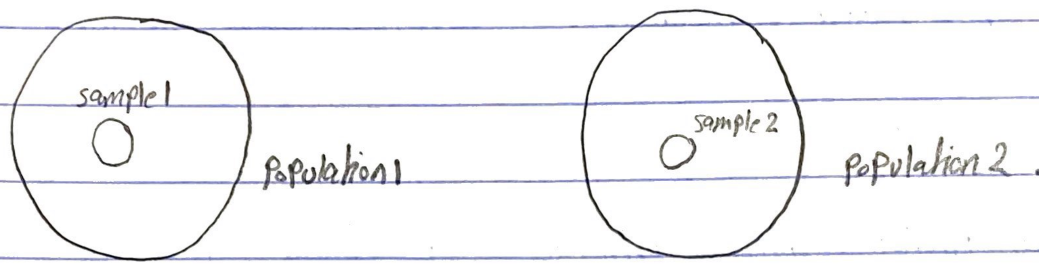
chapter 10 : statistical inference About means and proportions with two population.

10.1 : Inference about defference between two population means, σ_1 and σ_2 Known.



μ_1 : Mean of population 1.

μ_2 : Mean of population 2.

→ we want to study $\mu_1 - \mu_2$ assuming σ_1, σ_2 Known.

σ_1 : standard deviation of population 1.

σ_2 : standard deviation of population 2.

→ we take sample 1 from population 1.
and sample 2 from population 2.

- Assumptions :

1. sample 1 random
2. sample 2 random.
3. sample 1 and sample 2 are independent.

→ point estimator for $\boxed{\mu_1 - \mu_2 = \bar{x}_1 - \bar{x}_2}$

$\bar{x}_1$ = sample 1 mean.

$\bar{x}_2$ = sample 2 mean.

→ confidence interval / Interval Estimate for $\mu_1 - \mu_2$:

CI : $(\bar{x}_1 - \bar{x}_2) \mp E$, E : margin of error.

$$E = Z_{\frac{\alpha}{2}} \times \underbrace{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \rightarrow \text{standard error.}$$

Assuming :

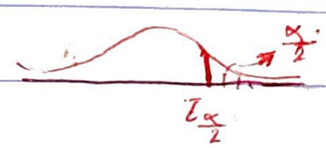
1. population 1 normal.
2. population 2 normal.

or Both sample are Large enough.

→ practical Advice :

Large enough samples : $n_1 \geq 30$, $n_2 \geq 30$ } بستخدمو لما يكون pop 1,2 مش Normal

→ $(1-\alpha)$ CI for $\mu_1 - \mu_2 = (\bar{x}_1 - \bar{x}_2) \mp Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$



→ Hypothesis test about $\mu_1 - \mu_2$

H_0 : Null hypothesis الي فيها يساوي

H_1 : Alternative hypothesis الي فيها لا يساوي

D_0 : Hypothesized value.

① $H_0 : \mu_1 - \mu_2 \geq D_0$ (Lower-Tailed Test).

$H_1 : \mu_1 - \mu_2 < D_0$

مع بعض اسمهم → one Tailed test.

② $H_0 : \mu_1 - \mu_2 \leq D_0$ (upper-Tailed Test).

$H_1 : \mu_1 - \mu_2 > D_0$

③ $H_0 : \mu_1 - \mu_2 = D_0$ (Two-Tailed Test).

$H_1 : \mu_1 - \mu_2 \neq D_0$

→ Test statistic for all cases in section 10.1

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Assumptions : same as for CI.

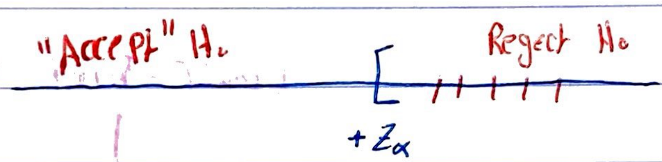
→ Critical value Approch :

①

"Accept" : Don't reject ما بقدر أرفض

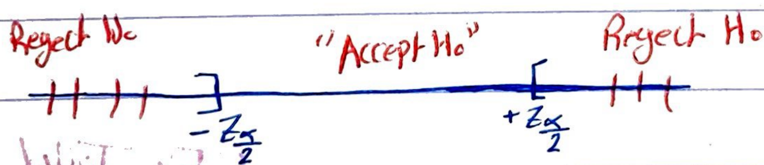
Regect H_0 if $Z \leq -Z_\alpha$ (Lower Tailed test) .

②



Regect H_0 if $Z \geq Z_\alpha$ (upper Tailed test) .

③

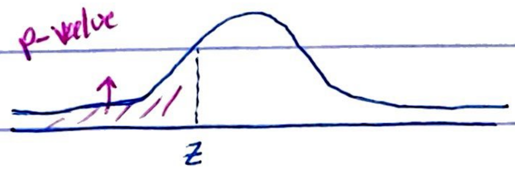


Regect H_0 if $Z \geq Z_{\frac{\alpha}{2}}$ or $Z \leq -Z_{\frac{\alpha}{2}}$

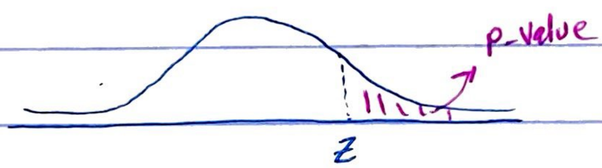
← الثلاث حالات هاي القاعدة الِي تختلف هو حساب P-value

→ **P-value Approach** : Regect H_0 if $p\text{-value} \leq \alpha$.

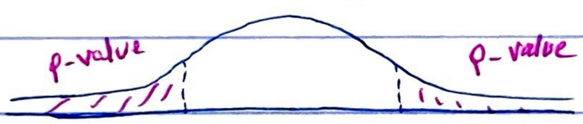
① Lower Tailed test .



② upper Tailed test :



③ Two Tailed test :



Exercise 1 on textbook:

sample 1	sample 2
$n_1 = 50$	$n_2 = 35$ → large enoghe.
$\bar{x}_1 = 13.6$	$\bar{x}_2 = 11.6$
$\sigma_1 = 2.2$	$\sigma_2 = 3.0$

Two independent random sample .

(i) Can we use the CI and testing procedures of sec. 10.1 ? اذا السؤال تحقق الشروط الي ب 10.1

yes

(2) Are the assumptions satisfied ?

sample 1 its Random.

sample 2 its Random.

sample 1 and 2 are independent.

Both sample are Large enough. $n_1, n_2 \geq 30$

(3) Find a point estimator for the deff. between the two population ?

$$\bar{x}_1 - \bar{x}_2 = 13.6 - 11.6 = 2$$

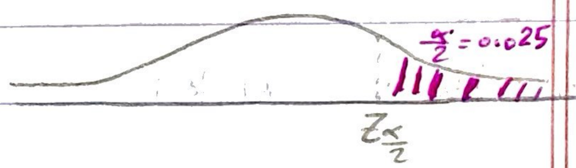
(4) provide a $\underset{1-\alpha}{95}$ percent CI for $\mu_1 - \mu_2$?

$$(1-\alpha)\,CI \text{ for } \mu_1 - \mu_2 = (\bar{x}_1 - \bar{x}_2) \mp Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$\rightarrow \quad 1-\alpha = 0.95 \rightarrow \alpha = 0.05 \rightarrow \frac{\alpha}{2} = 0.025$

$Z_{\frac{\alpha}{2}} = 1.96$

من الجدول بس للمادة
الي خالصين من جدول t



تعويض $\rightarrow$

$$= 2 \mp 1.96 \sqrt{\frac{4.84}{50} + \frac{9}{35}} =$$

(5) consider the following Hypothisis test : $H_0 : \mu_1 - \mu_2 \leq \underset{}{\overset{D_0}{0}}$, use $\alpha = 0.05$ to conclude ?

$H_1 : \mu_1 - \mu_2 > 0$

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = 3.36$$

$\rightarrow$

في المستقبل

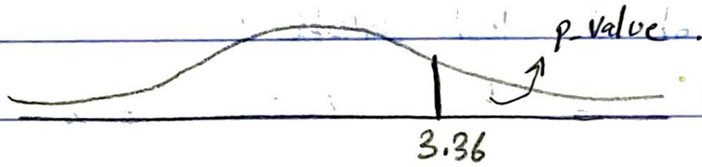
conti. → critical value Approach (Two tale) .

من الجدول t
$\alpha = 0.05$

⟹ conclusion Reject H_0 $(\alpha = 0.05)$.

→ $\mu_1 > \mu_2$ احتمال الخطأ 0.05

→ p-value



من الجدول

p-value < 0.005

p-value < α

Conclusion Reject H_0 . $(\alpha = 0.05)$

$\mu_1 > \mu_2$ $(\alpha = 0.05)$