Lecture Notes on **Discrete Mathematics**.
Birzeit University, Palestine, 2015
Modified in July 2021

First Order Logic

Mustafa Jarrar

&

Radi Jarrar

1. **Predicates and Quantified Statements I**
2. Predicates and Quantified Statements II
3. *Statements with Multiple Quantifiers*

 ,

Watch this lecture and download the slides

http://jarrar-courses.blogspot.com/2014/03/discrete-mathematics-course.html

More Lectures Courses at: http://www.jarrar.info

Acknowledgement:
This lecture is based on, but not limited to, chapter 3 in "Discrete Mathematics with Applications by Susanna S. Epp (3rd Edition)".

In this Lecture

We will learn

- **Part 1: What is a predicate, and Predicate Logic**
- Part 2: Universal and Existential Quantifiers: ∀ , ∃
- Part 3: Formalize and Verablize Statements;
- Part 4: Different Writings of Quantified Statements;
- Part 5: Tarski's World (Simple Example)

Keywords: Predicates, FOL, First Order Logic, Universal Quantifier, Existential Quantifier, Tarski's World.

First Order Logic

is also called:

- The Logic of Quantified Statements
- Predicate Logic
- First--Order Predicate Calculus
- Lower Predicate Calculus
- Quantification theory

 ,

First Order Logic

A **proposition** is basically a sentence that has a truth value that can either be true or false, but it needs to be assigned any of the two values and not be ambiguous. Propositional logic is used to analyze a statement or group of statements.

Predicates can be seen as properties or additional information to express the subject of the sentence.

A quantified predicate is a proposition, that is, when you assign values to a predicate with variables it can be made a proposition.

What is First Order Logic?

Propositional Logic

- Set of propositional symbols (e.g., Ahmed, Student, P, Q)
- No binding of variables (joined together by logical operators to form sentences)

$\neg P$ Negation
$P \wedge Q$ Conjunction
$P \vee Q$ Disjunction
$P \rightarrow Q$ Implication
$P \leftrightarrow Q$ Equivalence

We regard the world as *Propositions*

First Order Logic

P(x..y), Q(t,...s) Predicates

(Allows quantification over variables)

$\neg P$ Negation
$P \wedge Q$ Conjunction
$P \vee Q$ Disjunction
$P \rightarrow Q$ Implication
$P \leftrightarrow Q$ Equivalence
$\forall$ Universal quantification
$\exists$ *Existential* quantification

We regard the world as *Quantified Predicates*

What is Predicate?

• Definition

A **predicate** is a sentence that contains a finite number of variables and becomes a statement when specific values are substituted for the variables. The **domain** of a predicate variable is the set of all values that may be substituted in place of the variable.

$$P(x_1, x_2 \; , \ldots , x_n)$$

Examples

First order logic	Propositional logic
Person(Amjad)	Amjad is a person.
University(BZU)	BZU is a university.
StudyAt(Amjad, BZU)	Amjad studies at BZU.

,

Arity of Predicates

Arity is the number of arguments or operands taken by a function or relation in logic, mathematics, and computer science

$$P(x_1, x_2\ , \dots , x_n)$$

Predicate Name

Arguments

Examples:

Unary Predicates: Person(Amjad), University(BZU)

Binary Predicates: StudyAt(Amjad, BZU)

Ternary Predicates StudyAt(Amjad,BZU, CS)

Quaternary Predicate: StudyAt(Amjad,BZU, CS, 2015)

n-ary Predicate: StudyAt(Amjad,BZU, CS, 2015, BA, ...)

Truth of Predicates

- **Definition**

If $P(x)$ is a predicate and x has domain D, the **truth set** of $P(x)$ is the set of all elements of D that make $P(x)$ true when they are substituted for x. The truth set of $P(x)$ is denoted

$$\{x \in D \mid P(x)\}.$$

such that

$\{x \in Organization \mid University(x)\}$

The set of all organizations that are universities.

$\{x \in Person \mid student(x)\}$

The set of all persons that are students.

In this Lecture

We will learn

- Part 1: What is a predicate, and Predicate Logic
- **Part 2: Universal and Existential Quantifiers: ∀, ∃**
- Part 3: Formalize and Verablize Statements;
- Part 4: Different Writings of Quantified Statements;
- Part 5: Tarski's World (Simple Example)

Part 2

The Universal Quantifier: $\forall$

• Definition

Let $Q(x)$ be a predicate and D the domain of x. A **universal statement** is a statement of the form "$\forall x \in D, Q(x)$." It is defined to be true if, and only if, $Q(x)$ is true for every x in D. It is defined to be false if, and only if, $Q(x)$ is false for at least one x in D. A value for x for which $Q(x)$ is false is called a **counterexample** to the universal statement.

$\forall P \in$ Palestinian . Likes(p, Zatar) All Palestinians Like Zatar.

False

$\forall x \in \mathbf{R}, x^2 \geq x.$ False

$\frac{1}{4} \ngeq \frac{1}{2}$ ×

counterexample

Let $D = \{1, 2, 3, 4, 5\}$, $\forall x \in D, x^2 \geq x.$

True

Part 3

The Existential Quantifier: ∃

• Definition

Let $Q(x)$ be a predicate and D the domain of x. An **existential statement** is a statement of the form "$\exists x \in D$ such that $Q(x)$." It is defined to be true if, and only if, $Q(x)$ is true for at least one x in D. It is false if, and only if, $Q(x)$ is false for all x in D.

$$\exists p \in \text{Person} \ . \ \text{Likes}(p, \text{Zatar})$$

True

integers

$$\exists m \in \mathbf{Z}^{+} \text{ such that } m^2 = m.$$

True $1 = 1$

Let $E = \{5, 6, 7, 8\}$ $\quad \exists m \in E$ such that $m^2 = m$.

False

In this Lecture

We will learn

- Part 1: What is a predicate, and Predicate Logic
- Part 2: Universal and Existential Quantifiers: ∀, ∃
- **Part 3: Formalize and Verablize Statements;**
- Part 4: Different Writings of Quantified Statements;
- Part 5: Tarski’s World (Simple Example)

Verbalizing Formal Statements

Write the following formal statements in an informal language:

$\forall x \in \mathbf{R}, x^2 \geq 0.$

All real numbers have non-negative squared value

OR Every real number has a non-negative squared value

OR The square of any real number has a non-negative value

$\forall x \in \mathbf{R}, x^2 \neq -1.$ $(\neg \exists x \in R, x^2 = -1)$

All real numbers have squares that are not equal to -1

OR No real value has a square equals to -1

$\exists m \in \mathbf{Z}^+$ such that $m^2 = m.$

There is a positive integer whose square equals to itself

OR We can find at least one positive integer equal to its own square

OR some positive integer equals to its own square

$\forall x \in \mathbf{R}$, if $x > 2$ then $x^2 > 4.$

If a real number is greater than 2 then its square is greater than 4

OR The square of any real number is greater than 4 x

OR the squares of all real numbers greater than 2 are greater than 4

Formalize Statements

Write the following informal statements in a formal language:

All triangles have three sides

$\forall$ *triangles t, t has three sides*

OR $\forall t \in T$, *t has three sides*

No dogs have wings ($\forall d \in Dogs. \neg HasWings(d)$)

$\forall$ *dogs d, d has no wings* ($\neg \exists d \in Dogs. HasWings(d)$)

OR $\forall d \in D$, *d does not have wings*

Some programs are structured ($\exists p \in P. Structured(p)$)

$\exists$ *a program p such that p is structured*

OR $\exists$p ∈ P such that p is structured

If a real number is an integer, then it is a rational number

$\forall$ *real numbers x, if x is an integer, then x is a rational number*

OR $\forall x \in R$, *if* $x \in Z$ *then* $x \in Q$ ($\forall i \in Z. i \in Rational$)

All bytes have eight bits

$\forall$ *x, if x is a byte then x has eight bits*

($\forall x \in bytes. HaveEightBits(x)$)

No fire trucks are green

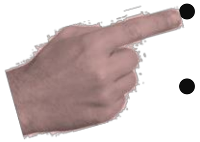
$\forall$ *x, if x is a firetruck then x is not green*

($\neg \exists x \in$ fire trucks, $Green(x)$) ($\forall t \in$ Fire trucks, $\neg Green(t)$)

In this Lecture

We will learn

- Part 1: What is a predicate , and Predicate Logic
- Part 2: Universal and Existential Quantifiers: $\forall$, $\exists$
- Part 3: Formalize and Verablize Statements;
- **Part 4: Different Writings of Quantified Statements;**
- Part 5: Tarski’s World (Simple Example)



Different Writings

$\forall\ x \in Square$. Rectangle (x)

$\forall x$. If x is as square then x is a rectangle

$\forall$ *Squares x* . *x* *is a* a rectangle

Although the book uses this notation but it's not recommended as predicates are not clear.

$\forall$ p$\in$Palestinian . Likes(p, Zatar)

$\forall$ P . Palestinian(p) $\wedge$ Likes(p, Zatar)

$\exists$p$\in$Person . Likes(p, Zatar)

$\exists$p. Person(p) $\wedge$ Likes(p, Zatar)

,

Quantifications might be Implicit

Formalize the following:

If a number is an integer, then it is a rational number.

$\forall n \cdot Integer(n) \rightarrow Rational(n)$

If a person was born in Palestine then s/he is Palestinian

$\forall x \in Person \cdot BornInPalestine(x) \rightarrow Palestinian(x)$

$\forall x \in Person \cdot BornIn(x, Palestine) \rightarrow Palestinian(x)$

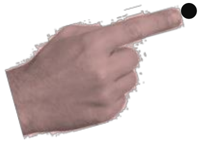
People like Hommos are smart

$\forall x \in Person \cdot Like(x, Homos) \rightarrow Smart(x)$

$\forall x \in Person \cdot LikeHomos(x) \rightarrow Smart(x)$

In this Lecture

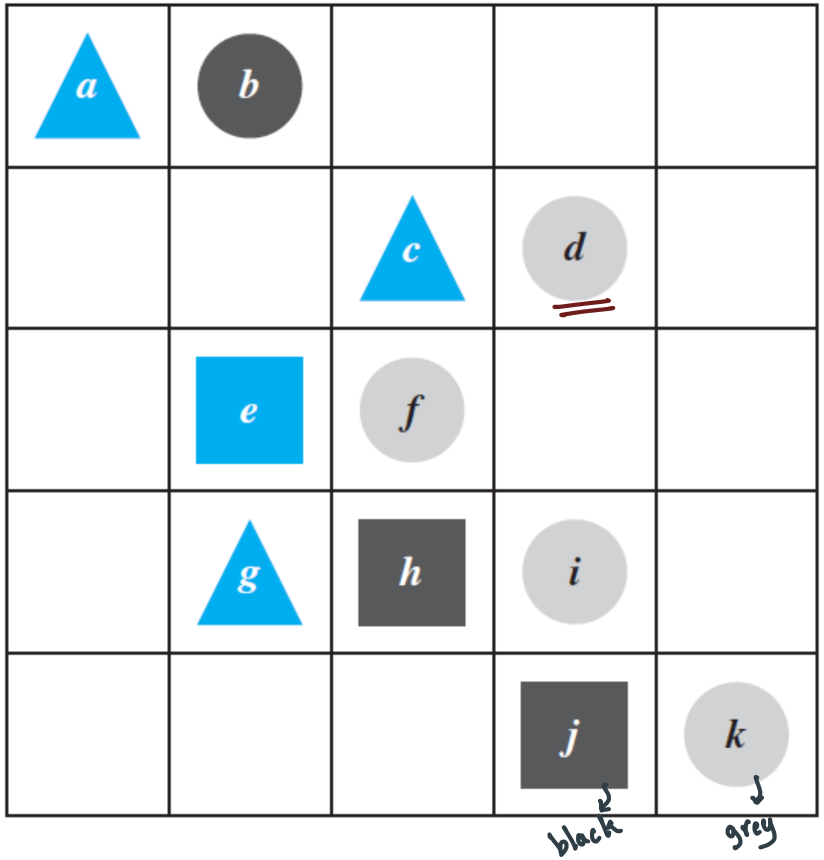
We will learn



- Part 1: What is a predicate, and Predicate Logic
- Part 2: Universal and Existential Quantifiers: $\forall$, $\exists$
- Part 3: Formalize and Verablize Statements;
- Part 4: Different Writings of Quantified Statements;
- **Part 5: Tarski’s World (Simple Example)**

Tarski's World Example

The following statements are true or false?



$\forall t \in$ triangle, Blue(t)

$\forall t \ .\ \text{Triangle}(t) \rightarrow \text{Blue}(t).$ True

$\forall x \in$ Blue, Triangle(x)

$\forall x \ .\ \text{Blue}(x) \rightarrow \text{Triangle}(x).$ False

object

$\exists y \ .\ \text{Square}(y) \wedge \text{RightOf}(d, y).$ True

مش شرط d على يمينه بنفس الصف.

$\exists z \ .\ \text{Square}(z) \wedge \text{Gray}(z).$ False

First Order Logic

Mustafa Jarrar

1. Predicates and Quantified Statements I

2. **Predicates and Quantified Statements II**

3. Statements with Multiple Quantifiers

In this Lecture

We will learn

- ❑ **Part1: Negations of Quantified Statements;**
- ❑ Part 2: Contrapositive, Converse and inverse Quantified Statements;
- ❑ Part 3: Necessary and Sufficient Conditions, Only If

Keywords: Predicates, FOL, First Order Logic, Universal Quantifier, Existential Quantifier, Negation, Truth of Universal Statements, Necessary and Sufficient Conditions

Negations of Quantified Statements

How to negate a universal statement:

All Palestinians like Zatar

Some Palestinians do not like Zatar

Theorem 3.2.1 Negation of a Universal Statement

The negation of a statement of the form

$$\forall x \text{ in } D, Q(x)$$

is logically equivalent to a statement of the form

$$\exists x \text{ in } D \text{ such that} \sim Q(x).$$

Symbolically, $\sim(\forall x \in D, Q(x)) \equiv \exists x \in D \text{ such that} \sim Q(x).$

,

Negations of Quantified Statements

How to negate an extensional statement:

Some Palestinians Like Zatar

All Palestinians do not like Zatar

Theorem 3.2.2 Negation of an Existential Statement

The negation of a statement of the form

$$\exists x \text{ in } D \text{ such that } Q(x)$$

is logically equivalent to a statement of the form

$$\forall x \text{ in } D, \sim Q(x).$$

Symbolically, $\sim(\exists x \in D \text{ such that } Q(x)) \equiv \forall x \in D, \sim Q(x).$

Negations of Quantified Statements

$\forall\ p \in$ Prime . Odd(p)

$\exists p \in$ Prime . ~Odd(p)

Some computer hackers are over 40

All computer hackers are not over 40

All computer programs are finite

Some computer programs are not finite

Negations of Quantified Statements

No politicians are honest

Some politicians are honest

$\forall x\,.\,P(x) \rightarrow Q(x)$ (~p ∨ q)

$\exists x\,.\,P(x) \wedge {\sim}Q(x)$

$\forall p \in \text{Person}\,.\,\text{Blond}(p) \rightarrow \text{BlueEyes}(p)$

$\exists p \in \text{Person}\,.\,\text{Blond}(p) \wedge {\sim}\text{BlueEyes}(p)$

If a computer program has more than 10000 lines then it contains a bug

A computer program has more than 10000 and does not contains a bug

or some

or There is

In this Lecture

We will learn

- ❑ Part1: Negations of Quantified Statements;
- ❑ **Part 2: Contrapositive, Converse and Inverse Quantified Statements;**
- ❑ Part 3: Necessary and Sufficient Conditions, Only If

Keywords: Predicates, FOL, First Order Logic, Universal Quantifier, Existential Quantifier, Negation, Truth of Universal Statements, Necessary and Sufficient Conditions

Variants of Universal Conditional Statements

- **Definition**

Consider a statement of the form: $\forall x \in D$, if $P(x)$ then $Q(x)$.

1. Its **contrapositive** is the statement: $\forall x \in D$, if $\sim Q(x)$ then $\sim P(x)$.
2. Its **converse** is the statement: $\forall x \in D$, if $Q(x)$ then $P(x)$.
3. Its **inverse** is the statement: $\forall x \in D$, if $\sim P(x)$ then $\sim Q(x)$.

$$\forall x \in \text{Person} \quad . \quad \text{Palestinian}(x) \rightarrow \text{Smart}(x)$$

Contrapositive: $\forall x \in \text{Person} \quad . \quad \sim \text{Smart}(x) \rightarrow \sim \text{Palestinian}(x)$

Converse: $\forall x \in \text{Person} \quad . \quad \text{Smart}(x) \rightarrow \text{Palestinian}(x)$

Inverse: $\forall x \in \text{Person} \quad . \quad \sim \text{Palestinian}(x) \rightarrow \sim \text{Smart}(x)$

Variants of Universal Conditional Statements

$$\forall x \in \mathbf{R}.\ \ x > 2 \rightarrow x^2 > 4.$$

$$\forall x \in \mathbf{R}.\ \text{MoreThan}(x,2) \rightarrow \text{MoreThan}(x^2,4)$$

Contrapostive: $\forall x \in \mathbf{R}\ .\ x^2 \leq 4 \rightarrow x \leq 2$

Converse: $\forall x \in \mathbf{R}\ .\ x^2 > 4 \rightarrow x > 2$

Inverse: $\forall x \in \mathbf{R}\ .\ x \leq 2 \rightarrow x^2 \leq 4$

Variants of Universal Conditional Statements

- **Definition**

Consider a statement of the form: $\forall x \in D$, if $P(x)$ then $Q(x)$.

1. Its **contrapositive** is the statement: $\forall x \in D$, if $\sim Q(x)$ then $\sim P(x)$.
2. Its **converse** is the statement: $\forall x \in D$, if $Q(x)$ then $P(x)$.
3. Its **inverse** is the statement: $\forall x \in D$, if $\sim P(x)$ then $\sim Q(x)$.

Logically equivalent

$$\forall x \in D, \text{ if } P(x) \text{ then } Q(x) \equiv \forall x \in D, \text{ if } \sim Q(x) \text{ then } \sim P(x)$$

$$\forall x \in D, \text{ if } P(x) \text{ then } Q(x) \not\equiv \forall x \in D, \text{ if } Q(x) \text{ then } P(x).$$

$$\forall x \in D, \text{ if } P(x) \text{ then } Q(x) \not\equiv \forall x \in D, \text{ if } \sim P(x) \text{ then } \sim Q(x).$$

In this Lecture

We will learn

- ❑ Part1: Negations of Quantified Statements;
- ❑ Part 2: Contrapositive, Converse and inverse Quantified Statements;
- ❑ **Part 3: Necessary and Sufficient Conditions, Only If**

Keywords: Predicates, FOL, First Order Logic, Universal Quantifier, Existential Quantifier, Negation, Truth of Universal Statements, Necessary and Sufficient Conditions

Necessary and Sufficient Conditions

- **Definition**
  - "$\forall x, r(x)$ is a **sufficient condition** for $s(x)$" means "$\forall x, r(x) \rightarrow s(x)$."
  - "$\forall x, r(x)$ is a **necessary condition** for $s(x)$" means "$\forall x, \sim r(x) \rightarrow \sim s(x)$" or, equivalently, "$\forall x, s(x) \rightarrow r(x)$."

Example:

Squareness is a sufficient condition for rectangularity.

If something is a square, then it is a rectangle.

$\forall$ x . Square(x) → Rectangular(x)

To get a job it is sufficient to be loyal.

If one is loyal (s)he will get a job

$\forall$ x . Loyal(x) → GotaJob(x)

Necessary and Sufficient Conditions

- **Definition**
  - "$\forall x, r(x)$ is a **sufficient condition** for $s(x)$" means "$\forall x, r(x) \rightarrow s(x)$."
  - "$\forall x, r(x)$ is a **necessary condition** for $s(x)$" means "$\forall x, \sim r(x) \rightarrow \sim s(x)$" or, equivalently, "$\forall x, s(x) \rightarrow r(x)$."

Example:

Being smart is necessary to get a job.

If you are not smart you don't get a job

If you got a job then you are smart

$\forall x$. ~Smart(x) → ~GotaJob(x)

$\forall x$. GotaJob(x) → Smart(x)

Being above 40 years is necessary for being president of Palestine

$\forall x$. ~Above(x, 40) → ~CanBePresidentOfPalestine(x)

$\forall x$. CanBePresidentOfPalestine(x) → Above(x, 40)

First Order Logic

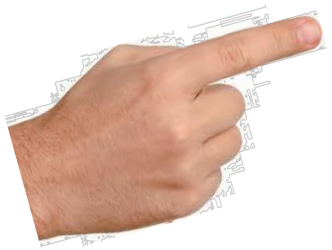
Mustafa Jarrar

&

Radi Jarrar

3.1 Predicates and Quantified Statements I

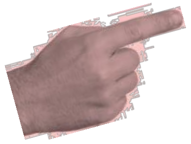
3.2 Predicates and Quantified Statements II



3.3 Statements with Multiple Quantifiers

In this Lecture

We will learn



- ❑ **Part1: Multiple and Order of Quantifiers**
- ❑ Part 2: Verbalization of Formal Statements
- ❑ Part 3: Formalization of informal Statements
- ❑ Part 4: Negations of Multiply-Quantified Statements
- ❑ Part 5: Example: Using FOL to formalize text (optional)

Keywords: Predicates, FOL, First Order Logic, Universal Quantifier, Existential Quantifier, Negation, Truth of Universal Statements, Formalization, verbalization, Order of quatifiers

Multiple and Order of Quantifiers

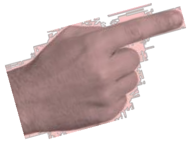
$\forall x\ \exists y\ .\ \text{Loves}(x,y)$ Everything loves something Each thing loves one or more things	$\exists x\ \forall y\ .\ \text{Loves}(x,y)$ Something loves everything
$\forall y\ \exists x\ .\ \text{Loves}(x,y)$ Everything is loved by something Everything has something that loves it	$\forall y\ \exists x\ .\ \text{Loves}(y,x)$ Everything loves something
$\exists y\ \forall x\ .\ \text{Loves}(x,y)$ Something is loved by everything Everyone love the same thing There exists something that everything loves it	$\forall x\ \exists y\ .\ \text{Loves}(x,y)\ ,\ x \neq y$ Everything loves something but not itself
$\forall x\ \forall y\ .\ \text{Loves}(x,y)$ $\forall x, y\ .\ \text{Loves}(x,y)$ Everything loves everything	$\exists x\ \exists y\ .\ \text{Loves}(x,y)$ $\exists x, y\ .\ \text{Loves}(x,y)$ something loves something

Multiple and Order of Quantifiers

Everyone loves all movies كل شخص يحب كل الافلام $\forall p_{\in Person}\ \forall m_{\in Movie} \cdot Loves(p,m)$	**Some people loves some movies** بعض الناس يحبون بعض الافلام $\exists p_{\in Person}\ \exists m_{\in Movie} \cdot Loves(p,m)$
There is a movie that everyone loves فلم يحبه كل الناس $\exists m_{\in Movie}\ \forall p_{\in Person} \cdot Lovedby(m,p)$ $\cdot Loves(p,m)$	**Some people love all movies** بعض الناس يحب كل الافلام $\exists p_{\in Person}\ \forall m_{\in Movie} \cdot Loves(p,m)$
Everyone loves some movies كل شخص يحب بعض الافلام $\forall p_{\in Person}\ \exists m_{\in Movie} \cdot Loves(p,m)$	**All movies are loved by someone** كل فلم له بعض المحبين $\forall m_{\in Movie}\ \exists p_{\in Person} \cdot Lovedby(m,p)$

In this Lecture

We will learn

- ❑ Part1: Multiple and Order of Quantifiers

- ❑ **Part 2: Formalise/Verbalization of Formal Statements**
- ❑ Part 3: Negations of Multiply-Quantified Statements
- ❑ Part 4: Example: Using FOL to formalize text (optional)

Keywords: Predicates, FOL, First Order Logic, Universal Quantifier, Existential Quantifier, Negation, Truth of Universal Statements, Formalization, verbalization, Order of quatifiers

Multiple Quantifiers with Negated Predicates

$\exists x \ \exists y \ . \ \sim Love(x,y)$

Somebody does not love somebody

بعض اشخاص لا يحبون بعض الاشخاص

$\forall x \ \forall y \ . \ \sim Love(x,y)$

Everyone does not love anyone

كل شخص لا يحب أي شخص

No one love any one.

لا احد يحب احد

$\exists x \ \forall y \ . \sim Love(x,y)$

Someone does not love anyone

يوجد شخص لا يحب أي الاشخاص

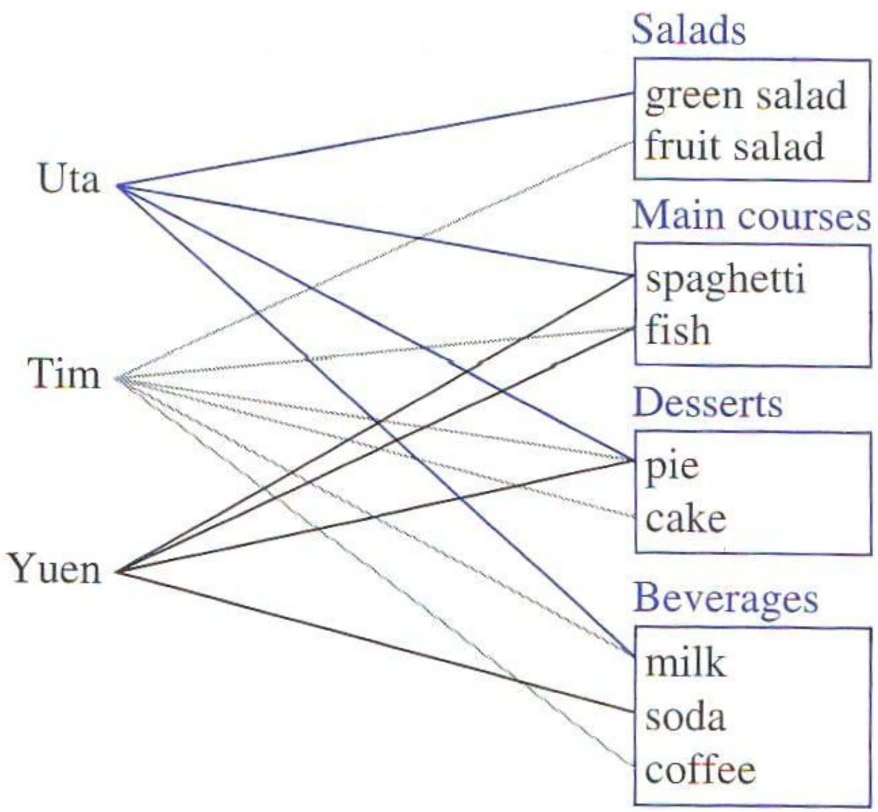
$\forall x \ \exists y \ . \ \sim Love(y,x)$

Everyone is not loved by someone

Everyone has some people who do not love him

كل شخص يوجد اخرين لا يحبونه

Verbalize and Test Statements



a. $\exists$ an item I such that $\forall$ students S, S chose I.

b. $\exists$ a student S such that $\forall$ items I, S chose I.

c. $\exists$ a student S such that $\forall$ stations Z, $\exists$ an item I in Z such that S chose I.

d. $\forall$ students S and $\forall$ stations Z, $\exists$ an item I in Z such that S chose I.

There is an item that was chosen by every student. → true

There is a student who chose every available item. →false

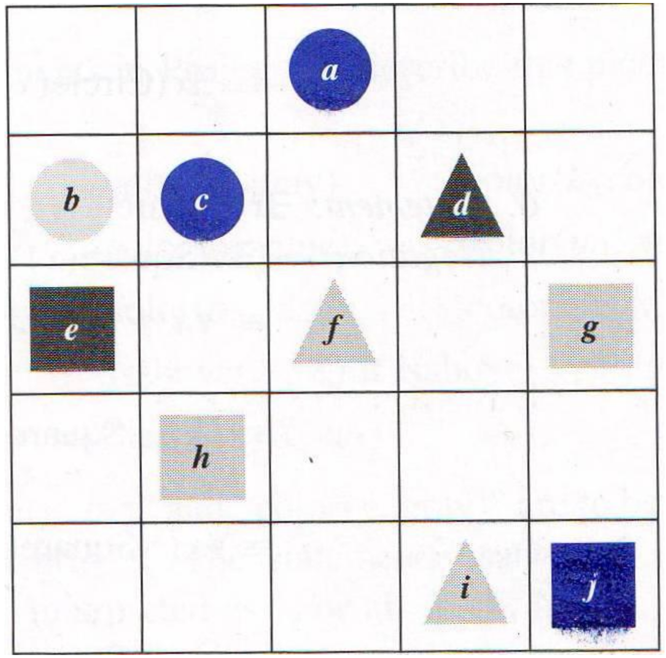
There is a student who chose at least one item from every station. → true

Every student chose at least one item from every station → false.

Tarski's world - Formalizing Statements

Describe Tarski's world using universal and external quantifiers

using Formal FOL Notation



a. For all circles x, x is above f.

$$\forall x(\text{Circle}(x) \rightarrow \text{Above}(x, f)).$$

OR $\forall x \in \text{Circle}, \text{Above}(x,f)$

b. There is a square x such that x is black.

$$\exists x(\text{Square}(x) \wedge \text{Black}(x)).$$

c. For all circles x, there is a square y such that x and y have the same color.

$$\forall x(\text{Circle}(x) \rightarrow \exists y(\text{Square}(y) \wedge \text{SameColor}(x, y))).$$

d. There is a square x such that for all triangles y, x is to right of y.

$$\exists x(\text{Square}(x) \wedge \forall y(\text{Triangle}(y) \rightarrow \text{RightOf}(x, y))).$$

Formalize these statements

The **reciprocal (نظير ضربي)** of a real number *a* is a real number *b* such that *a.b* = 1. The following two statements are true. Rewrite them formally using quantifiers and variables:

Every nonzero real number has a reciprocal.

$\forall u \in \text{NonZeroR}, \exists v \in \text{R} \;.\; uv = 1.$

There is a real number with no reciprocal.

The number 0 has no reciprocal.

$\exists c \in \text{R} \;\; \forall d \in \text{R},. \;\; cd \neq 1.$

Formalize these statements

There Is a Smallest Positive Integer

$\exists m \in Z^+ \ \forall n \in Z^+ .\ LessOrEqual(m,n)$

In the book:

∃ a positive integer m such that ∀ positive integers n, $m \leq n$.

There Is No Smallest Positive Real Number

$\forall x \in R^+ \ \exists y \in R^+ \ .\ Less(y,x)$

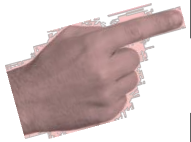
In the book:

∀ positive real numbers x, ∃ a positive real number y such that $y < x$.

In this Lecture

We will learn

- ❑ Part1: Multiple and Order of Quantifiers
- ❑ Part 2: Formalise/Verbalization of Formal Statements
- ❑ **Part 3: Negations of Multiply-Quantified Statements**
- ❑ Part 4: Example: Using FOL to formalize text (optional)



Keywords: Predicates, FOL, First Order Logic, Universal Quantifier, Existential Quantifier, Negation, Truth of Universal Statements, Formalization, verbalization, Order of quatifiers

Negations of Multiply-Quantified Statements

$$\sim(\forall x \text{ in } D, \exists y \text{ in } E \text{ such that } P(x, y)) \equiv \exists x \text{ in } D \text{ such that } \forall y \text{ in } E, \sim P(x, y).$$

$$\sim(\exists x \text{ in } D \text{ such that } \forall y \text{ in } E, P(x, y)) \equiv \forall x \text{ in } D, \exists y \text{ in } E \text{ such that } \sim P(x, y).$$

Examples:

$\sim (\forall x \, \exists y \, . \, \text{Loves}\,(x,y))$

$\exists x \, \forall y \, . \, \sim\text{Loves}\,(x,y)$

$\sim(\exists x \, \forall y \, . \, \text{Loves}\,(x,y))$

$\forall x \, \exists y \, . \, \sim\text{Loves}\,(x,y)$

Negations of Multiply-Quantified Statements

Not all people love someone.

~ (all people love someone)

$\sim(\forall x\ \exists y\ .\ Love(x,y))$

$\exists x\ \ \forall y\ .\ \sim Love(x,y))$

Some people do not love everyone

Not all people love everyone.

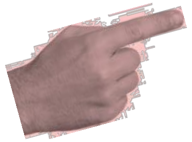
~ (All people love everyone)

$\sim \forall x \forall y\ Like(x, y)$

In this Lecture

We will learn

- ❑ Part1: Multiple and Order of Quantifiers
- ❑ Part 2: Formalise/Verbalization of Formal Statements
- ❑ Part 3: Negations of Multiply-Quantified Statements
- ❑ **Part 4: Example: Using FOL to formalize text (optional)**



Keywords: Predicates, FOL, First Order Logic, Universal Quantifier, Existential Quantifier, Negation, Truth of Universal Statements, Formalization, verbalization, Order of quatifiers

Example: Using FOL to formalize text

Example from: Russell & Norvig Book

The law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

Example: Using FOL to formalize text

... it is a crime for an American to sell weapons to hostile nations:

$\forall x,y,z \,.\, American(x) \wedge Weapon(y) \wedge Sells(x,y,z) \wedge Hostile(z) \Rightarrow Criminal(x)$

if then

Nono … has some missiles, i.e.,

$\exists x \,.\, Owns(Nono,x) \wedge Missile(x)$

… all of its missiles were sold to it by Colonel West

$\forall x \,.\, Missile(x) \wedge Owns(Nono,x) \Rightarrow Sells(West,x,Nono)$

Missiles are weapons:

$\forall x \,.\, Missile(x) \Rightarrow Weapon(x)$

An enemy of America counts as "hostile“:

$\forall x \,.\, Enemy(x,America) \Rightarrow Hostile(x)$

West, who is American …

$American(West)$

The country Nono, an enemy of America …

$Enemy(Nono,America)$