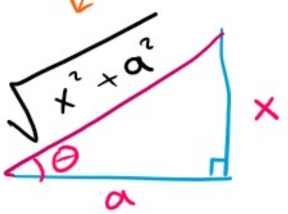
Exp How to evaluate $\int \sqrt{9+x^2}\,dx$?

$\int \sqrt{x^2-9}$? $\int \sqrt{9-x^2}$?

$\int \frac{dx}{9+x^2}$?

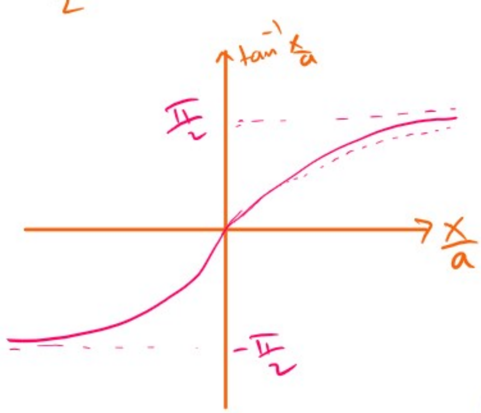


$x = a\tan\theta$

$dx = a\sec^2\theta\,d\theta$

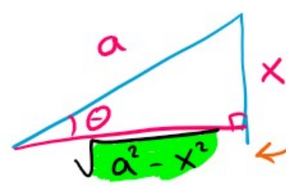
$\tan\theta = \frac{x}{a} \Rightarrow \theta = \tan^{-1}\frac{x}{a}$

$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$



$\sqrt{x^2+a^2} = \sqrt{a^2\tan^2\theta + a^2}$

$= \sqrt{a^2(\tan^2\theta + 1)}$

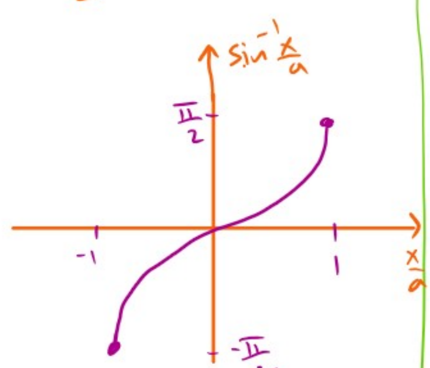


$x = a\sin\theta$

$dx = a\cos\theta\,d\theta$

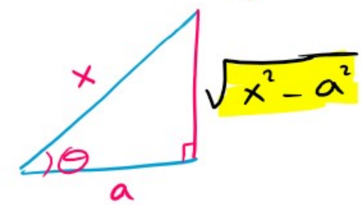
$\sin\theta = \frac{x}{a} \Rightarrow \theta = \sin^{-1}\frac{x}{a}$

$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$



$\sqrt{a^2-x^2} = \sqrt{a^2 - a^2\sin^2\theta}$

$= \sqrt{a^2(1-\sin^2\theta)}$



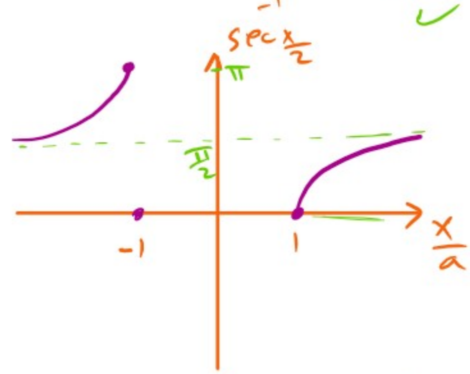
$x = a\sec\theta$

$dx = a\sec\theta\tan\theta\,d\theta$

$\sec\theta = \frac{x}{a} \Rightarrow \theta = \sec^{-1}\frac{x}{a}$

$\boxed{0 \le \theta < \pi}$

$\frac{x}{a} \ge 1$



$\frac{\pi}{2} < \theta \le \pi$ X

$\frac{x}{a} \le -1$ X

$\sqrt{x^2-a^2} = \sqrt{a^2\sec^2\theta - a^2}$

$=\sqrt{a^2(\tan^2\theta+1)}$

$= a\sqrt{\sec^2\theta}$

$= a|\sec\theta|$

$= a\sec\theta$ ✓

$=\sqrt{a^2(1-\sin^2\theta)}$

$= a\sqrt{\cos^2\theta}$

$= a|\cos\theta|$

$= a\cos\theta$ ✓

$\sqrt{x^2-a^2}=\sqrt{a^2\sec^2\theta-a^2}$

$=\sqrt{a^2(\sec^2\theta-1)}$

$= a\sqrt{\tan^2\theta}$

$= a|\tan\theta|$

$= a\tan\theta$

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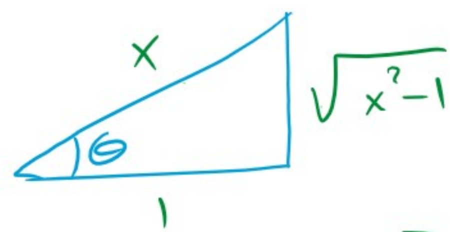
(14) Exp

$$\int \frac{2\,dx}{x^3\sqrt{x^2-1}}\ , \quad x>1$$

$a=\sqrt{1}=1$

$a=1$



$x = a\sec\theta$

$= \sec\theta$

$dx = \sec\theta\tan\theta\, d\theta$

$\sqrt{x^2-1} = a\tan\theta = \tan\theta$

$$\int \frac{2\,\cancel{\sec\theta}\,\cancel{\tan\theta}\,d\theta}{\sec^{\cancel{3}\,2}\theta\ \cancel{\tan\theta}} = 2\int \frac{d\theta}{\sec^2\theta} = 2\int \cos^2\theta\, d\theta$$

$$= \cancel{2}\int \left(\frac{1+\cos 2\theta}{\cancel{2}}\right) d\theta = \int (1+\cos 2\theta)\,d\theta$$

$$= \theta + \frac{\sin 2\theta}{2} + C$$

$$= \sec^{-1} x + \frac{\cancel{2} \sin\theta \cos\theta}{\cancel{2}} + c$$

$$= \sec^{-1} x + \frac{\sqrt{x^2-1}}{x} \quad \frac{1}{x} + c$$

24 Exp 2

$$\int_0^1 \frac{dx}{(4-x^2)^{\frac{3}{2}}} = \int_0^1 \frac{dx}{\left(\sqrt[2]{4-x^2}\right)^3}$$

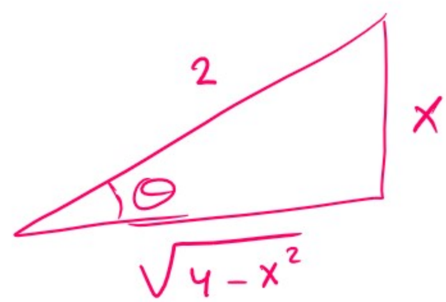
$a = \sqrt{4} = 2$

$x = a \sin\theta$

$= 2 \sin\theta$

$dx = 2 \cos\theta \, d\theta$

$\sqrt{4-x^2} = a \cos\theta = 2 \cos\theta$



$$\int_0^{\frac{\pi}{6}} \frac{2\cos\theta \, d\theta}{(2\cos\theta)^3}$$

$x = 2 \sin\theta$

$\theta = \sin^{-1} \frac{x}{2}$

$x = 0 \Rightarrow \theta = \sin^{-1} 0 = 0$

$x = 1 \Rightarrow \theta = \sin^{-1} \frac{1}{2} = \frac{\pi}{6}$

$$\int_0^{\frac{\pi}{6}} \frac{\cancel{2\cos\theta} \, d\theta}{\underset{4}{\cancel{8}} \cos^{\cancel{3}\,2}\theta} = \frac{1}{4} \int_0^{\frac{\pi}{6}} \sec^2\theta \, d\theta$$

$$= \frac{1}{4} \tan\theta \Big|_0^{\frac{\pi}{6}} = \frac{1}{4} \left[\tan\frac{\pi}{6} - \cancel{\tan 0} \right] \quad 0$$

$$= \frac{1}{4} \quad \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{4\sqrt{3}}$$

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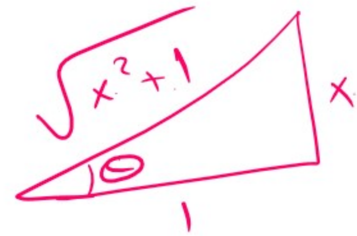
1 0

$$= \frac{1}{4} \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{4\sqrt{3}}$$

18 Exp

$$\int \frac{dx}{x^2 \sqrt{x^2+1}}$$

$a = \sqrt{1} = 1$



$x = a \tan\theta$

$= \tan\theta$

$dx = \sec^2\theta \, d\theta$

$\sqrt{x^2+1} = a\sec\theta = \sec\theta$

$$\int \frac{\cancel{\sec^2}\theta \, d\theta}{\tan^2\theta \, \cancel{\sec\theta}} = \int \frac{\frac{1}{\cancel{\cos\theta}} \, d\theta}{\frac{\sin^2\theta}{\cancel{\cos^2}\theta}}$$

$$= \int \frac{\cos\theta \, d\theta}{\sin^2\theta}$$

$u = \sin\theta$

$du = \cos\theta \, d\theta$

$$= \int \frac{du}{u^2} = \int u^{-2} du = \frac{u^{-1}}{-1} + C$$

$$= -\frac{1}{\sin\theta} + C$$

$$= -\csc\theta + C$$

$$= -\frac{\sqrt{x^2+1}}{x} + C$$

dx

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Exp (10)

$$\int \frac{5\,dx}{\sqrt{25x^2-9}} = \int \frac{\cancel{5}\,dx}{\sqrt{\cancel{25}\left(x^2-\frac{9}{25}\right)}} = \int \frac{dx}{\sqrt{x^2-\frac{9}{25}}}$$

$a = \sqrt{\frac{9}{25}} = \frac{3}{5}$

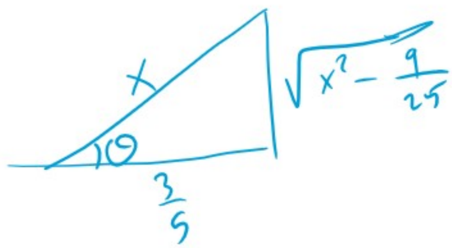
$x = a\sec\theta = \frac{3}{5}\sec\theta$

$dx = \frac{3}{5}\sec\theta\tan\theta\,d\theta$

$\sqrt{x^2-\frac{9}{25}} = a\tan\theta = \frac{3}{5}\tan\theta$

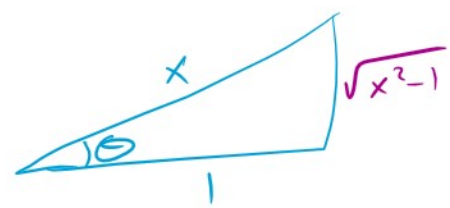
$$\int \frac{\cancel{\frac{3}{5}}\sec\theta\,\cancel{\tan\theta}\,d\theta}{\cancel{\frac{3}{5}}\,\cancel{\tan\theta}} = \int \sec\theta\,d\theta = \ln\left|\sec\theta+\tan\theta\right| + c$$

$$= \ln\left|\frac{x}{\frac{3}{5}} + \frac{\sqrt{x^2-\frac{9}{25}}}{\frac{3}{5}}\right| + c$$



Exp 26

$$\int \frac{x^2\,dx}{(x^2-1)^{\frac{5}{2}}}, \quad x>1$$



$$\int \frac{x^2\,dx}{\left(\sqrt{x^2-1}\right)^5}$$

$a = \sqrt{1} = 1$

$x = a\sec\theta = \sec\theta$

$dx = \sec\theta\tan\theta\,d\theta$

$\sqrt{x^2-1} = a\tan\theta = \tan\theta$

$$\int \frac{\sec^2\theta\,\sec\theta\,\cancel{\tan\theta}\,d\theta}{(\tan\theta)^{\cancel{5}\,4}}$$

$$\int \frac{1}{\ldots}\;\frac{\cos^4\theta}{\ldots}\,d\theta = \int \frac{\cos\theta\,d\theta}{\sin^4\theta}$$

$\int \sec^3\theta \ldots$

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$$\int \frac{\sec^3\theta}{\tan^4\theta}\,d\theta = \int \frac{1}{\cos^3\theta}\cdot\frac{\cos^4\theta}{\sin^4\theta}\,d\theta = \int \frac{\cos\theta\,d\theta}{\sin^4\theta}$$

$$\tan^4\theta = \left(\frac{\sin\theta}{\cos\theta}\right)^4$$

$u = \sin\theta$

$du = \cos\theta\,d\theta$

$$\int \frac{du}{u^4} = \int u^{-4}\,du = \frac{u^{-3}}{-3} + C$$

$$= -\frac{1}{3}\,\frac{1}{u^3} + C = -\frac{1}{3}\,\frac{1}{\sin^3\theta} + C$$

$$= -\frac{1}{3}\csc^3\theta + C$$

$$= -\frac{1}{3}\left(\frac{x}{\sqrt{x^2-1}}\right)^3 + C$$