$\frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = v \frac{dv}{dx}$$

$$\int mv \frac{dv}{dx} dx$$

$$m \int_{v_i}^{v_f} v\, dv$$

$$W_{net} = m \left[\frac{v^2}{2}\Big|_{v_i}^{v_f}\right]$$

$$= \frac{1}{2} m (v_f^2 - v_i^2) = \boxed{\Delta K}$$

* Power (P)

$P_{avg} = \frac{\Delta W}{\Delta t} = \frac{J}{s}$

$[P] = \frac{J}{s} = watt$

$w = mgh$

$\Rightarrow P_{inst} = \frac{dW}{dt}$

$\boxed{P_{avg} = \vec{F} \cdot \vec{V}}$

$P = F \cdot V$ مهمة ($\frac{J}{s}$)

Discussion:

P_1: $m_p = 1{,}67 \times 10^{-27}$ kg

$a = 3{,}6 \times 10^{15}$ m/s²

$V_i = 2{,}4 \times 10^{7}$ m/s

$\Delta X = 3{,}5$ cm

@ final speed ⓑ ΔK

حل

@ $V_f^2 = V_i^2 + 2a\Delta X$

$V_f = \sqrt{(2{,}4 \times 10^{7})^2 + 2(3{,}6 \times 10^{15})(3{,}5 \times 10^{-2})}$

$= 2{,}9 \times 10^{7}$ m/s

ⓑ $\Delta K = K_f - K_i$

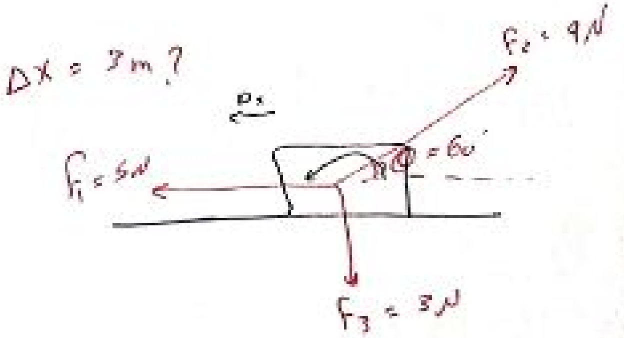
$= \frac{1}{2} m (v_f^2 - v_i^2)$ $= \frac{1}{2}[1{,}67 \times 10^{-27}][(2{,}9 \times 10^{7})^2 - (2{,}4 \times 10^{7})^2]$

$= \boxed{2{,}1 \times 10^{-13}\ J}$

P_{15}

@ $W_{net} = ??$ — $W_1 + W_2 + W_3$ / $W_{net} = f_{net} \cdot \Delta X$

$\Delta X = 3m$?



$W_1 = \vec{F} \cdot \Delta X = |F||\Delta x| \cos\theta$

$= 5(3) \cos 0 = 15\ J$

$W_2 = fd\cos\theta = 9(3)\cos 120 = -13{,}5\ J$

$W_3 = fd\cos\theta = 3(3)\cos 90 = 0\ J$

$W_{net} = 1{,}5$

(b) $\Delta K = W_{net}$

$= 1,5\ J$

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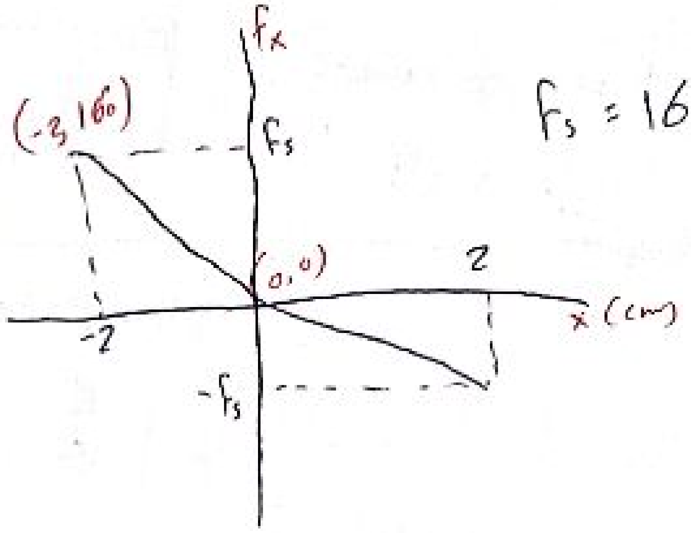
P32

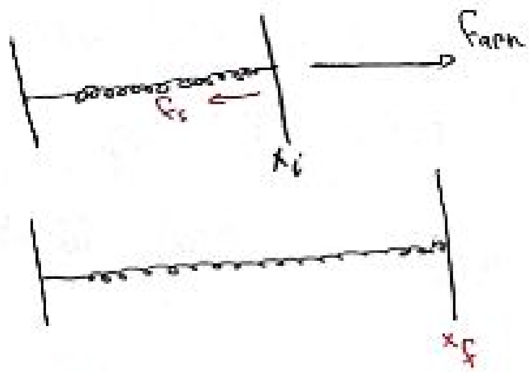
how much work does the spring do on the block moves from $x_i = 8$ cm to

(a) $x_f = 5$ cm (b) $x_f = -5$ cm

(c) $x_f = -8$ cm



$f_s = 160N$



$W_s = \frac{1}{2}K\left(x_i^2 - x_f^2\right)$

$f_s = -Kx$

$f_s = -K\ x$
y , slope x

slope $= -K = \frac{160-0}{-2-0} = -80$

$f_a = -f_s$

$\Rightarrow W_s = \int_{x_i}^{x_f} f\,dx$

$W_s = -W_a$

$K = \boxed{80}$ N/cm $\times 10^{-2}$

$K = 8000\ \frac{N}{m}$

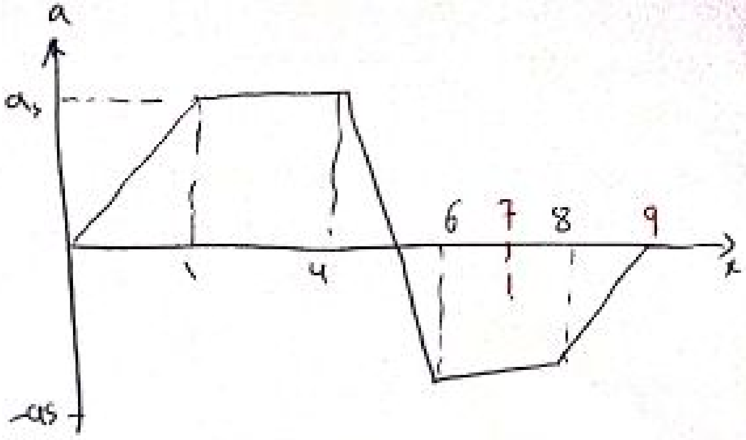
(a) $W_s = \frac{1}{2}(8000)\left[(8\times10^{-2})^2 - (5\times10^{-2})^2\right] = \boxed{15,6\ J}$

(b) $W_s = 15,6$

(c) $W_s = 0$

P 37

$m = 2\,kg$



$W_{net} = mv_f$

$W = \int f_{net}(x)\,dx$

$= \int m\,a(x)\,dx$

$W_{net} = m \int a(x)\,dx \rightarrow$ area

(a) $W_{net(4)} = 2\left[\frac{1}{2}(1)(6) + (3)(6)\right] = 42\,J$

(b) $W_{net(7)} = 30\,J$

(c) $W_{net(9)} = 12\,J$

(d) V at $x = 4$

$W_{net} = \Delta K$

$= K_f - \cancel{K_i}$

$42 = \frac{1}{2} m v_f^2 \Rightarrow v_f = \boxed{6.5\ m/s}$

V at $x = 7$

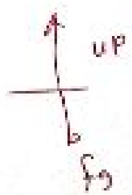
$30 = K_f - \cancel{K_i} \Rightarrow 30 = \frac{1}{2}(2) v_f^2 \Rightarrow v_f = \boxed{5.5}$

V at $x = 9$

$12 = \frac{1}{2}(2) v_f^2 \Rightarrow v_f = \boxed{3.5\ m/s}$

$P_{avg} = \frac{\Delta W}{\Delta t}$, $P_{inst} = \frac{dw}{dt}$, $P = \vec{f} \cdot \vec{v}$

P 45 $m = 3 \times 10^3\,kg$, $\Delta y = 210\,m$ ↑up, $\Delta t = 23\,sec$



$P = f \cdot v = fv\cos\theta$

$= mg\,V$

$|P| = mg \frac{\Delta y}{\Delta t}$

$= 3 \times 10^3 (9.8)\left(\frac{210}{23}\right) = 2.68 \times 10^3$

Good Luck

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~~$E_{tot} = E_m + E_{int}$~~

~~$m = 2kg$~~
~~$v = 3 m/s$~~
~~$\mu_k = 0.5$~~

Chapter 8

Potential energy (U)

$\Delta U = U_2 - U_1$ $\boxed{U_1 = 0}$

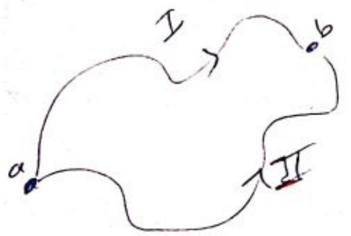
$\boxed{\Delta U = -W}$

① Isolated system ② Conservetev force

forces → conservative
القوة المحافظة

→ Non - conservative
القوة الغير محافظة

* الشغل ما بعتمد على المسار يعني الشغل ثابت — Conservative force : ① work dosenot depend on the Path



$-W_I = -W_{II}$ نفس المقدار وعكس الاشارة

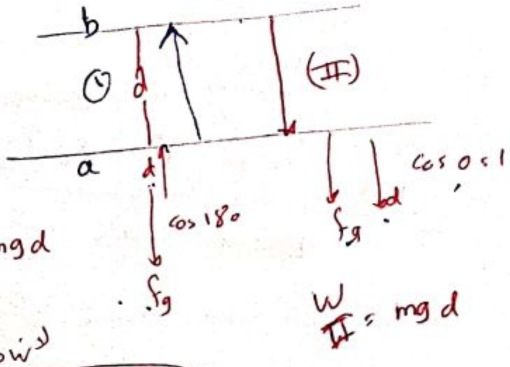
② W_{net} [~~clsd~~ closd Path] = 0

- Non - conservative forces : work depends on the Path الشغل بعتمد على المسار

K. friction — Drag force

انواع Conservavative forces

① gravitational force قوة الجاذبية الارضية



$W_{(I)} = -mgd$

$W_{II} = mgd$

لانها قوة محافظة conservative $\boxed{W_{net} = 0}$

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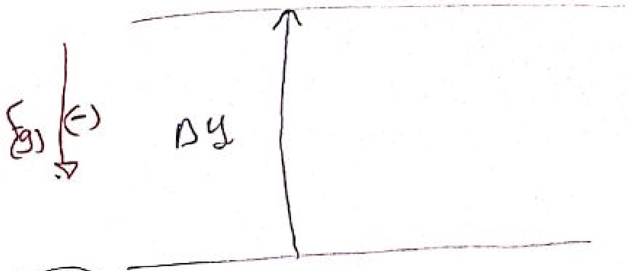
$$W = \int_{x_i}^{x_f} f(x)\,dx$$

$$\Delta U = -W = -\int_{x_i}^{x_f} f(x)\,dx$$

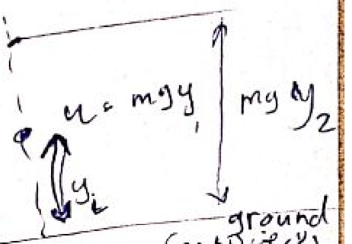
$$\boxed{U_2 - U_1 = -\int_{x_0}^{x_f} f(x)\,dx}$$

Case (I): gravitational force



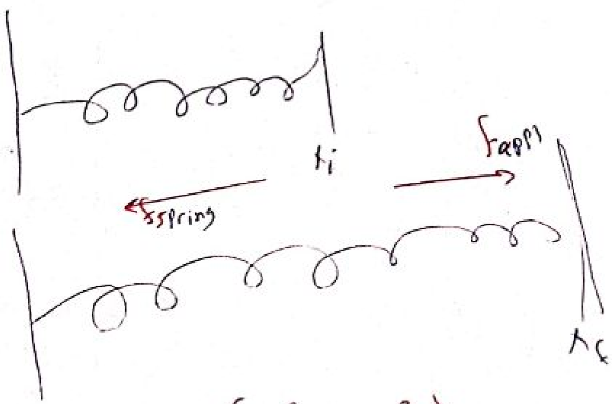
$$\Delta U = -\int_{y_i}^{y_f} f(y)\,dy$$

$$= \int_{y_i}^{y_f} mg\,dy \Longrightarrow \boxed{\Delta U = mg\,(y_f - y_i)}$$



Case (II) spring force

طردي مع x $\quad f_{spring} = -KX$ "Hook's law"



$$\Delta U = -\int_{x_i}^{x_f} f(x)\,dx$$

$$= \int_{x_i}^{x_f} \underline{Kx}\,dx$$

$$\boxed{\Delta U = \frac{1}{2}K\left(x_f^2 - x_i^2\right)}$$

$$\Delta U = -W_s$$

$$W_s = \frac{1}{2}K\left(x_i^2 - x_f^2\right)$$

$$\boxed{W_{app} = -W_s}$$

* Mechanical Energy (E)

$$\boxed{E_{mec} = K + U}$$

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- Isolated [no external forces]
- Conservative forces

E_{mec} = Conserved 'محفوظة'

$W = \Delta K \Rightarrow \Delta K = -\Delta U$

$W = -\Delta U$

$K_f - K_i = -(U_f - U_i)$

$K_i + U_i = K_f + U_f$

الطاقة الكلية ثابتة $\boxed{E_i = E_f}$ $\Delta E_{mec} = 0$

$E = \frac{1}{2} m w^2 a^2$

$w = \sqrt{\frac{K}{m}}$

$E = \frac{1}{2} K a^2$

$a = \sqrt{\frac{2E}{K}}$

$v = w a$

$h = \frac{u^2 \sin^2\theta}{2g}$

$v = \sqrt{2gL}$, $\frac{4v^2}{8}$, $5\checkmark$

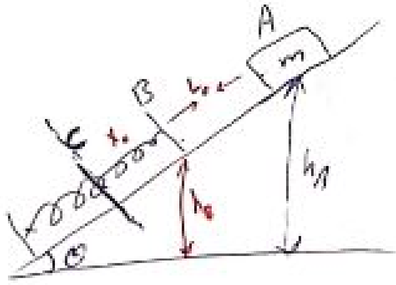
P(29) $m = 12 kg$

$v_i = 0$

$\theta = 30$, $X_o = 5,5$ cm

$X = 2$ cm, $f = 270 N$

$L_o + X_o = ??$



$f = -kx \Rightarrow k = \frac{f}{x}$

$= \frac{270}{2 \times 10^{-2}}$

$= \boxed{1,35 \times 10^4}$

نعوض

$\sin\theta = \frac{h_a}{L_o + X_o}$

$\sin 30 = \frac{0,174}{L_o + 5,5 \times 10^{-2}}$

$L = 0,292$ m

$E_A = E_C$

$\cancel{K_A} + U_A = \cancel{K_C} + U_C$

$0 + mgh_{(a)} = 0 + \frac{1}{2} K X^2$

$mgh_{(a)} = \frac{1}{2} K X_o^2$

$12(9,8) h_a = \frac{1}{2}(1,35 \times 10^4)(5,5)$

$\boxed{h_{(a)} = 0,174 \text{ m}}$

انظر $E_A = E_B$

$\cancel{K_A} + u_A = K_B + u_B$

0

$mgh_{(a)} = \frac{1}{2} m v_B^2 + mgh_{(B)}$

$Dy = h_A - h_B$

$l_0 \sin \theta = Dy$

$l_0 \sin \theta = h_A - h_B$

$\boxed{h_B =}$

ونعوض فوق ونرجع (م)

$\boxed{v_B = 1.7 \ m/s}$

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Chapter 8, 2

Reading a potential energy curve :-

ΔU(x)

$$\Delta U(x) = -W = -f(x)\cdot\Delta x$$

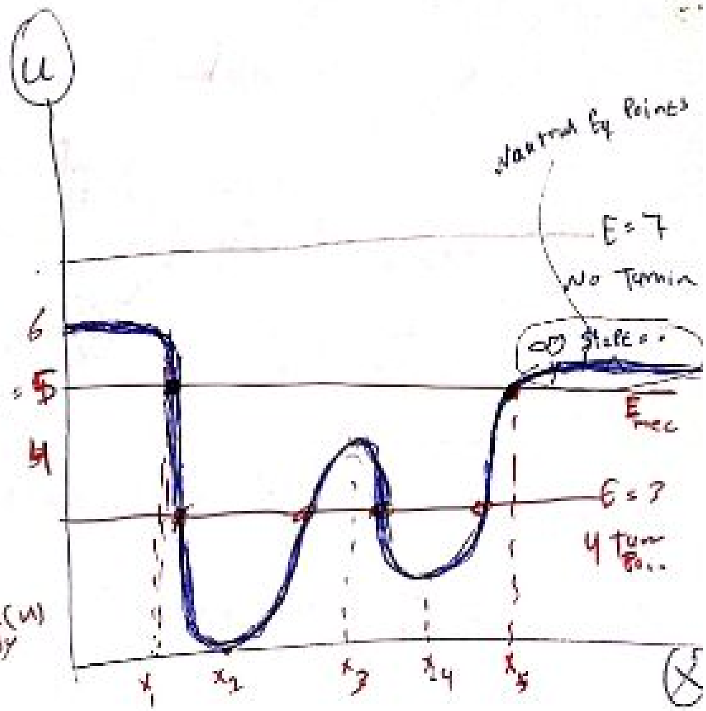
$$f(x) = \frac{-\Delta U}{\Delta x}$$

$$f(x) = \frac{-dU(x)}{dx}$$

فرضا

$E_{mec} = 5\ J$

$$E = K(x) + U(x)$$



* turning Point

النقطة التي تتحول كل E_{mec} الى Potential(U) energy

K = 0

$E = U$

$K(x) = 0$

X_5, X_1 = turning Points

$U = E$

$K(x) = 0$

$V(x) = 0$

$\boxed{V = 0}$

التقاطع مع U لذا $E = U$

Equilibrium Points :-

نقطة التعادل/الاتزان محصلة القوة صفر

$$\boxed{\vec{F}_{net} = 0}, \quad f = -\frac{du}{dx}$$

slope(u - x)

هي نقطتين max وال min لا لا

$$-\frac{du}{dx} = 0 \Rightarrow \left[\text{local min, local max}\right]$$

if $\vec{f} = 0$, slope = 0

الميل يساوي صفر X اتجاه مع

$x_2, x_3, x_4 \Rightarrow E_q$ points

* Eq. Points

① Stable Eq Points (min)

X_2, X_4 (stable)

② un-stable Eq Points. (غير ثابت) (max)

X_3 (un-stable)

Exp $f(x) = x^2 - 2x$

$\left.\frac{df}{dx}\right|_{x_0} > 0$ max

$\left.\frac{df}{dx}\right|_{x_0} < 0$ min

$u(x)$

① Eq. Points $\left(f = 0 \Rightarrow \frac{du}{dx} = 0\right)$

→ stable (مستقر)

→ unstable

② Turning Points $(K = 0)$ $(v = 0)$

$E = u(x)$; $x \in$ turning Points

Exp: if $u(x) = x^2 - 2x$, and $E_{mec} = 4\,J$ find

① Turning Points, ② Eq. Points.

الحل ① $E_{mec} = u(x)$

$4 = x^2 - 2x$

$x^2 - 2x - 4 = 0$

$x = \frac{b \mp \sqrt{b^2 - 4ac}}{2a}$

$= \frac{2 \mp \sqrt{4 - 4(1)(-4)}}{2}$

$\frac{2 \mp \sqrt{20}}{2} \Rightarrow x_1 = \frac{2 + \sqrt{20}}{2} = \boxed{3.2}\,m$

$x_2 = \frac{2 - \sqrt{20}}{2} = \boxed{-1.2}\,m$

② Eq. Points

$f = 0 \Rightarrow \frac{du}{dx} = 0$

$2x - 2 = 0 \rightarrow \boxed{x = 1\,m}$

لو تجمعون 4 ما توجد

u': $- - - - -$ | $+ + + +$

local min [Stable Points]

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$$f(x) = -\frac{du}{dx}$$

$$\Delta u = \int f\,dx = -W$$

$\boxed{\Delta u = -W}$ - Isolated
- Conservative forces.

Exp $u(x,y) = x^2 y + 2x$

$$f(x) = \frac{-du}{dx} = -(2xy + 2)$$

$$\vec{F} = (-2xy + 2)\hat{i} - x^2\hat{j}$$

$$f(y) = \frac{-du}{\partial y} = -(x^2 + 0)$$

إذا $\left(\begin{array}{l} x=1 \\ y=2 \end{array}\right.$

$$\vec{F} = -4\hat{i} - \hat{j}$$

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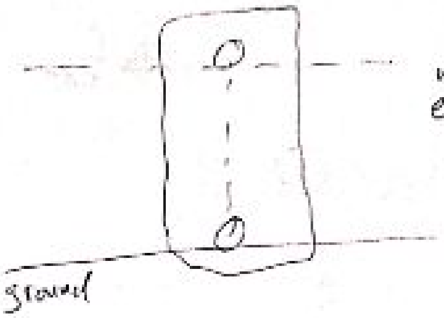
review :-

* Isolated system : نظام معزول

[conservative force] قوة محافظة

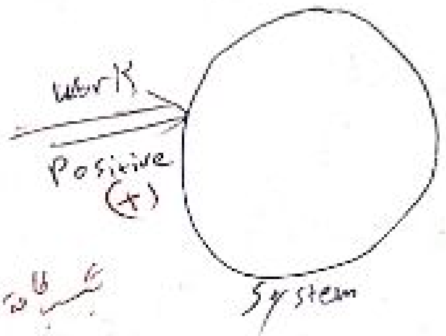
$E_{mec} = K + U$ $W = 0$

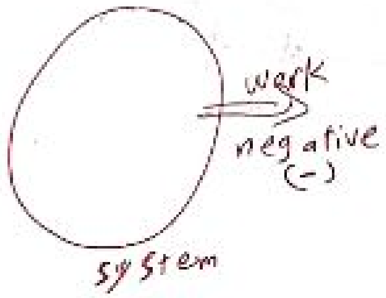
$\Delta E_{mec} = \Delta K + \Delta U = 0$



with not external force.

* work : is energy transfered to or from the system when external force acts on that system:





* non - Isolated system

($F_{external}$)

case I → No. fraction [conservative force]

case II → friction [non-conservative force]

c ← u

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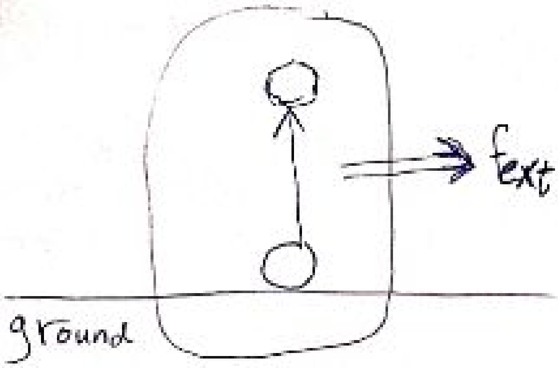
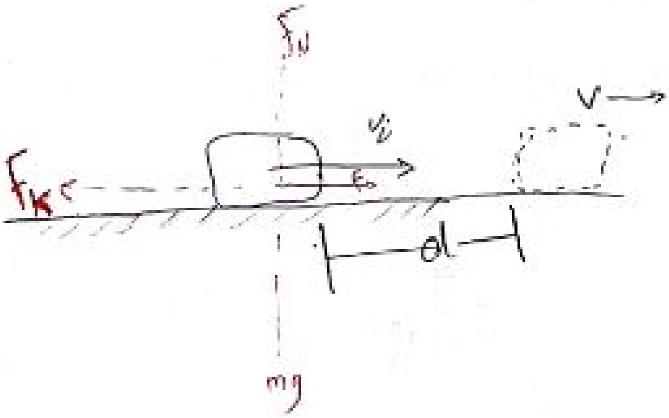
⇒ Non-Isolated system (without friction).
غير معزول بدون احتكاك



$W = \Delta E_{mec}^{\neq 0}$

$= \Delta K + \Delta U$

⇒ Non-Isolated system with friction:



$F_{net} = ma$

$\boxed{F - F_k = ma}$

to find (a) → معادلات حركة

$v^2 = v_0^2 + 2ad$

$2ad = v^2 - v_0^2$

$\boxed{a = \frac{1}{2}\frac{(v_f^2 - v_0^2)}{d}}$

$\boxed{Fd = \underbrace{\tfrac{1}{2}mv^2 - mv_0^2}_{\Delta K} + F_k d}$

$Fd = \Delta K + F_k d$

$Fd = \Delta E_{mec} + F_k d$

$\boxed{\Delta E_{mec} = \Delta K}$

$\boxed{W = \Delta E_{mec} + F_k d}$

$-W_{friction}$

ΔE_{th}

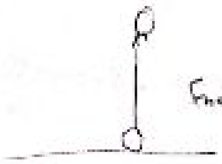
thermal energy

تلخيص

$\boxed{\text{Isolated System}}$ ($W = 0$)

No friction

$\Delta E_{mec} = 0$



friction

$\Delta E_{mec} + \Delta E_{th} = 0$

$\boxed{\text{NoN - Isolated system}}$

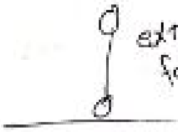
$W \neq 0$

$\Delta E_{int} = 0$

No friction

$W = \Delta E_{mec}$

$= \Delta K + \Delta U$



friction

$W = \Delta E_{mec} + \Delta E_{th}$

---

* Conservation of energy

$W = \Delta E_{mec} + \Delta E_{th} + \Delta E_{int}$

* Isolated system (friction)

$W = 0$

$\Delta E_{mec} + \Delta E_{th} = 0$

$\Delta U + \Delta K + f_k d = 0$

$U_f - U_i + K_f - K_i + f_k d = 0$

$\boxed{U_f + K_f = U_i + K_i} - f_k d \rightarrow W_{friction}$

لا يوجد friction — مع friction

Power = P [watt]

$P_{int} = \frac{dE}{dt}$ $P = \frac{\Delta W}{\Delta t}$

$P_{avg} = \frac{\Delta E}{\Delta t}$

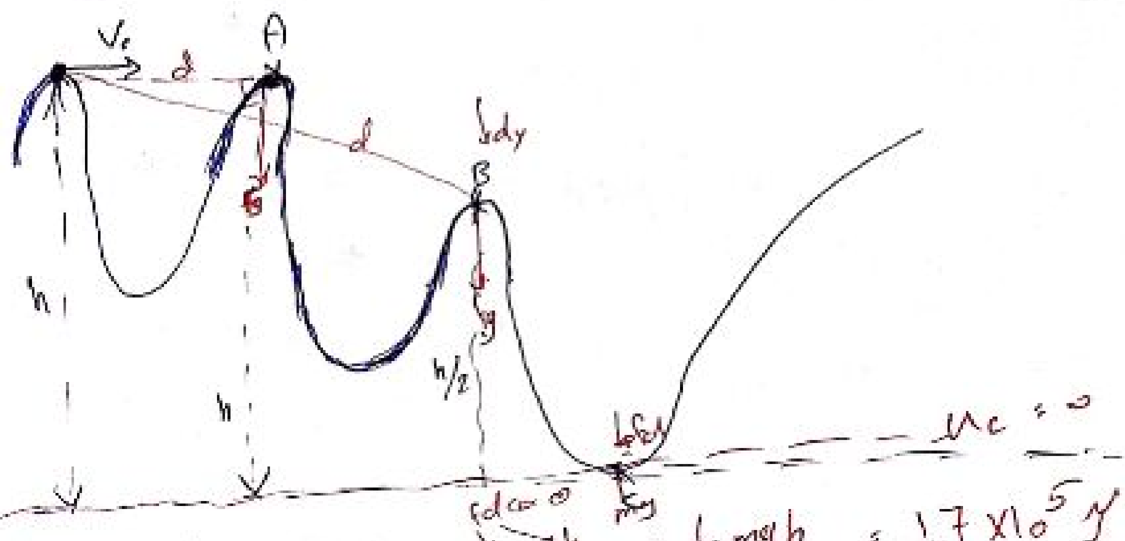
Dissecatione

P2

$v_0 = 17\ m/s$

$m = 825\ kg$

$h = 42\ m$



(a) W_A, W_B, W_C $W_B = mg\frac{h}{2} = \frac{1}{2}mgh = 1.7 \times 10^5\ J$

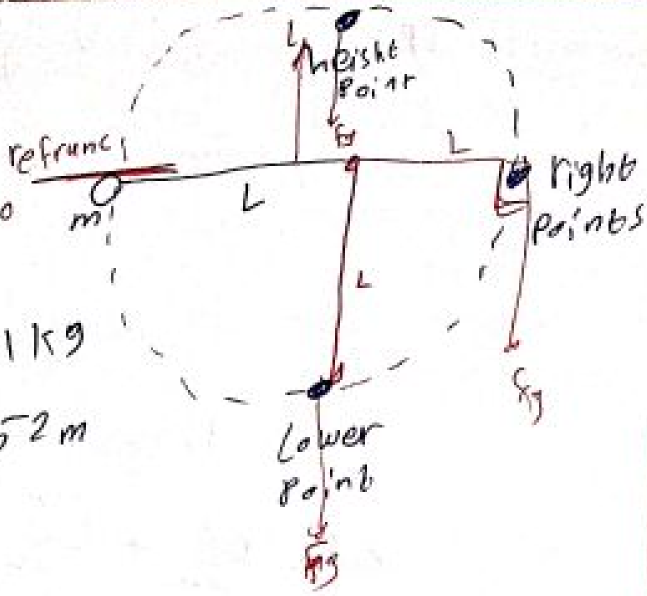
$W = Fd\cos 90 = \boxed{0\ J}$ $W_C = Fd\cos\theta = mgh = 3.4 \times 10^5\ J$

(b) U_B, U_A والنقطة c مرجع

$U_C = 0$ $\Delta U_{BC} = -W$ OR $U = mg\Delta y$

$U_B = mg\frac{h}{2} = 1.7 \times 10^5\ J$

$U_A = mgh = 3.4 \times 10^5\ J$



$m = 0.341\ kg$

$L = 0.452\ m$

(a)

$W_{lower} = F_g \cdot d = mgL = 1.51\ J$

$W_H = -mgh = -1.51\ J$

$W_R = 0$

(b) $\Delta U = -W$

$\Delta U_L = -1.51\ J$

$\Delta U_R = 0$

$\Delta U_H = 1.51\ J$

$K_i + U_i = K_f + U_f$

$K_i = \frac{1}{2}(0.9)(7)^2 = \boxed{22.1\ J}$

$22.1 + 15 = \frac{1}{2}(0.9)(v_f^2) + 35$

$v_f^2 = \frac{(37.1 - 35) \times 2}{0.9}$

$\boxed{v_f = 2.1\ m/s}$

(b) $F_x = -\frac{\Delta U}{\Delta x} = -\text{slope}$

from $x=0$ to $x=2$ $\boxed{F = 0}$

$F_x = -\left(\frac{15 - 35}{4 - 2}\right) = \boxed{10\ N}$

(c) $K_i + U_i = K_f + U_f$

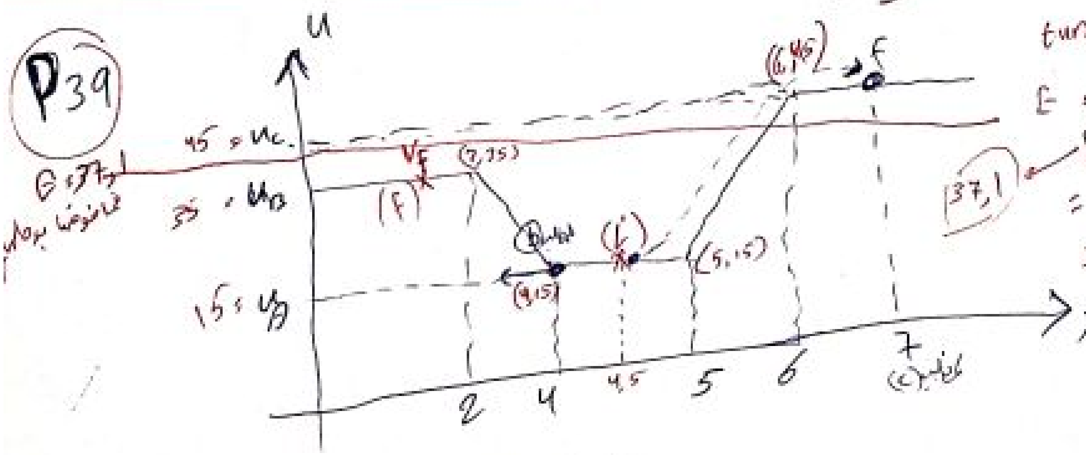
$37.1 = \frac{1}{2}(0.9)(v_f^2) + 45$

$v_c^2 = \frac{(37.1 - 45) \times 2}{0.9} < 0$

$\cancel{v_f}$

الحل

P39



turning points ($v = 0$, $K = 0$)

$E = \cancel{K}^{0} + U$

$\boxed{37.1} \leftarrow E = U(x)$

$= 15 + 30(x - 5)$

$y - y_1 = m(x - x_1)$ معادلة الخط المستقيم

$37.1 = 15 + 30(x - 5)$

$\boxed{x = 5.7\ m}$

$U_A = 15\ J$

$U_B = 35\ J$

$U_C = 45\ J$

$m = 0.9\ kg$

at $x = 4.5 \Rightarrow \boxed{v_i = 7\ m/s}$

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Isolated system ($f_{ext} = 0$)

$W_{ext} = 0$

no-friction

$W = \Delta E_{mec} = 0$

$K_i + U_i = K_f + U_f$

friction

$W = \Delta E_{mec} + \Delta E_{th} = 0$

$\Delta E_{th} = f_k \cdot d$

$K_i + U_i = K_f + U_f + f_k d$

Non Isolated ($f_{ext} \neq 0$, $W_{ext} \neq 0$)

No friction

$W = \Delta E_{mec}$

$= \Delta K + \Delta U$

friction

$W = \Delta E_{mec} + \Delta E_{th}$

$= \Delta K + \Delta U + f_k d$

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P19

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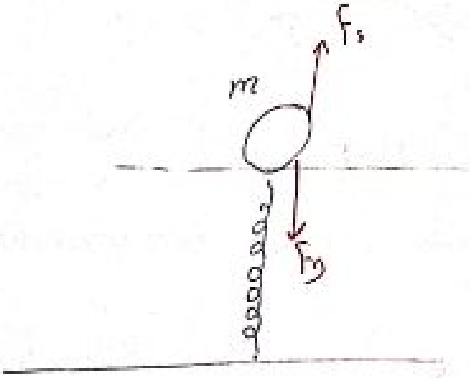


$m = 8\,Kg$,

$v_i = 0$ (at rest a=0)

$\Delta y = 10\,cm$

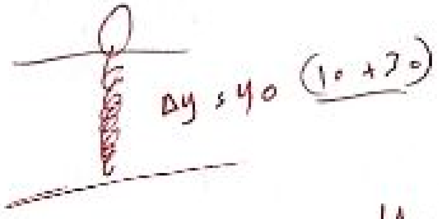
ⓐ $K = ??$

$f_s - f_g = 0$

$K\Delta y = mg$

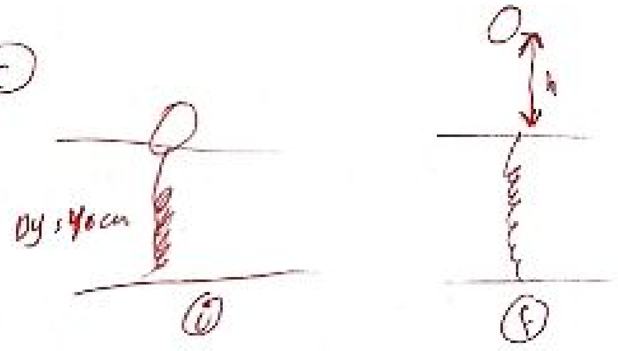
$K = \frac{mg}{\Delta y} = 7,84\ N/m$

ⓑ



$U_s = \frac{1}{2}Ky^2 = \boxed{62,7\ J}$

ⓒ



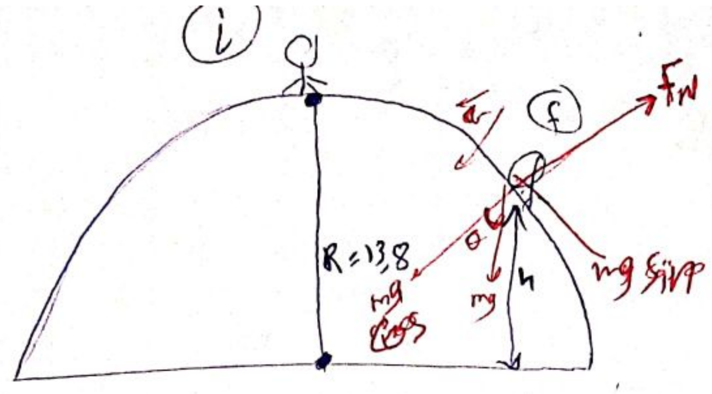
$\boxed{\cancel{K_i} + U_i = \cancel{K_f} + U_f}$ محفوظ بدون احتكاك

0 0

$62,7 = mgh$

$h = \frac{62,7}{mg} = \boxed{0,8\,m}$

34



at rest

$\cancel{K_i} + U_i = K_f + U_f$

0

$\cancel{m}gR = \frac{1}{2}\cancel{m}v_f^2 + \cancel{m}gh$??

$\cancel{mg}R = \frac{1}{2}\cancel{m}R\cancel{g}\cos\theta + \cancel{mg}h$

$R = \frac{1}{2}R\cos\theta + h$

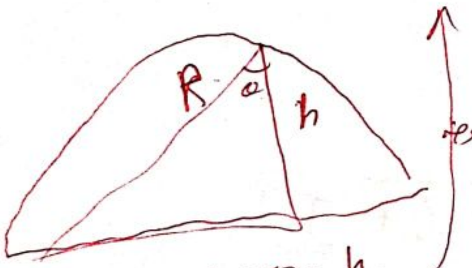
$(f=0)$ نعد منفلت

$a_r \frac{v^2}{R}$

$mg\cos\theta - F_N = \frac{mv^2}{R}$

$\cancel{m}g\cos\theta = \frac{\cancel{m}v^2}{R}$

$v^2 = Rg\cos\theta$



$\cos\theta = \frac{h}{R}$

نعوض

$\boxed{h = 9.2m}$

الحل

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Chapter 8

Potential Energy & Conseration of Energy

forces in Nature :

قوة المحافظة

① Gravitational force = mg
② Spring force = $-Kx$

→ Conservative forces

U Potential ~~Force~~ Energy
طاقة الوضع.

$U_s = \frac{1}{2}Kx^2$ Jon
الزنبرك

$U_g = mgy$

③ Normal force = $f_n = N$
④ friction force $f_s = f_{sm} = \mu_s N$
$f_k = \mu_k N$
⑤ Air Drag force = $\frac{1}{2} c \rho A v^2$

→ Non ~~conser~~ conservative force

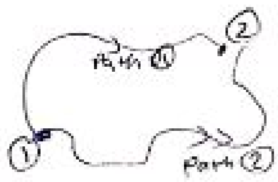
خواص/ صفات

* Properties of conservative forces :-

① Work done by $f_{\text{conservation}}$ is Path independent

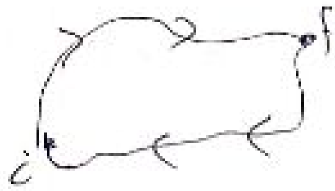
شغل القوة المحافظة لا يعتمد على المسار

$W_{con\,1} = W_{con\,2}$



② Work done by $f_{con} = 0$
$i \to i$

$W_{cons} = 0$ around a closed path



③ Work done against $f_{\text{conservative}}$ do not ~~...~~ lost ~~...~~ But it Stored as
لا يضيع
Energy called Patential Energy.

الشغل المبذول ضد القوى المحافظة لا يضيع وإنما يخزن على شكل طاقة وضع

$$W_{app\,(i \to f)} = \Delta U = U_f - U_i$$

(A) W done by $f_{conn} = -\Delta U$
شغل القوة المحافظة

$$W_{\substack{f_{cons}\\ i\to f}} = -\Delta U = -[U_f - U_i]$$

(B) Gravitational Potential Energy.

$$W_{\substack{f_{con}\\ i\to f}} = -\Delta U = -[U_f - U_i] ;$$

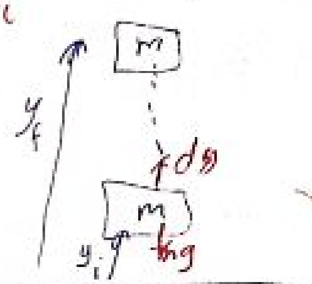
$$\boxed{\int_{r_i}^{r_f} \vec{F}_{cons} \cdot \vec{dr} = -\Delta U}$$
شغل القوة المحافظة = طاقة الوضع (الكامنة)

$$\Delta U = -\int_{r_i}^{r_f} \vec{F}_{cons} \cdot \vec{dr}$$

$$\boxed{U_f - U_i = -\int_{r_0}^{r_f} \vec{F}_{cons} \cdot \vec{dr}}$$

(B)

$$\int_{r_i}^{r_f} \vec{F} \cdot dr = \int_{x_i}^{x_f} F_x\,dx + \int_{y_i}^{y_f} F_y\,dy + \int_{z_i}^{z_f} F_z\,dz$$



$$U_f - U_i = -\int_{r_i}^{r_f} \vec{F_g} \cdot \vec{dr}$$

$$= -\int_{y_i}^{y_f} (-mg\,dy)$$

$$= mg \int_{y_i}^{y_f} dy$$

$$U_f - U_i = mg\,y_f - mg\,y_i$$

let $y_i = 0$, $U_0 = 0$

$$\boxed{U_g = mgy} \quad \text{Joul}$$

y is above zero level

(C) Spring Potential Energy ;

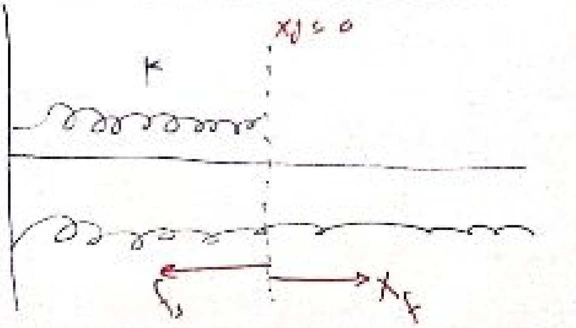
$$U_f - U_i = -\int_{r_i}^{r_f} \vec{F_s} \cdot \vec{dr}$$

$$U_f - U_i = -\int_{x_i}^{x_f} (-Kx)\,dx = K \int_{x_i}^{x_i} x\,dx$$

$$U_f - U_i = \frac{1}{2} K x_f^2 - \frac{1}{2} K x_i^2$$

let $x_i = 0 \Rightarrow U_0 = 0$

$$\boxed{U_s = \frac{1}{2} K x^2} \quad x_i = 0$$



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$$E_{mechanical} = \text{Kinetic Energy} + \text{Potential Energy}$$

$$E_m = \frac{1}{2}mv^2 + U$$

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✱ Conservation of Mechanical Energy :-

قانون حفظ الطاقة الميكانيكية

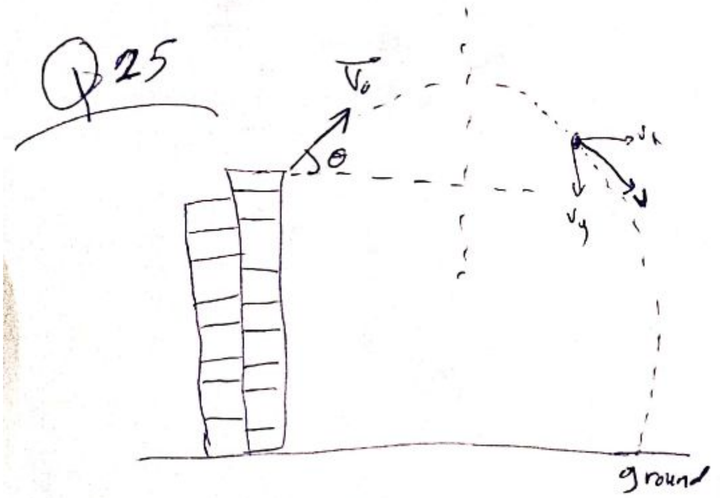
If the only force Acting on the System is Conservative force.

$$\boxed{\begin{array}{l} W_{cons} = -\Delta U \\ W_{net} = \Delta K \end{array}} \Longrightarrow \begin{array}{l} \Delta K = -\Delta U \\ \Delta K + \Delta U = 0 \end{array}$$

قانون حفظ الطاقة الميكانيكية

$$\boxed{\begin{array}{l} \Delta(K+U) = 0 \\ K + U = \text{constant} \\ (K+U)_i = (K+U)_f \end{array}}$$

Q25



الي at 6s

$v_x = 18\,m/s$

$v_y = v_{0y} + a_y t$
$= 24 + (-10)(6)$
$= 24 - 60$
$= -36\,m/s$

$V_6 = 18\hat{i} - 36\hat{j}\ m/s$

$m = 1\,kg$ At $t=0$, $\vec{V_0} = 18\hat{i} + 24\hat{j}\ m/s$
After 6s find ΔU?

$(K+U)_i = (K+U)_f = K_0 + U_0 = K_6 + U_6$

$= K_0 - K_6 = U_6 - U_0 \Longrightarrow U_6 - U_0 = 450 - 768 = -318\,J$

$K_0 = \frac{1}{2}mv_0^2$
$= \frac{1}{2}(1)(18^2+24^2) = \boxed{450\,J}$

$K_6 = \frac{1}{2}mv_6^2$
$= \frac{1}{2}(1)(18^2+(-36)^2)$
$\boxed{768\,J}$

$w_g = -\Delta U = 318\,J$

Chapter (8)
Lecture (2)

$$F_{\text{conservation}} \begin{cases} mg \Rightarrow U_g = mgy \ (J) \\ -Kx \Rightarrow U_s = \frac{1}{2}Kx^2 \ (J) \end{cases}$$

$$E_{\text{initial}} = E_{\text{Final}}$$

$$\boxed{\frac{1}{2}mV_i^2 + U_i = \frac{1}{2}mV_f^2 + U_f}$$

F is conservative

$$\boxed{W_{\text{conservative}} = -\Delta U}$$

مهمة

✱ Finding F_{cons} from U :-

$$W_{\text{con}} = -\Delta U$$

$$dU = -F_x\, dx$$

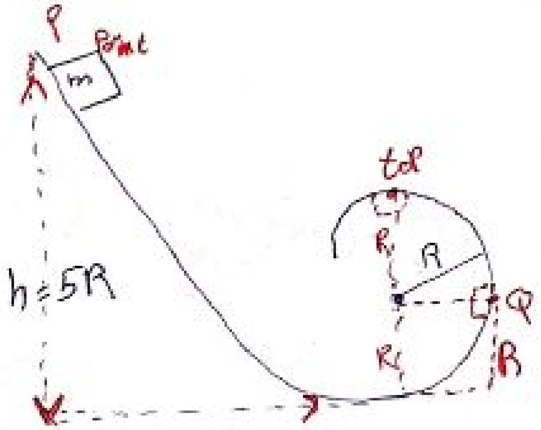
إذا بعطيني u مطلوب f

$$\boxed{F_{x\,\text{cons}} = -\frac{dU}{dx}}$$

مهمة

Problem 8, 6 .

$m = 0.32\text{ kg}$
$R = 12\text{ cm}$



ⓐ $W_g\,(P \to Q)$

$$W_g\,(P \to Q) = -\Delta U$$

$$= -[U_Q - U_P]$$

$$= -[mgR - mg(5R)]$$

$$= +4mgR = \boxed{0.150\ J}$$

ⓑ $W_g\,(P \to \text{top}) = -\Delta U$

$$= -[U_{\text{top}} - U_P]$$

$$= -[mg(2R) - mg(5R)]$$

$$= +3mgR = \boxed{0.113\ J}$$

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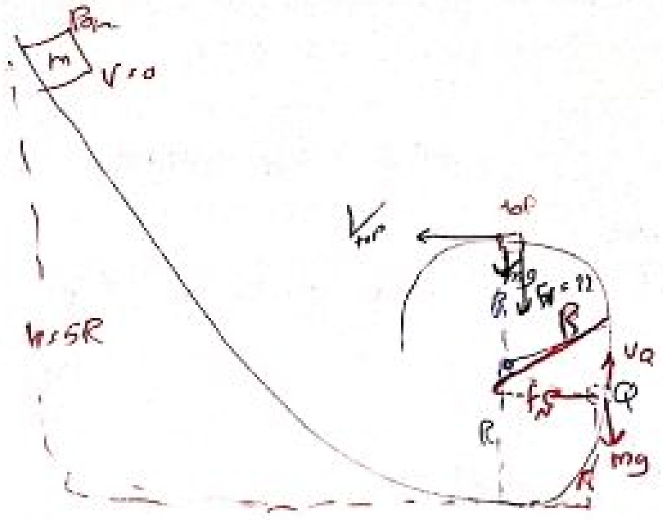
Problem 8 - 17



ⓐ At Q find $\vec{F_N}$? $\vec{F_V}$?

$$\boxed{F_y = -mg}$$

$$\cancel{K} = \frac{1}{2} m v_p^2 + U_p = \frac{1}{2} m v_Q^2 + U_Q$$

$$0 + 5\cancel{m} g R = \frac{1}{2} \cancel{m} v_Q^2 + m g R$$

$$\frac{1}{2} V_Q^2 = 4Rg \Rightarrow V_Q = \sqrt{8Rg}$$

$$F_N = \frac{m V_Q^2}{R} = \frac{m(8Rg)}{R} = 8mg = (\quad)(-\hat{i})$$

$$\vec{F_Q} = -8mg\hat{i} - mg\hat{j}$$

find the Normal force acting on m at the top point

at the top

$$\boxed{F_N + mg = \frac{m V_t^2}{R}}$$

$$(K + U)_p = (K + U)_t$$

$$(0 + 5mgR) = \frac{1}{2} m v_t^2 + mg(2R) \Rightarrow \boxed{V_t = \sqrt{6Rg}}$$

At the top

$$F_N = \frac{m V_t^2}{R} - mg$$

$$F_N = \frac{m}{R} 6Rg - mg \Rightarrow F_N = 5mg \text{ at the top}$$

$$\boxed{\vec{F_N} = -5mg\hat{j}}$$

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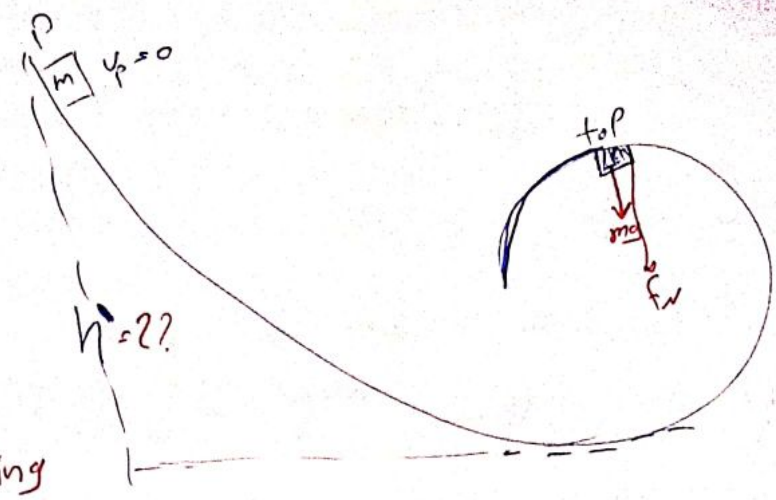
(c) find h ??



At the top

$$\frac{mv^2}{R} = mg + f_N$$

on the verge of losing contact at the top means $\boxed{f_N \longrightarrow 0}$

$$\frac{mv_t^2}{R} = mg \Rightarrow v_t = \sqrt{Rg}$$

$$K_P + U_P = K_t + U_t$$

$$0 + mgh' = \tfrac{1}{2}mv_t^2 + 2mgR$$

$$mgh' = \tfrac{1}{2}m(Rg) + 2mgR$$

$$\cancel{mg}h' = 2.5\cancel{mg}R \Rightarrow \boxed{h' = 2.5R}$$

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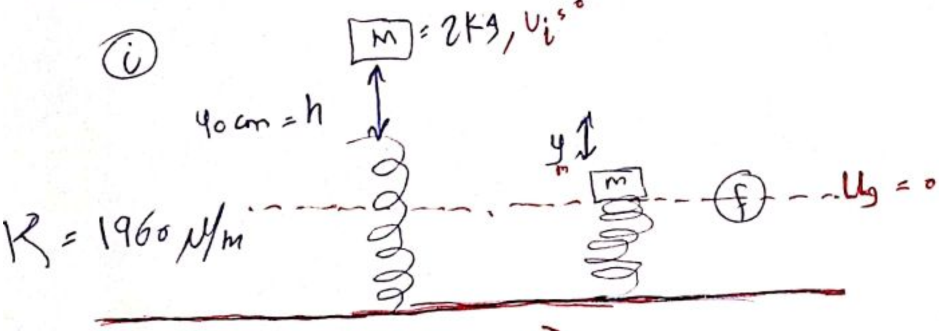
Problem 8 - 24

(i)



$$(K+U)_i = (K+U)_F$$

$$\left(0 + mg(h + y_m)\right) = \left(0 + \tfrac{1}{2}Ky_m^2\right)$$

$$2(9.8)[0.4 + y_m] = \tfrac{1}{2}(19600)y_m^2 = \boxed{}$$

Problem 8 - 104

$m = 20kg$, $f_{con} = -3x - 5x^2$

at $x = 0$, $U_0 = 0$

(a) find U at $x = 2m$?

$W_{con} = -\Delta U \longrightarrow \Delta U = -\int_i^f f_{con}\, d_x$

$U_f - U_i = -\int(-3x - 5x^2)\,dx$

$U = \frac{3}{2}x^2 + \frac{5}{3}x^3 + C$

$0 = 0 + 0 + C$

$\boxed{C = 0}$

$U = \frac{3}{2}x^2 + \frac{5}{3}x^3$

$U(2) = \frac{3}{2}(2)^2 + \frac{5}{3}(2)^3 = 19,6\ J$

(b) At $x = 5m$, $V_x = -4\ m/s$

find V_x at $x = 0$??

$(K + U)_{x=0} = (K + U)_{x=5}$

$\frac{1}{2}mV_0^2 + 0 = \frac{1}{2}m(-4)^2 + \left[\frac{3}{2}(5)^2 + \frac{5}{3}(5)^3\right]$

$V_0 = \sqrt{}$

$V_0 = -6,37\ m/s$

(c) Repeat a & b for $U = -8J$ at $x = 0$

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Problem $m = 0,2 kg$, $U(x) = 8x^2 + 2x^4$ Joule

at $x = 1m$, $v = 5$ m/s

find v at the origin ?

$$(K+U)_1 = (K+U)_0$$

$$\frac{1}{2}(0.2)(5)^2 + (8(1)+2(1)) = \left(\frac{1}{2}(0.2)v_0^2 + 0\right)$$

b) find F_{cons} ?

$$F_{cons} = \frac{-dU}{dx} = -\left[6x + 8x^3\right]$$

$$F_{cons} = -16x - 8x^3$$

End ch8

Good Luck

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Chapter 9

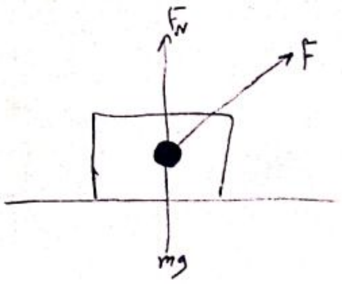
Center of mass, linear momentum:

مركز الكتلة — كمية التحرك الخطي (الزخم)

* center of mass: is the points that moves as though, all of the system's mass concentrated there, and all external force applied there.

- محصلة القوة لها عاي النقطة
- كتلة الجسم مركزة في تلك النقطة

مجموعة الأجسام — مركزة — القوى تؤثر عند النقطة



* Two cases:
- → system of Particl (مجموعة من الأجسام)
- → Rigid (solid) bodies. (أجسام صلبة)

Case I: system of particles

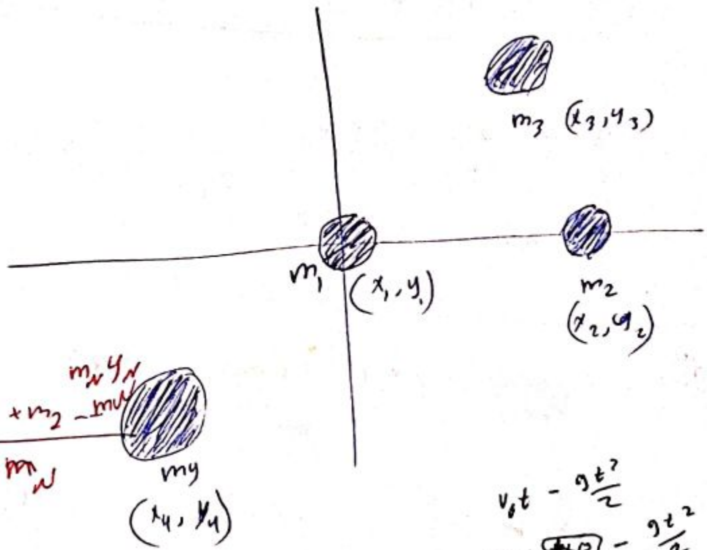
$$X_{com} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$$

$$\Rightarrow X_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i x_i$$

(M: total mass)

$$\Rightarrow y_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i y_i = \frac{m_1y_1 + m_2y_2 + m_2 \cdots m_N y_N}{m_1 + m_2 \cdots m_N}$$

$$\Rightarrow Z_{com} = \frac{1}{m}\sum_{i=1}^{N} m_i Z_i$$



$v_0 t - \frac{gt^2}{2}$

$25 + \boxed{\times 0} - \frac{gt^2}{2}$

$v = gt$

$$\vec{r}_{com} = x_{com}\hat{i} + y_{com}\hat{j} + Z_{com}\hat{k}$$

$$r_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i \vec{r}_i$$

$\underline{P_1}$ $m_1 = 2kg \longrightarrow (-1, 5) \longrightarrow r_1 = -\hat{i} + 5\hat{j}$

$m_2 = 4kg \longrightarrow (6, -7.5)$

$m_3 = 3kg \longrightarrow ??\ (x_3, y_3)$

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$r_{com} = (-0.5, -0.7)$

$\downarrow x_{com}$ $\downarrow y_{com}$

$$X_{com} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3^{??}}{m_1 + m_2 + m_3}$$

$$\frac{-0.5}{1} = \frac{-2 + 24 + 3x_3}{2+4+3} \Longrightarrow \boxed{X_3 = -1.5\,m}$$

$-4.5 = 22 + 3x_3$

$-22 \quad -22$

$$Y_{com} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{9}$$

$$-0.7 = \frac{10 + 30 + 3y_3}{9} \Longrightarrow \boxed{y_3 = -1.43\,m}$$

$-6.3 = 40 + 3y_3$

$-40 \quad -40$

$\frac{-46.3}{3} = \frac{3y_3}{3}$

$y_3 =$

OR

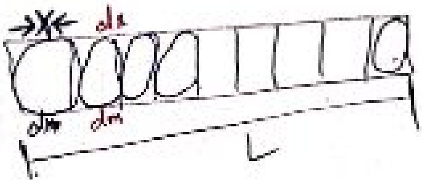
$$r_{com} = \frac{m_1 r_1 + m_2 r_2 + m_3 r_3}{M}$$

$$-0.5\hat{i} - 0.7\hat{j} = \frac{2(-\hat{i} + 5\hat{j}) + 4(6\hat{i} - 7.5\hat{j}) + 3(x_3\hat{i} + y_3\hat{j})}{M \quad 9}$$

Case II: Solid bodies (Rigid)

الأجسام الصلبة

$$\boxed{X_{com} = \frac{1}{M}\int x\,dm}\ ,\ \boxed{Y_{com} = \frac{1}{M}\int y\,dm}\ ,\ \boxed{Z_{com} = \frac{1}{M}\int z\,dm}$$



$$\frac{dm}{dx} = \frac{M}{L} \Rightarrow \boxed{dm = \frac{M\,dx}{L}}$$

$$\boxed{dm = \lambda\,dx}$$

uniform منتظم $\Longrightarrow$ $\boxed{\lambda = \text{constant}}$ ثابت

[illegible] $\frac{dm}{dx}$ ...

$$\boxed{\frac{dm}{dx} = \frac{M}{L} = \lambda_{L}}$$

uniform linear dincity

كثافة خطية

يعني: كتلة في وحدة طول

1-D

$$\left.\begin{aligned} X_{com} &= \frac{1}{M}\int x\,dm \\ Y_{com} &= \frac{1}{M}\int y\,dm \\ Z_{com} &= \frac{1}{M}\int z\,dm \end{aligned}\right] \Longrightarrow r_{com} = \frac{1}{M}\int \vec{r}\,dm$$

$\boxed{dm = \lambda\,dx}$, $\underline{\lambda = \text{uniform}}$, $\boxed{\lambda = \frac{M}{L}}$ λ = linear dincity

2-D
ابعاد

$dm = \sigma\,dA$, σ = surface densitndy

$\boxed{\sigma = \frac{M}{A}}$ مساحة

3-D

$dm = \rho\,dv$, ρ = volume dincity $= \boxed{\frac{M}{V_{vol}}}$

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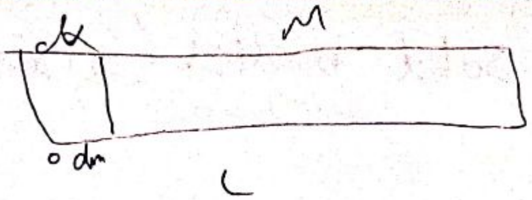
Q5

$$\boxed{X_{com} = \frac{1}{M}\int x\,dm}$$



$$dm = \lambda\,dx = \frac{M}{L}dx$$

$$\Rightarrow X_{com} = \frac{1}{\cancel{M}}\int_0^L x\left(\frac{\cancel{M}}{L}\right)dx = \frac{1}{L}\int_0^L x\,dx$$

$$= \frac{1}{L}\left[\frac{x^2}{2}\Big|_0^L\right] = \frac{1}{L}\cdot\frac{L^2}{2} = \boxed{\frac{L}{2}}$$

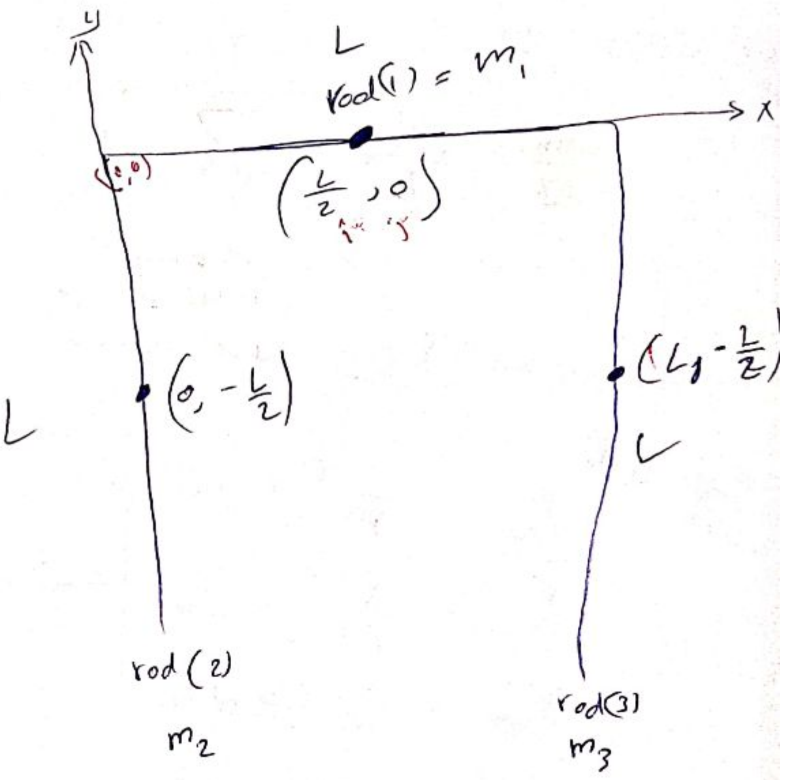
جسم منتظم (uniform rod)

بمنتصف النص

$L = 10\,cm$

$$X_{com} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$Y_{com} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$



Q6

مساحة A

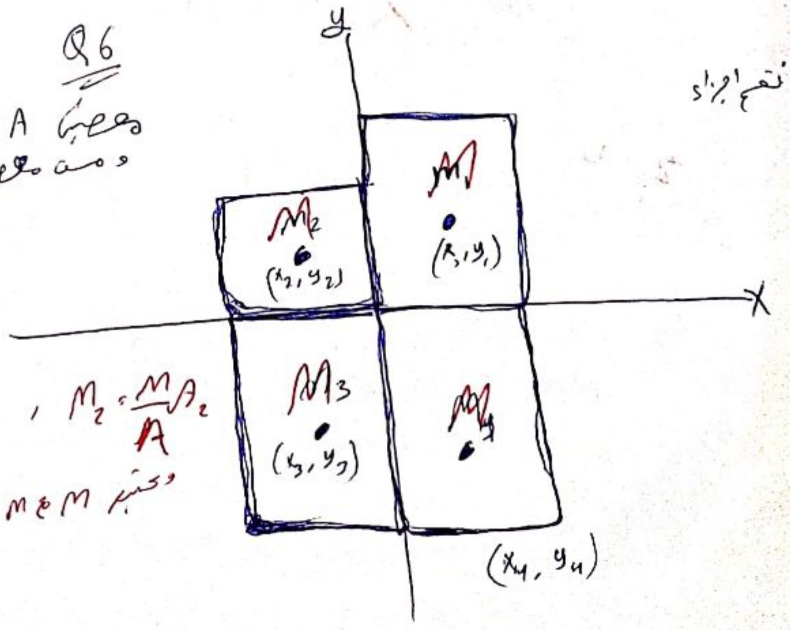
m_1 m_2 m_3 m_4 ومساحة

$$\frac{M_1}{A_1} = \frac{M}{A}$$

$$M_1 = \frac{M}{A}A_1 \quad , \quad M_2 = \frac{M}{A}A_2$$

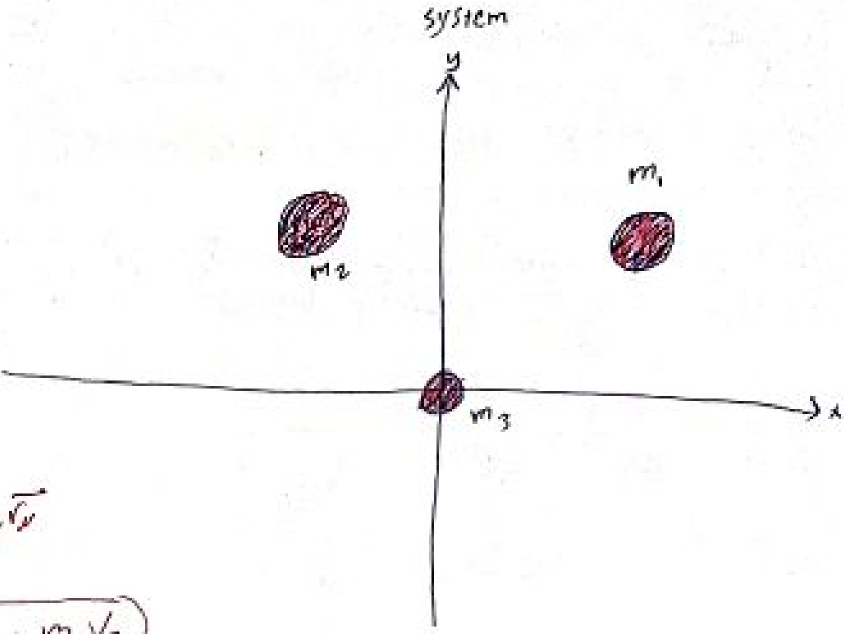
M & M وحنبر

نقسم أجزاء



Newton's 2nd Law for system of Partical.

$$\boxed{\vec{F}_{net} = \underset{\text{الكتلة الكلية}}{M}\ \vec{a}_{com}}$$



$$\vec{r}_{com} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 \cdots\cdots + m_N\vec{r}_N}{M}$$

نشتق له للزمن $M r_{com} = m_1 r_1 + m_2 r_2 + \cdots m_N \vec{r}_N$

نشتق $M V_{com} = m_1 V_1 + m_2 V_2 \cdots + m_N V_N$

$$M a_{com} = \frac{\overset{F_1}{m_1 a_1} + \overset{F_2}{m_2 a_2} \cdots m_N a_N}{M}$$

$$\boxed{M a_{com} = F_{net}}$$ اثبات القانون المشروح

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Chapter 9, 2

center of mass

system of Partical

$$X_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i x_i$$

$$y_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i y_i$$

$$Z_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i z_i$$

$$\vec{r}_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i \vec{r}_i$$

rigid bodies

$$X_{com} = \frac{1}{M}\sum_{i=1}^{N} \int x\, dm$$

$dm = \lambda dx$

$\lambda = \frac{M}{L}$

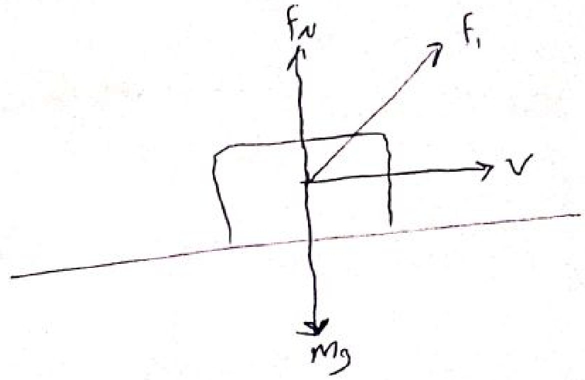
$\sigma = \frac{M}{A}$

$\rho = \frac{M}{V}$

* linear momentum كمية التحرك $(\vec{P})$

* $\vec{P} = m\vec{V}$, m = mass
  V = Velocity

* P in the same direction of $\vec{V}$.
  بنفس اتجاه (V) يكون P



$$\boxed{\frac{dP}{dt} = f_{net}}$$
محصلة القوة

* Newton's 2nd law

$f_{net} = \frac{dP}{dt}$, $\vec{P} = m\vec{V}$

$= \frac{d}{dt}[mV] = m\frac{dv}{dt} + v\frac{dm}{dt}$

if m = constant (لا يتغير) $\Rightarrow \frac{dm}{dt} = 0$

$f_{net} = m\frac{dv}{dt} = \boxed{m\vec{a}}$

$$\boxed{f_{net} = m\vec{a} \quad m = \text{constant (ثابت)}}$$

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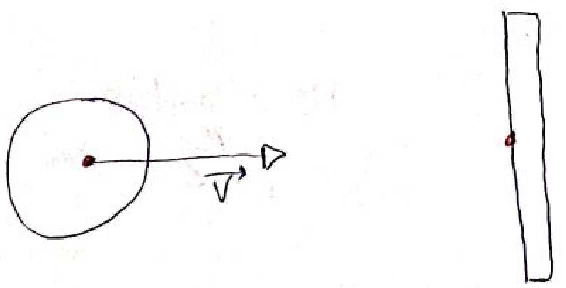
* Collision (التصادم) and Impulse (الدفع) ~~الدفع~~



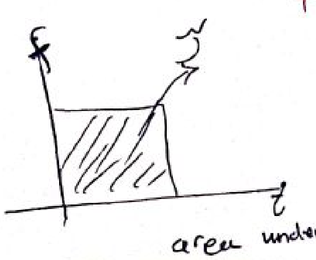
* Impulse $(\vec{J})$

$$\boxed{J = F_{Avg} \cdot \Delta t}$$

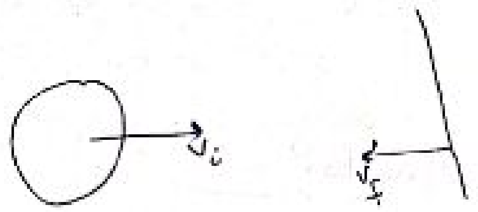
, $\vec{F}$ constant الشرط

الثاني

* $\vec{J} = \int_{t_1}^{t_2} f(t)\, dt$ (f varable) متغيرة
depend on time
يعتمد على الزمن



area under the curve



$$J = \Delta P$$
$$J = m \Delta V$$
$$= m(\vec{v}_f - \vec{v}_i)$$

اثباته

$$f_{net} = \frac{dP}{dt}$$

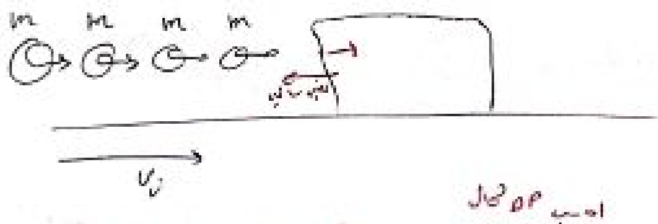
$$f_{avg} = \frac{\Delta P}{\Delta t}$$

$$\vec{J} = f_{avg} \cdot \Delta t = \Delta P = m(v_f - v_i) \quad , \; f \text{ Constant}$$

$$J = \int f\,dt = \Delta \vec{P} \quad , \text{ not Constant.}$$

---

special case



$$\vec{J} = -n \Delta \vec{P}$$

n = # of partical

ΔP = change in linear momentum of 1. partical.

اصبح ΔP كامل

$$* \; J = -n \Delta P = n \, m \Delta V$$
$$f_{avg} \, \Delta t = -n \, m \Delta V$$
$$f_{avg} = \frac{-n \, m \Delta V}{\Delta t}$$

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حفظ كمية الحركة

* Conservation of linear momentum

* From Newton's 2^{nd} law

$F_{net} = \frac{dP}{dt}$ → P is conserved if $\boxed{\Delta P = 0}$

(بشرط النظام معزول)

* $\vec{P}$ is conserved (محفوظة) $(\Delta \vec{P} = 0)$, if the system is Isolated and closed (ومغلق)

$\vec{\Delta P} = 0$

$\vec{P_f} - \vec{P_i} = 0$

$\boxed{\vec{P_f} = \vec{P_i}}$

Isolated system

$\Delta P = 0 \Rightarrow \boxed{\vec{P_f} = \vec{P_i}}$

$\boxed{P_{ix} = P_{fx}}$ $\boxed{\vec{P_{iy}} = \vec{P_{fy}}}$

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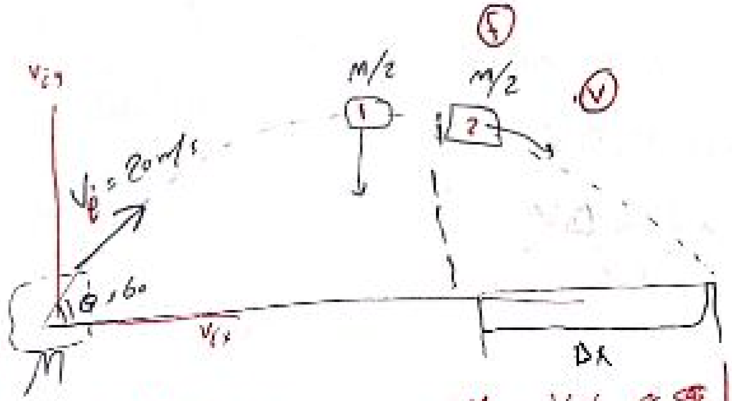
P13



$P_{ix} = P_{f,x(2)} + \cancel{P_{f,x(1)}}$

$M_i v_i \cos\theta = \frac{M}{2} V$

$V = 2 V_i \cos\theta$

$= 2(20)\cos 60 = \boxed{20\,m/s}$

$\Delta X = V_0 t$??

$V_f^2 = V_{0y}^2 + 2a\Delta y$

$= V_0^2 \sin^2\theta + 2g\Delta y$

$\Delta y = \frac{-V_0^2 \sin\theta^2}{2g} = \boxed{15.3}$

$\Delta y = \cancel{V_{0y}} t + \frac{1}{2} g t^2$

$15.3 = \frac{1}{2} g t^2 \Rightarrow t = \boxed{}$

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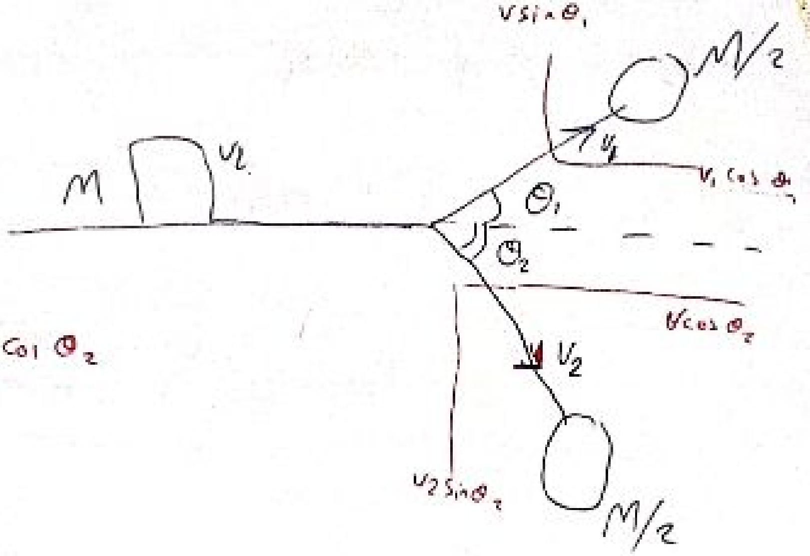

$\vec{P}_{ix}$, $\vec{P}_{f_1}$

$$Mv_i = \frac{M}{2} v_1 \cos\theta_1 + \frac{M}{2} v_2 \cos\theta_2$$

$$\vec{P}_{iy} = \vec{P}_{iy}$$

$$0 = \frac{M}{2} v\sin\theta_1 - \frac{M}{2} v_2 \sin\theta_2$$

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chapter 9, 3

momentum and Kinetic energy in collision

✱ Type of collisions : أنواع التصادمات

① elastic collision تصادم مرن • $\vec{P}_i = \vec{P}_f$ $(\Delta\vec{P}) = 0$ في جميع الأنواع $(\Delta K = 0)$ محفوظة

② inelastic collision غير مرن $(\Delta K \neq 0)$

③ completely inelastic collision. عديم المرونة $(\Delta K \neq 0)$

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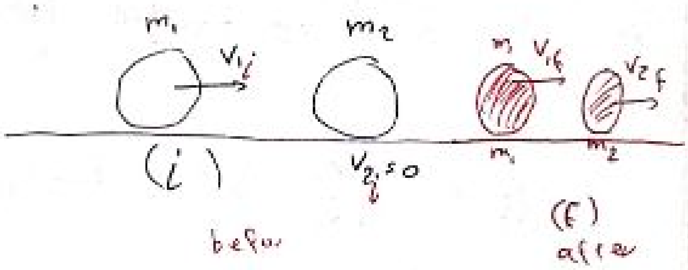
1 Elastic collision :

$$\vec{P}_i = \vec{P}_f$$



$$K_i = K_f$$

$$m_1 v_{1i} + m_2 \cancel{v_{2i}} = m_1 v_{1f} + m_2 v_{2f}$$
0

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \longrightarrow ①$$

$$\frac{1}{2} m v_{1i}^2 + 0 = \frac{1}{2} m v_{1f}^2 + \frac{1}{2} m v_{2f}^2 \longrightarrow ②$$

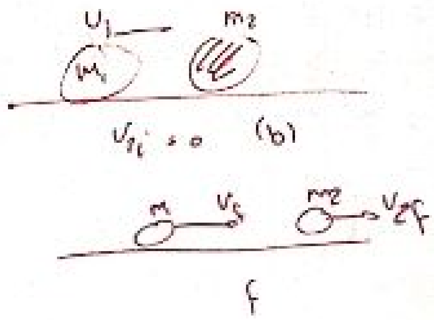
find V_{2f} from equ ①
subsitute in equ ②

$$m_1 V_{1i} = m_1 V_{1f} + m_2 V_{2f} \quad ①$$

$$\frac{1}{2} m V_{1i}^2 = \frac{1}{2} m V_{1f}^2 + \frac{1}{2} m V_{2f}^2 \quad ②$$

حيث شرط m_2 ثابتة

$$\boxed{V_{1f} = \frac{m_1 - m_2}{m_1 + m_2} V_{1i}} \qquad \boxed{V_{2f} = \frac{2m_1}{m_1 + m_2} V_{1i}}$$



اذا تغير m_2 تتحرك ⟸

$$m_1 V_{1i} + m_2 V_{2i} = m_1 V_{1f} + m_2 V_{2f}$$

$$\frac{1}{2} m_1 V_{1i}^2 + \frac{1}{2} m_2 V_{2i}^2 = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2$$

thrust(F) = mass rate of fuel exhaust velocity
F = (m) × (V_f)

if $m_1 = m_2$ $\xRightarrow{①}$ $\boxed{V_{1f} = 0}$

$\xRightarrow{②}$ $V_{2f} = \frac{2m}{m_1 + m_2} V_{1i}$

$$\boxed{V_{2f} = V_{1i}}$$

يندفع الثاني بعد التصادم بسرعة
يساوي الاول قبل التصادم

② if massive target ($m_2 >> m_1$) neglect m_1

الكتلة الأكبر الهدف
اهمل

$$V_{1f} = \frac{-m_2}{m_2} V_{1i} = -V_{1i}$$

$$V_{2f} = \frac{2m_1}{m_2} V_{1i}$$

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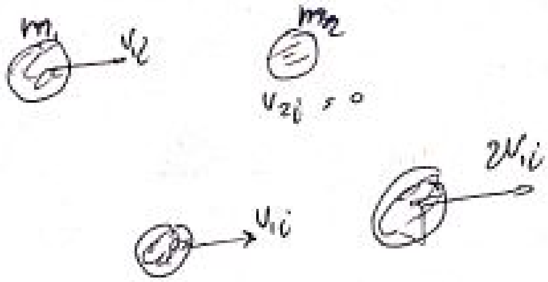
③ massive projectile ($m_1 >> m_2$)

$V_{1f} = V_{1i}$

$V_{2f} = \frac{2m_1}{m_1} V_{1i} = 2V_{1i}$



- ملخص

* target at rest ($V_{2i} = 0$)

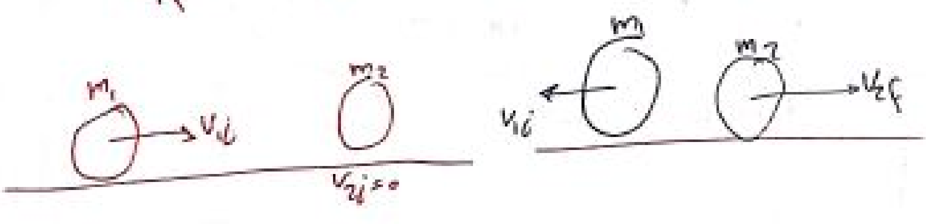
[illegible]

$V_{1f} = \frac{m_1 - m_2}{m_1 + m_2} V_{1i}$ $V_{2f} = \frac{2m_1}{m_1 + m_2} V_{1i}$

Case ① $m_1 = m_2$ ($V_{1f} = 0$, $V_{2f} = V_{1i}$)

Case ② massive target ($m_2 >> m_1$)

$V_{1f} = -V_{1i}$, $V_{2f} \simeq \frac{2m_1}{m_2} V_{1i}$

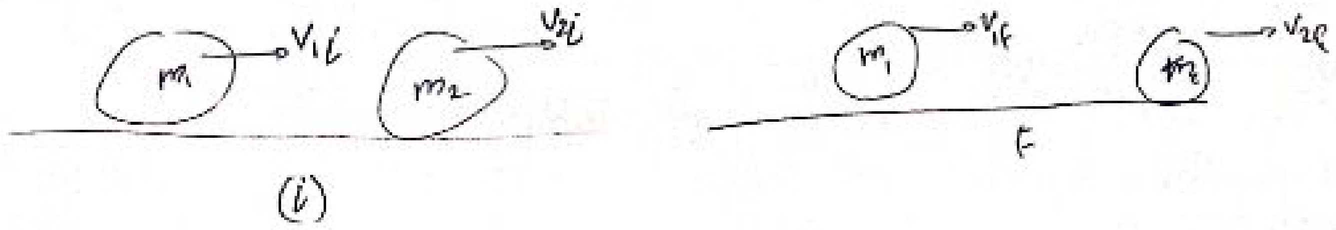


Case ③ massive projectile ($m_1 >> m_2$)

$V_{1f} \simeq V_{1i}$, $V_{2f} \simeq 2V_{1i}$

الهدف متحرك

* moving targer



* $\vec{P_i} = \vec{P_f}$

$$m_1 V_i + m_2 V_{2i} = m_1 V_{1f} + m_2 V_{2f}$$

* $K_i = \vec{K_f}$

$$\frac{1}{2} m v_{1i}^2 + \frac{1}{2} m v_{2i}^2 = \frac{1}{2} m V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2$$

moving targit

$$V_{1f} = \frac{m_1 - m_2}{m_1 + m_2} V_{1i} + \frac{2m_2}{m_1 + m_2} V_{2i}$$

$$V_{2f} = \frac{2m_1}{m_1 + m_2} V_{1i} + \frac{m_2 - m_1}{m_1 + m_2} V_{2i}$$

* Elastic collision
  - target at rest
  - moving target

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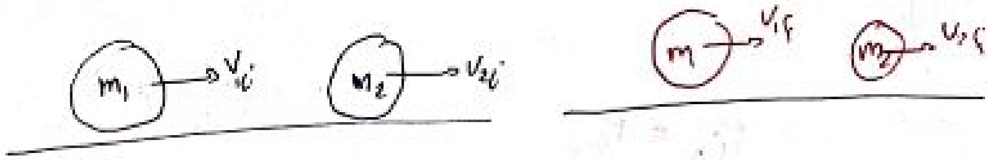
② Inelastic collision:

$\vec{P}_i = \vec{P}_f \quad , \quad K_i \neq K_f \Rightarrow K_i = K_f + \overset{\Delta E}{E_{dissip}}$

$\boxed{\Delta K = K_i - K_f}$



$P_i = P_f$

$m_1 v_{1i} + m_2 v_{2i} = m v_{1f} + m v_{2f}$ — ①

$\Delta E = \left(\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m v_{2i}^2\right) - \left(\frac{1}{2} m v_{1f}^2 + \frac{1}{2} m v_{2f}^2\right)$ — ②

③ Complitly Inelactic Collision

$M = m_1 + m_2$

$\vec{P_i} = \vec{P_f} \Rightarrow m_1 v_{1i} + m_2 v_{2i} = M v_f$

$\Delta E = K_i - K_f \Rightarrow \Delta E = \left(\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2\right) - \frac{1}{2} M v_f^2$

$P = M \vec{v}_{com}$

$V_{com} = \frac{\vec{P}}{M} = \frac{m_1 v_{1i} + m_2 v_{2i}}{M}$

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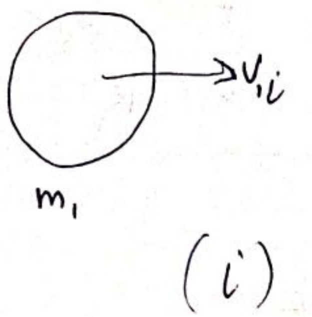
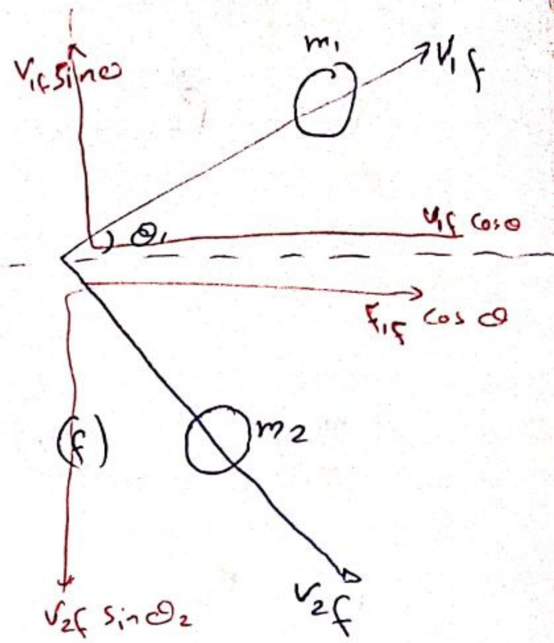
* Collision in 2D :





$\overline{P_i} = \overline{P_f}$

$P_{ix} = P_{fx}$

$P_{iy} = P_{fy}$

لأي نوع تصادم

$P_{ix} = P_{fx}$

$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2$ — (1)

$P_{iy} = P_{fy}$

$0 = m_1 v_{1f} \sin\theta_1 - m_2 v_{2f} \sin\theta_2$ — (2)

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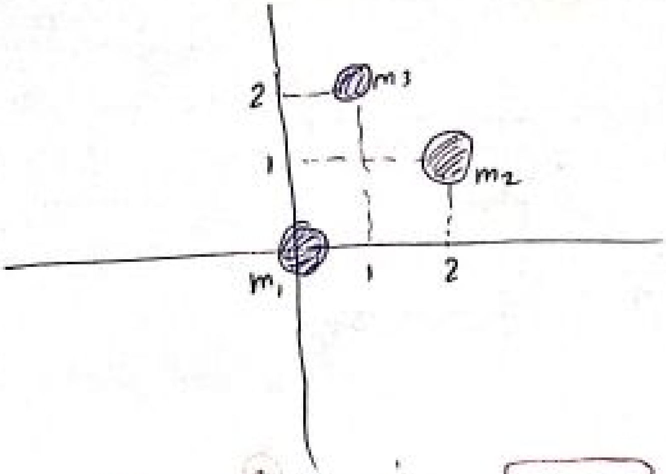
P②

$m_1 = 3Kg$

$m_2 = 4Kg$

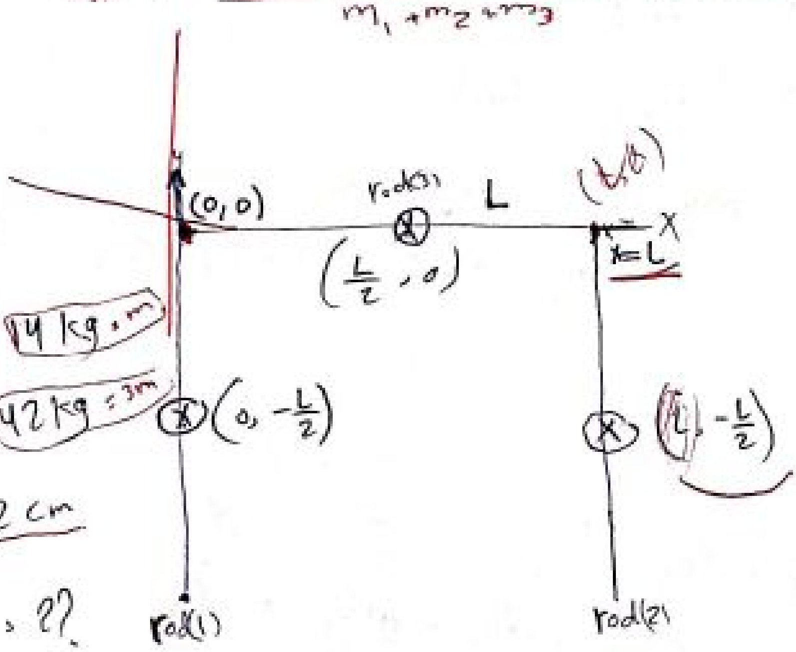
$m_3 = 8Kg$



$$X_{com} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \boxed{1,1m}$$

$$Y_{com} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \boxed{1,3 m}$$

P④



$m_1 = m_2 = \boxed{14 Kg = m}$

$m_3 = \boxed{42 Kg = 3m}$

$L = \underline{22 cm}$

$y_{com}, x_{com} = ??$

$$X_{com} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{M} = \frac{m(0) + mL + 3m\frac{L}{2}}{5m}$$

$$= \frac{L}{2} = \frac{22}{2} = \boxed{11}$$

$$Y_{com} = \frac{m\left(\frac{-L}{2}\right) + m\left(\frac{-L}{2}\right) + 3m(0)}{5m} = -\frac{mL}{5m} = \frac{-L}{5} = \boxed{-4,4 cm}$$

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P(22)

17

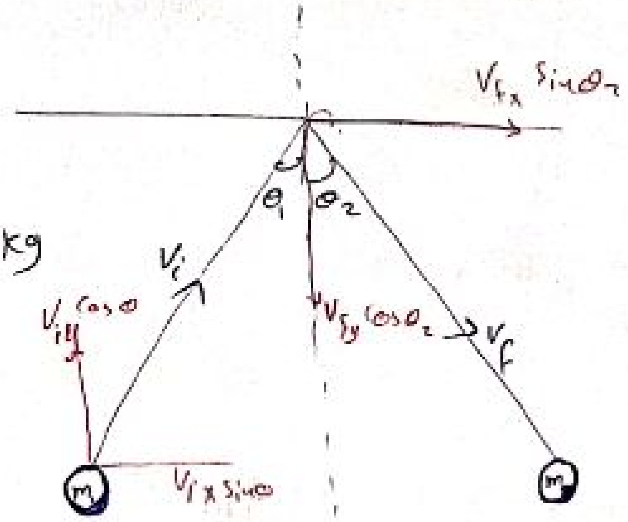
$\theta_1 = 30^\circ$

$m = 0{,}165$ kg

$v_i = 2$ m/s

$\boxed{V_{xi} = V_{xf}}$

y = Verabile



(a) angle $\theta_2 = ??$

(b) $\Delta \vec{P} = ??$

(a) $\vec{P}_x = m\vec{v}_x$

$\vec{P}_{x(i)} = \vec{P}_{x(f)}$

$\cancel{m}\cancel{v_i}\sin\theta_1 = \cancel{m}\cancel{v_f}\sin\theta_2$

$\sin\theta_1 = \sin\theta_2 \Rightarrow \theta_2 = \theta_1 = \boxed{30^\circ}$

(b) ΔP — $\Delta P_x = 0$, $\Delta P_y = ??$

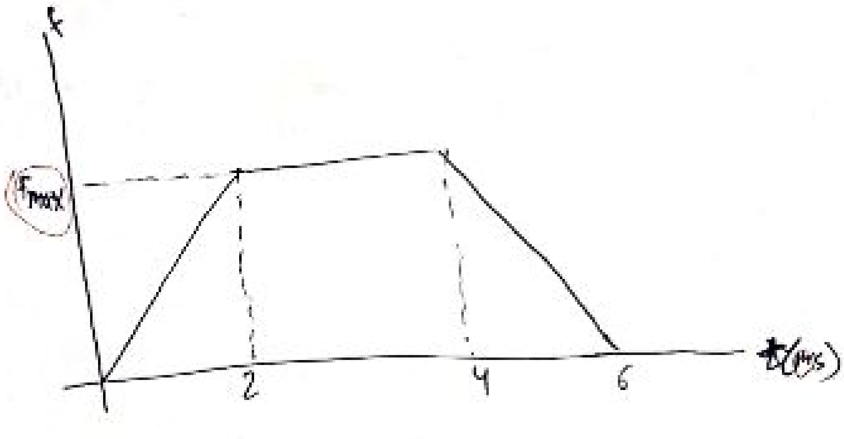
$\Delta P_y = P_{fy} - P_{iy}$

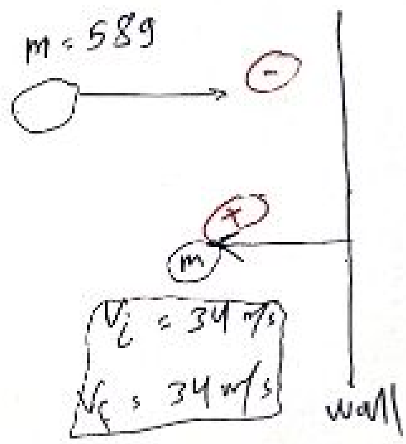
$= -m\, v_f\cos\theta_2 - m\, v_i\cos\theta_1$

$= -2m\, v_i\cos\theta$

$\Delta P = -0{,}572$ kg m/s

P(35)





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$\vec{J} = \Delta P = \int F\, dt$

المساحة تحت المنحنى

نحسب المساحة

t(ms)

$\Delta P = \vec{P_f} - \vec{P_i}$

$= mv_f + mv_i$

$\Delta P = 2mv_i$

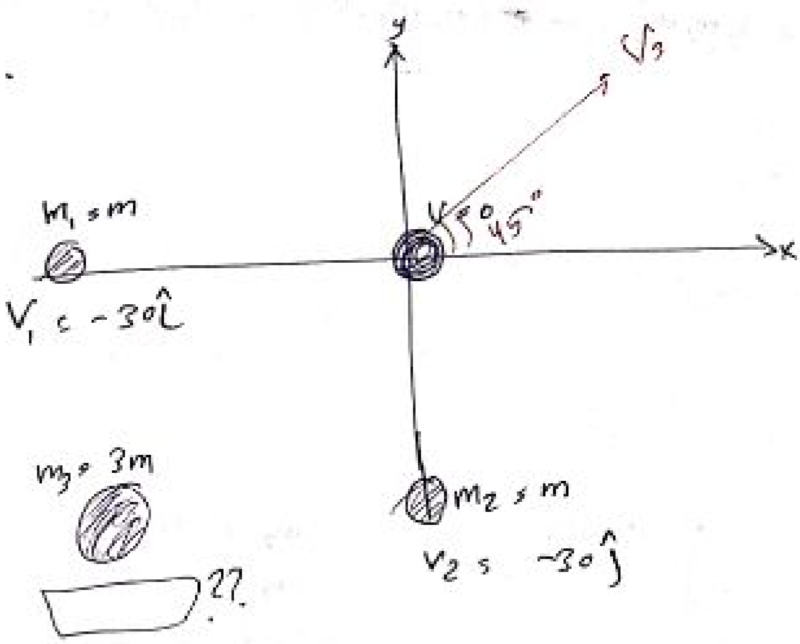
$\int F dt = \frac{1}{2}(0{,}002)F_{max} + 0{,}002\, F_{max} + \frac{1}{2}(0{,}002)\, F_{max}$

$= 0{,}004\, F_{s\,max}$

$2mv_i = 0{,}004\, F_{s\,max}$

$F_{s\,max} = 9{,}9 \times 10^{2}\ N$

P47



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$\vec{P_i} = \vec{P_f}$

$0 = m_1\vec{V_1} + m_2\vec{V_2} + m_3\vec{V_3}$

$V_3 = \dfrac{-m_1\vec{V_1} - m_2\vec{V_2}}{m_3} = \dfrac{30m\hat{i} + 30m\hat{j}}{3m} = \boxed{10\hat{i} + 10\hat{j} = V_3}$

$\tan\theta = \frac{10}{10} = 1 = 45$

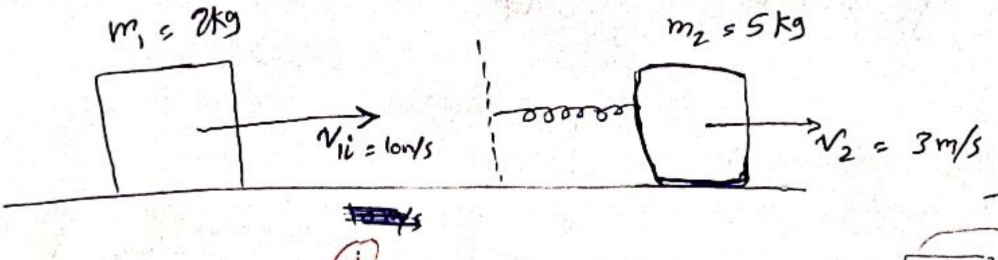
$\times\ 225$

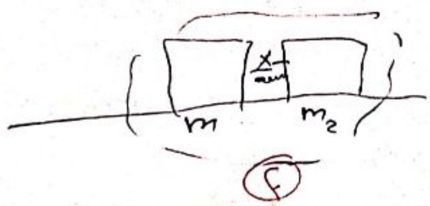
$\theta = 45$

P 59

$m_1 = 2kg$ $\quad$ $m_2 = 5kg$

$K = 1120$ N/m





$X_{s\,max} = ??$

" completely inelastic collision "
عديم المرونة

$K_i + U_i = K_f + U_{f\,spring}$ $\quad$ $U_s = \frac{1}{2}Kx^2$??

$U_i = 0$

$K_i - K_f = U_s$

$\left(\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2\right) - \frac{1}{2}(m_1+m_2)v^2 = \frac{1}{2}KX^2 \Rightarrow \boxed{X = 0{,}25\ m}$

لإيجاد v $\quad$ $P_i = P_f$

$m_1 v_{1i} + m_2 v_{2i} = (m_1+m_2)v_f$

$v_f = \boxed{}$

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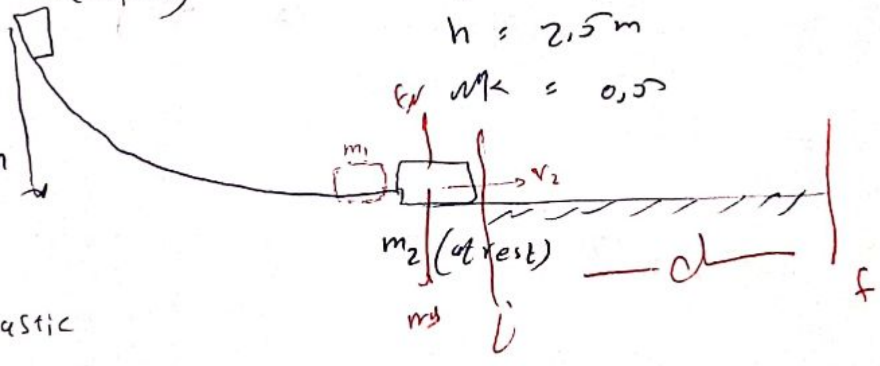
P 68



$m_2 = 2m_1$

$h = 2{,}5\ m$

$\mu_k = 0{,}5$

@ elastic

(b) Completely inelastic

$V_{1i} = \sqrt{2gh} = \sqrt{2(9.8)(2.5)} = 7\ m/s$

بعد التصادم $V_2 = \frac{2m_1}{m_1+m_2}V_{1i}$ قبل التصادم

$= \frac{2m_1}{m_1+2m_1}(7) = \frac{2}{3}(7) = \boxed{4{,}67\ m/s}$

$K_i + \cancel{U_i} = \cancel{K_f} + \cancel{U_f} + \underbrace{\Delta E_{th}}_{W_{friction}}$

$\frac{1}{2} m v_2^2 = \Delta E_{th}$

$\frac{1}{2} m v_2^2 = f_K \cdot d$

$\frac{1}{2} \cancel{m} v_2^2 = \mu_K (\cancel{m} g) d$

$\boxed{d = 3,22\ m}$

② $V_2 = \frac{m_1}{m_1 + m_2} v_{1i} \Rightarrow d = 0,556\ m$

$m_1 v_1 = (m_1 + m_2) V_2$

$F_{net} = \frac{dP}{dt} \Rightarrow f_{net} = m\vec{a}$

$P = mv$

$F_{net} = \frac{d}{dt}[mv] = m\frac{dv}{dt} + v\frac{dm}{dt}$

$= ma + \cancel{v\frac{dm}{dt}}$ → 0 at m constant

$F_{net} = m\vec{a}$

✓

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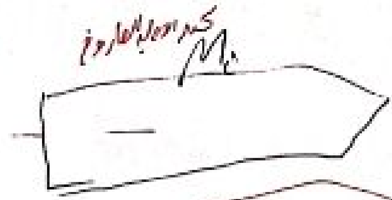
* $Ma = R V_{rel}$ "is equation of rocket"

* $V_f - V_i = V_{rel} \ln \frac{M_i}{M_f}$ "2nd rocket equation"

عبر بالنسبة للصاروخ

V_{rel} = velocity of gas with respect the rocket.

m_{gass}



$M_f = M_i - m$

$R = \frac{dm}{dt}$ يفقدها الصاروخ بالنسبة للزمن

P76

$M_i = 6090$ kg

$V_i = 105$ m/s

$m_{gass} = 80$ kg

$V_{rel} = 253$

$V_f = ??$

??

$V_f - V_i = V_{rel} \ln \frac{M_i}{M_f}$

$M_f = M_i - m_{gass}$

$6090 - 80 = \square$

$V_f - 105 = 253 \ln \frac{6090}{\square}$

$V_f = \square$

End ch9

Good Luck

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Chapter 10

Rotation الحركة الدورانية

motion الحركة → translation انتقالية (x position, $\Delta x = \vec{x}_2 - \vec{x}_1$, $v_{avg} = \frac{\Delta \vec{x}}{\Delta t}$, $a_{avg} = \frac{\Delta v}{\Delta t}$, speed

motion الحركة → Rotation دائرية (θ, $\Delta\vec{\theta} = \vec{\theta}_2 - \vec{\theta}_1$, ω_{Avg}, α_{Avg}

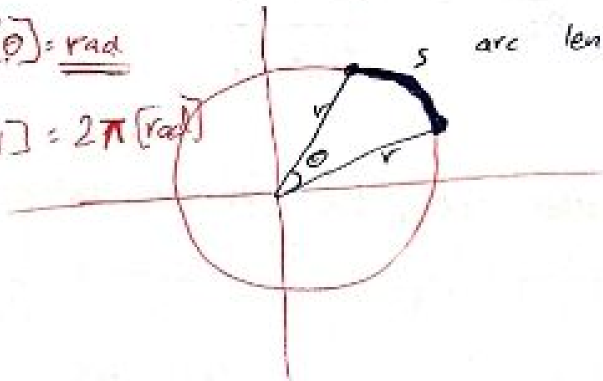
* Rotational Variables : متغيرات الحركة الدورانية

① angular position : الموقع الزاوي/الزاوي (θ)

$$\boxed{\theta = \frac{s}{r}}$$

$[\theta] = \text{rad}$

$1\,[\text{rv}] = 2\pi\,[\text{rad}]$

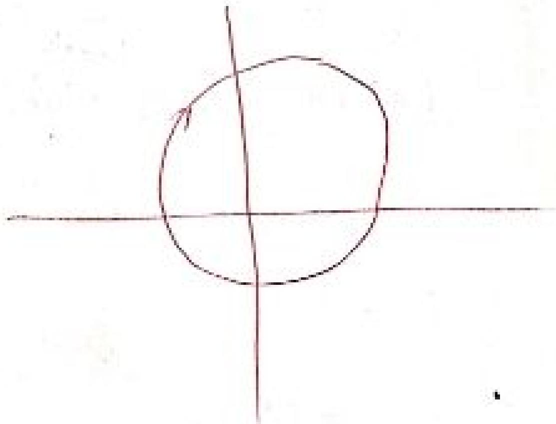


s arc length طول القوس

② angular displacement الإزاحة الزاوية ($\Delta\vec{\theta}$).

$\Delta\vec{\theta} = \vec{\theta}_2 - \vec{\theta}_1$

$[\Delta\theta = \text{rad}]$



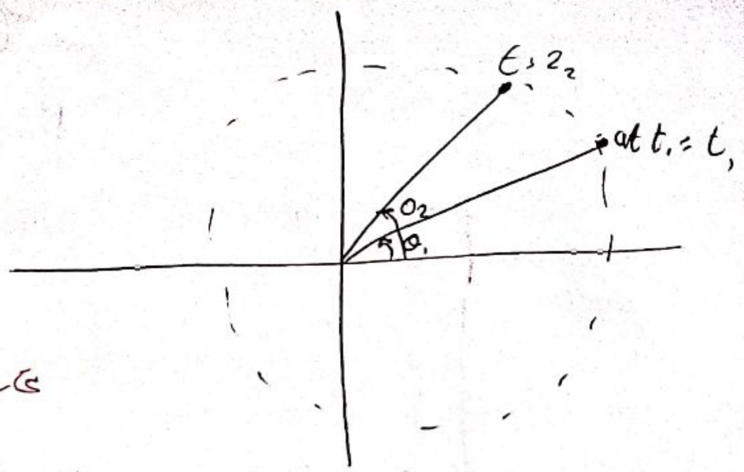
* clock wise ⇒ $\Delta\theta$ is negative (−)

* Counter clockwise ⇒ $\Delta\theta$ is Positive (+)

السرعة الزاوية

$\boxed{3}$ angular Velocity (ω_{avg})

$$\omega_{Avg} = \frac{\Delta \theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$$



$\omega_{avg} \xrightarrow{+}$ Counter clockwise عكس عقارب

$\omega_{Avg} \xrightarrow{-}$ clockwise مع عقارب

التسارع الزاوي

$\boxed{4}$ angular acceleration (α_{Avg})

$$\alpha_{Avg} = \frac{\Delta \omega}{\Delta t} = \frac{\omega_2 - \omega_1}{t_2 - t_1}$$

* angular speed = | angular velocity (ω) |

عند لحظة معينة

Translation $\longrightarrow x(t) \xrightarrow{\frac{dx}{dt}} v(t) \xrightarrow{\frac{dv}{dt}} a(t)$ بالعكس تكامل

Rotation $\longrightarrow$ (rad) $\theta(t) \xrightarrow{\frac{d\theta}{dt}}$ rad/sec $\omega(t) \xrightarrow{\frac{d\omega}{dt}}$ rad/sec^2 (بالإنجليزي) $\alpha(t)$ بالعكس تكامل

Instantaneous angular Possition — Instantaneous angular velocity — Instantaneous angular acceleration.

$\cancel{\int \omega dt}$ $\theta(t) \xleftarrow{\int \omega dt} \omega(t) \xleftarrow{\int \alpha(t)} \alpha(t)$

اتجاه

(ω) direction (right hand rule).

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P(4)$_2$ $\theta(t) = 2 + 4t + 2t^3$

at $t = 0$ ⟹ ⓐ angular Position

ⓑ angular velocity ⓒ w at $t = 4$ sec ⓓ α at $t = 2$ sec ?

a $\theta\Big|_{t=0} = 2$ rad ⓑ $w(t) = \frac{d\theta}{dt} = 4 + 6t^2 = w(0) = \boxed{4}$ rad/sec

ⓒ $w\Big|_{t=4} = 4 + 96 = \boxed{100 \text{ rad/s}}$

ⓓ $\alpha = \frac{dw}{dt} = \boxed{12t} = \alpha\Big|_{t=2} = \boxed{24 \text{ rad/sec}^2}$

واجب (w_{avg} = $t = 1$, $t = 4$ $w_{Avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$

---

* Rotation motion at constant α :- (w) [illegible]

translation ($a \equiv$ cons)	Rotation ($\alpha \equiv$ cons)
$v = v_0 + at$	$w = w_0 + \alpha t$
$v^2 = v_0^2 + 2a\Delta x$	$w^2 = w_0^2 + 2\alpha\Delta\theta$
$\Delta x = v_0 t + \frac{1}{2}at^2$	$\Delta\theta = w_0 t + \frac{1}{2}\alpha t^2$
Δx, v, a	$\Delta\theta$, w, α

$v = \frac{2\pi r}{T}$

$a = v^2/r$

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P(9)$_2$ $\omega_0 = 12,6$ rad/sec , $\alpha = -4,2$ rad/sec^2

ⓐ t until t stop ??

$\cancel{\omega}^{\,0} = \omega_0 + \alpha t$

$t = \frac{-\omega_0}{\alpha} = \frac{-12,6}{-4,2} = \boxed{3\,sec}$

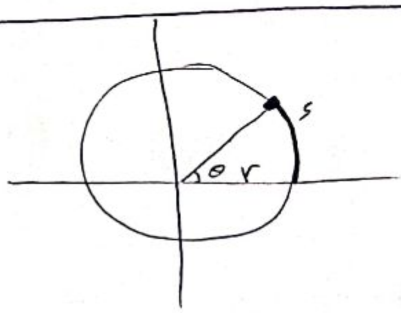
ⓑ $\cancel{\omega^2}^{\,0} = \omega_0^2 + 2\alpha\Delta\theta$

$\Delta\theta = \frac{-\omega_0^2}{2\alpha} = \frac{-(12,6)^2}{-(2)(4,2)} = \boxed{18,9\ rad}$

1 rev → 2π rad

x ← 18,9 rad

translation Rotation



$x, v, a \longleftrightarrow \theta, \omega, \alpha$

r·

s

$\boxed{s = r\theta}$ → angular position.

linear position $\frac{ds}{dt} = r\frac{d\theta}{dt} \Rightarrow \boxed{v = r\omega}$

$\frac{ds^2}{dt^2} = r\frac{d\theta^2}{dt^2}$

$\boxed{a = r\alpha}$

$\boxed{\begin{matrix}\alpha \\ a\end{matrix} = \frac{v^2}{r}}$

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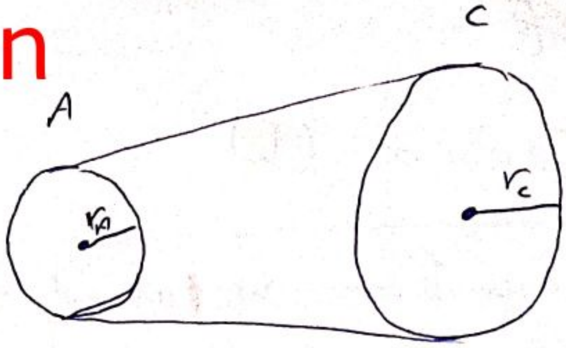


$r_A = 10\,cm$, $r_C = 25\,cm$

$\omega_{A(i)} = 0$, $\alpha_A = 1.6\ rad/sec^2$

$V_a = V_c$

(a) $t \longrightarrow \omega_C = 100\ rev/min$??

$\Longrightarrow 100 \times \frac{2\pi}{60} = 10.67\ \frac{rad}{s}$

حل : $V_A = V_C$

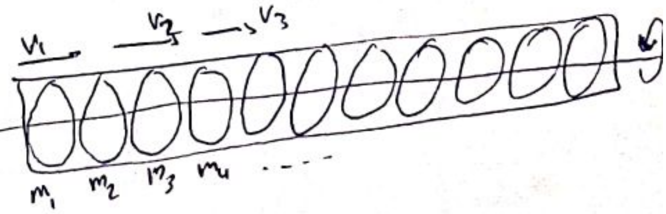
$\omega_a r_a = \omega_c r_C$

$\omega_A = \frac{\omega_C r_C}{r_A} = (10.47)\left(\frac{25}{10}\right) = \boxed{26.16\ rad/sec}$

$\omega_A = \omega_{0A} + \alpha t$

$26.16 = 0 + 1.6\,(t) \Rightarrow \boxed{t = 16\,sec}$

✱ Kientic energy (K.E) in Rotation :-



$K.E = \sum_{i=1}^{N} \frac{1}{2} m v_i^2$

$v_i = r_i \omega_i \Longrightarrow K.E = \sum_{i=1}^{N} \frac{1}{2} m_i (r\omega_i)^2$

$= \frac{1}{2} \sum_{i=1}^{N} (m_i r_i^2)\, \omega_i^2 = \boxed{\frac{1}{2} I \omega_i^2}$ $(m \propto I)$

انرشيا

Inetria (I)

~~KE~~

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* Inetria (I)

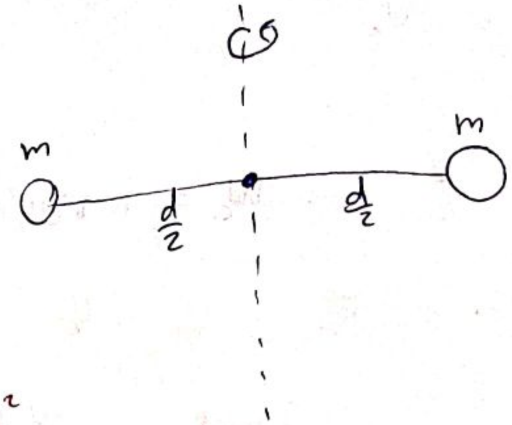
Case I → Rotational axis through the center of mass

محور الدوران يمر من المركز

$$I_{com} = \sum_i m_i r_i^2$$

$$= m\left(\frac{d}{2}\right)^2 + m\left(\frac{d}{2}\right)^2$$

$$= \frac{md^2}{2}$$



$6ma^2$

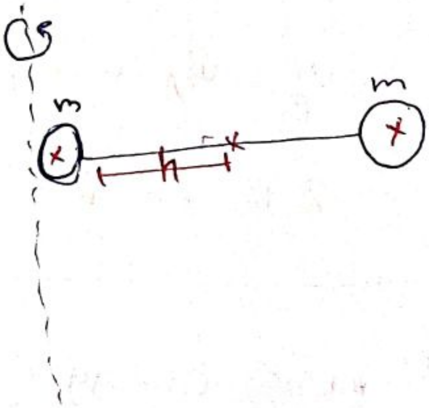
→ Rotational axis through apart from the com

$$\boxed{I = I_{com} + Mh^2}$$

M → total mass

h → distanse of Center of mass Rotationall axis.

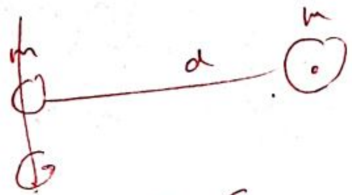
محور الدوران



$$= \frac{md^2}{2} + 2m\left(\frac{d}{2}\right)^2$$

$$= md^2$$ للتأكد

OR

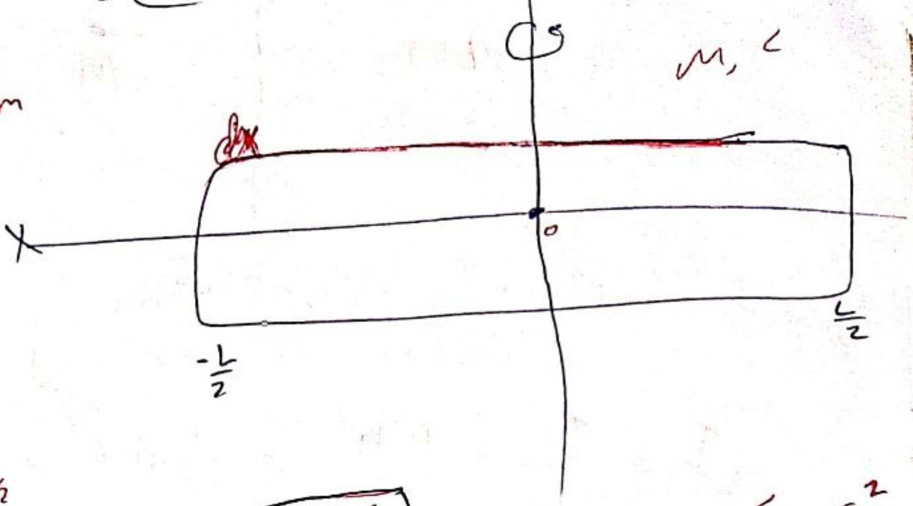


$$I = I_1 + I_2$$

$$= 0 + md^2 = md^2$$

Case II : rigid body (solid)

$$I = \int r^2\,dm = \int x^2\,dm$$



$$\lambda = \frac{dm}{dx} = \frac{M}{L}$$

$$\Rightarrow dm = \frac{M}{L}\,dx$$

$$I = \int_{-L/2}^{L/2} x^2 \frac{M}{L}\,dx = \frac{M}{L}\int_{-L/2}^{L/2} x^2\,dx = \boxed{\frac{1}{12} M L^2}$$

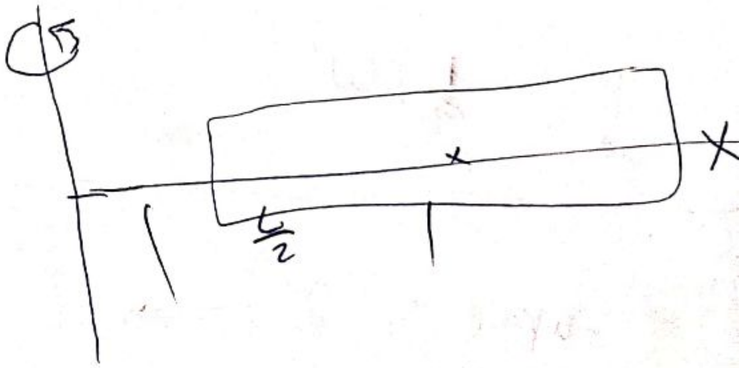
$I = \sum m_i r_i^2$

$$\boxed{I_{rod} = \frac{1}{12} m L^2}$$

شرط محور الدوران
يمر بـ Com

OR

Parrallel - axis theorem



$$I = I_{com} + M h^2$$

$$= \frac{1}{12} m L^2 + m\left(\frac{L}{2}\right)^2$$

$$= \frac{1}{12} m L^2 + \frac{M L^2}{4} = \frac{4 m L^2 + 12 M L^2}{12 \times 4} = \frac{16 M L^2}{48}$$

$$= \boxed{\frac{M L^2}{3}}$$

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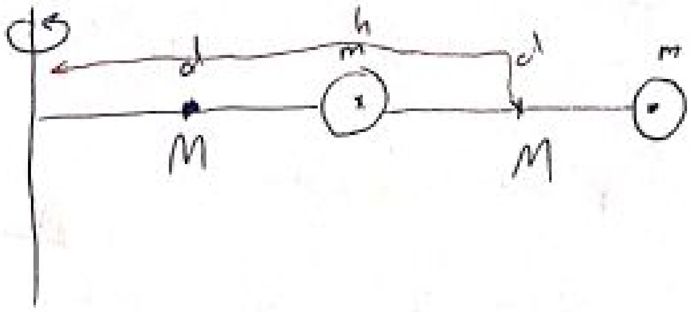
P41

$m = 0{,}85\,kg$

$M = 1{,}2\,kg$

$w = 0{,}3\ rad/sec$



$I = I_1 + I_2 + I_3 + I_4 = \boxed{0{,}023\ kg/m^2}$

$I_1 = I_{com} + Mh^2 = \frac{1}{12} M \overset{d}{l^2} + M\left(\frac{d}{2}\right)^2 =$

$I_2 = \frac{1}{12} M \overset{d}{l^2} + M\left(\frac{3d}{2}\right)^2 =$

$I_3 = m d^2 =$

$I_4 = m(2d)^2 =$

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$K_{tot} = \frac{1}{2} I W^2 =$

$\underline{\tau\text{ourqe}} : (\vec{\tau}) \Longrightarrow \vec{\tau} = \vec{r} \times \vec{F} = |\vec{r}||\vec{F}| \sin\theta$

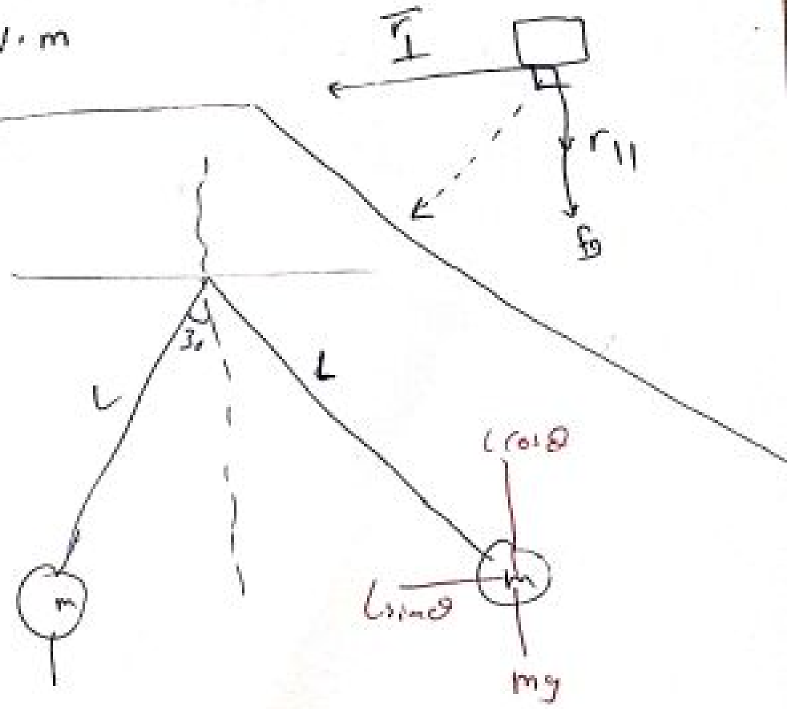
$[\tau] = N \cdot m$

P47 $m = 0.75\ kg$

$L = 1.25\ m$

$\tau, f_g = ??$

$\tau = mg\, d \sin\theta = 46\ N\cdot m$



* Newtun's second law: $f_{net} = m\vec{a}$

$$\boxed{\tau_{net} = I\alpha}$$

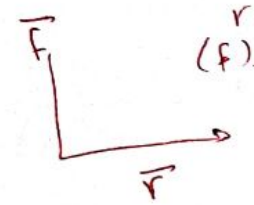
, $\vec{\tau}_{net} \equiv$ net tourqe

$I \equiv$ Intertica

$\alpha \equiv$ angular acceleration.

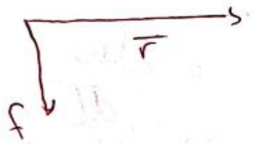
$$\vec{\tau} = \vec{r} \times \vec{F}$$

اصابع ع r وبتحرك نحو (f)



$t = \boxed{+Z}$

ابهام

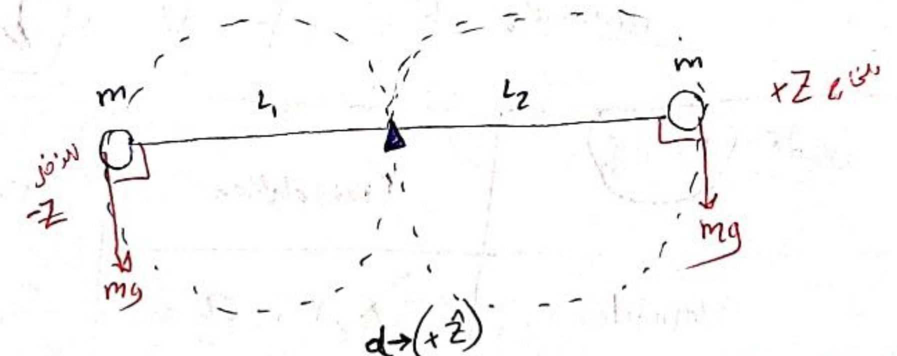


$t = \boxed{-Z}$

$\tau = \hat{i} \times -\hat{j} = (-Z)$

P56) $l_1 = 20$ cm , $l_2 = 80$ cm

a_1 , a_2 ??



$a = r\alpha$

$a_1 = l_1 \alpha$

$a_2 = l_2 \alpha$

$$\tau_{net} = I\alpha$$

$$\tau_{net} = \vec{\tau}_1 + \vec{\tau}_2 \Rightarrow \tau_1 = mg\,l_1, \quad \tau_2 = mg\,l_2 \Rightarrow \tau_{net} = mg(l_2 - l_1)$$

$$I = ml_1^2 + ml_2^2$$

$$mg(l_2 - l_1) = (ml_1^2 + ml_2^2)\alpha \Rightarrow \boxed{\alpha = 8.65 \text{ rad/sec}^2}$$

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$W_{net} = \Delta K \Rightarrow K = \frac{1}{2} I w^2$

④ Translation motion : $W = \int_{x_0}^{x_f} F\, dx$

Rotational $\Rightarrow W = \int_{\theta_i}^{\theta_f} \tau\, d\theta$

$\tau(\theta) = \theta^2$

$W = \tau_{avg} \cdot \Delta\theta$

④ Power $\xrightarrow{translation} \vec{P} = \frac{dW}{dt}$

rotational $\vec{P} = \vec{F} \cdot \vec{v} = \boxed{\vec{\tau} \cdot \vec{w}}$

ملخص Part I:	translation	rotation
Varables	x, v, a	θ, w, α
Constant acceleration	$v = v_0 + at$ $v^2 = v_0^2 + 2a\Delta x$ $\Delta x = v_0 t + \frac{1}{2}at^2$	$w = w_0 + \alpha t$ $w^2 = w_0^2 + 2\alpha\Delta\theta$ $\Delta\theta = w_0 t + \frac{1}{2}\alpha t^2$
Part 2 K	$K = \frac{1}{2}mv^2$	$K = \frac{1}{2} I w^2$ Part 3
Part 4	$W = \int F\, dx$ $W_{net} = \Delta K$ $P = \vec{F} \cdot \vec{v}$ $F_{net} = ma$	$W = \int \tau\, d\theta$ $W_{net} = \Delta K$ $P = \tau \cdot w$ $F_{net} = I\alpha$

$w_f^2 = w_i^2 + 2\alpha\theta$

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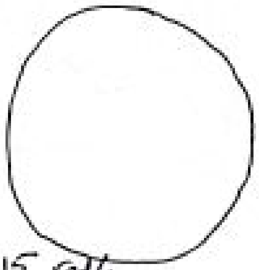
Discussion chapter 10

2
13
23
34

P(2)

ω _ angular velocity



$\omega = \frac{\Delta\theta}{\Delta t}$

$\Delta\theta = \frac{1}{2}(\omega_0 + \omega_f)t$
$\omega = \omega_0 + \alpha t$
$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$
$\Delta\theta = \omega t - \frac{1}{2}\alpha t^2$

$\omega_{\text{second hand}} = \frac{2\pi}{60} = 0{,}15 \text{ rad/s}$

$\omega_{\text{min-hand}} = \frac{2\pi}{60 \times 60} = 1{,}75 \times 10^{-3} \text{ rad/s}$

$\omega_{\text{hour hand}} = \frac{2\pi}{60 \times 60 \times 60} = 1{,}45 \times 10^{-4} \text{ rad/sec}$

P13 : $\theta_0 = 0$, $\omega_0 = 1{,}5$ rad/sec $\theta = 40$ rev , $\omega_f = 0$

1 rev $\rightarrow$ 2π rad
40 rev $\rightarrow$ x

(a) t (b) α

(a) $\omega_{avg} = \frac{\Delta\theta}{\Delta t} \Rightarrow \Delta t = \frac{\Delta\theta}{\omega_{avg}} = \frac{2\Delta\theta}{\omega_0} = \boxed{3{,}4 \times 10^2 \text{ sec}}$

$\omega_{avg} = \frac{1}{2}(\omega_0 + \cancel{\omega_f}) = \frac{\omega_0}{2}$

(b) $\cancel{\omega} = \omega_0 + \alpha t$

$\alpha = \frac{-\omega_0}{t} = \boxed{-7{,}12 \times 10^{-4}} \text{ rad/s}^2$

1 rev $\rightarrow$ 2π rad
20 rev $\rightarrow$ x

(c) t for first 20 rev

$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ $t = 98$ sec ✓ , $t = 572$ sec ✗

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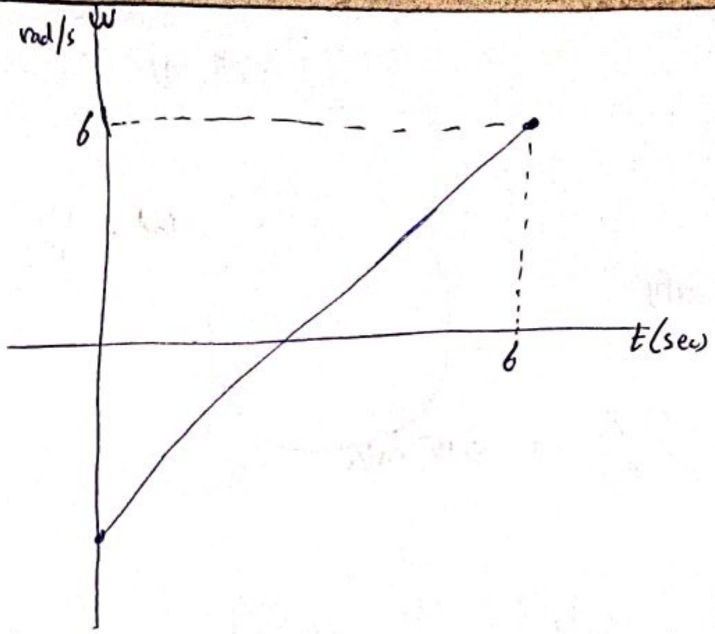
P(34)



(a) α_{rod}

$\frac{4-1}{4-2} = 1,5$

$\theta \xrightarrow{d/dt} w \xrightarrow{dw/dt} \alpha$

$\alpha_{rod} = \frac{dw}{dt} = \text{slope} = 1,5\ rad/s^2$

(b) $t = 4\,sec \longrightarrow K_{(4)} = 1,6\,J$ w_4^{-2}

$K_0 \longrightarrow (t=0) = ??$

$K = \frac{1}{2} I w^2$

$\frac{K_0}{K_4} = \frac{\cancel{\frac{1}{2} I} w_0^2}{\cancel{\frac{1}{2} I} w_{(4)}^2} =$

$K_0 = \frac{w_0^2}{w_4^2} K_{(4)} = \boxed{0,4\,J}$

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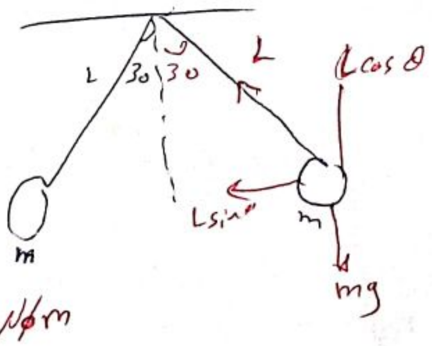
P(47) $m = 0.75\,kg$

$L = 1.25\,m$



العمودي

$\tau = r \times F$

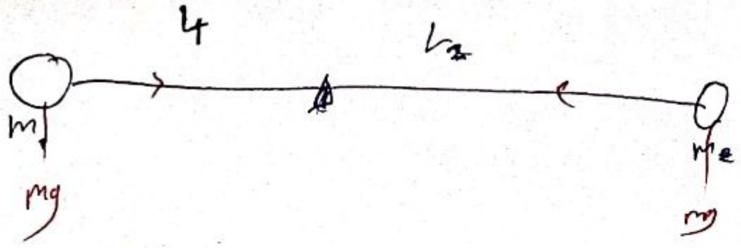
$= r f \sin\theta$

when $\theta = 0$, $180 \to \tau = 0$

τ (Taarqe gravitintional force)

$\tau = mgL\sin\theta = 4.6\,N/m$

P56



$L_1 = 20$ cm
$L_2 = 80$ cm

$a_1, a_2 = ??$

$a = \alpha r$
$V = r\omega$
$S = r\theta$

$a_1 = \alpha L_1$
$a_2 = \alpha L_2$

$\tau_{net} = I\alpha = (m_1 L_1^2 + m_2 L_2^2)\alpha$

$mgL_1 - mgL_2 = (m_1 L_1^2 + m_2 L_2^2)\alpha$

$\alpha = \boxed{}$

عوض

P23 $D = 1.2$ $r = 0.6$ $\omega_0 = 200$ rev/min

(A) $\omega_0 = \frac{200 \text{ rev} \times 2\pi}{60} = \boxed{20.9 \text{ rad/sec}}$

(B) $V = \omega_0 r = 20.9(.6) = \boxed{12.5 \text{ m/s}}$

(C) $\omega = 1000$ rev/min , $t = 1$ min

ie, 12 معادلات

$\omega = \omega_0 + \alpha t$
$1000 = 200 + \alpha(1) \longrightarrow \boxed{\alpha = 800 \text{ rev/min}^2}$

(D) $\Delta\theta = .5(\omega_0 + \omega)t$
$\Delta\theta = .5(200 + 1000)(1) \longrightarrow \boxed{\Delta\theta = 600 \text{ rev}}$

End ch10
Good Luck
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