(5.1) 6 $\int_1^e x^3 \ln x \, dx$

Let $u = \ln x \qquad dv = x^3 \, dx$

$du = \frac{1}{x} dx \qquad v = \frac{x^4}{4}$

$-\int$

$$\int_1^e x^3 \ln x \, dx = \frac{x^4}{4} \ln x \Big]_1^e - \int \left(\frac{x^4}{4}\right) \cdot \left(\frac{1}{x}\right) dx$$

$$= \frac{x^4}{4} \ln x \Big]_1^e - \frac{1}{4} \int_1^e x^3 \, dx$$

$$= \frac{e^4}{4} - \frac{1}{4} \frac{x^4}{4} \Big]_1^e = \frac{e^4}{4} - \frac{e^4}{16} + \frac{1}{16} = \frac{3e^4+1}{16}$$

11 $\int \tan^{-1} y \, dy$.

Let $u = \tan^{-1} y \qquad , dv = dy$

$du = \frac{1}{1+y^2} dy \qquad , v = y$

$$\int \tan^{-1} y \, dy = y \tan^{-1} y - \int \frac{y}{1+y^2} dy$$

$$= y \tan^{-1} y - \frac{1}{2} \ln |1+y^2| + C.$$

(8.1) (16) $\int p^4 e^{-p}\,dp = -e^{-p}(p^4) - 4p^3 e^{-p} - 12p^2 e^{-p} - 24pe^{-p} - 24e^{-p}$

p^4		e^{-p}
$4p^3$	$+$	$-e^{-p}$
$12p^2$	$-$	e^{-p}
$24p$	$+$	$-e^{-p}$
24	$-$	e^{-p}
0	$+$	$-e^{-p}$

$= e^{-p}\left(-p^4 - 4p^3 - 12p^2 - 24p - 24\right) + C_1$

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(22) $\int e^{-y}\cos y\,dy$

Let $u = \cos y$, $dv = e^{-y}dy$

$du = -\sin y$, $v = -e^{-y}$

$$\int e^{-y}\cos y\,dy = -e^{-y}\cos y - \underbrace{\int e^{-y}\sin y\,dy}_{*} \quad \dots (**)$$

for $(*)$ Let $u = \sin y$, $dv = e^{-y}dy$

$du = \cos y\,dy$, $v = -e^{-y}$

$$\int e^{-y}\cos y\,dy = -e^{-y}\cos y - \left(-e^{-y}\sin y + \int e^{-y}\cos y\,dy\right)$$

$$\Rightarrow 2\int e^{-y}\cos y\,dy = -e^{-y}\cos y + e^{-y}\sin y + C_1$$

$$\int e^{-y}\cos y\,dy = -\frac{1}{2}e^{-y}\cos y + \frac{1}{2}e^{-y}\sin y + C_1 \quad (2)$$

8.1 (11) $\int \tan^{-1} y \, dy$

Let $u = \tan^{-1} y$, $dv = dy$

$du = \frac{1}{1+y^2} dy$, $v = y$

$$\int \tan^{-1} y \, dy = y \tan^{-1} y - \frac{1}{2} \int \frac{2y}{1+y^2} dy$$

$$= y \tan^{-1} y - \frac{1}{2} \ln(1+y^2) + C_1$$

(25) $\int e^{\sqrt{3s+9}} \, ds$

Let $x = \sqrt{3s+9}$, then $dx = \frac{1(3)}{2\sqrt{3s+9}} ds$

$dx = \frac{3}{2x} ds \Rightarrow ds = \frac{2}{3} x \, dx$

$$\int e^{\sqrt{3s+9}} ds = \int \frac{2}{3} x e^{x} dx.$$

Now: Let $u = x$ $dv = e^x dx$

$du = dx$ $v = e^x$

$$\int \frac{2}{3} x e^x dx = \frac{2}{3}\left[x e^x - \int e^x dx\right] = \frac{2}{3}\left[x e^x - e^x\right] + C_1$$

$$= \frac{2}{3}\left[\sqrt{3s+9}\, e^{\sqrt{3s+9}} - e^{\sqrt{3s+9}}\right] + C_1$$

(8.1) $\boxed{\begin{matrix} 30 \\ = 33 \end{matrix}}$ $\int z(\ln z)^2 \, dz$

Let $u = \ln z$, then $du = \frac{1}{z} dz$

& $e^u = z$

$$\int z(\ln z)^2 \, dz = \int e^u \cdot u^2 \cdot e^u \, du = \int u^2 e^{2u} \, du$$

$$= \frac{u^2}{2} e^{2u} - \frac{1}{2} u e^{2u} + \frac{1}{4} e^{2u} + C$$

$$= \left[\frac{1}{2}(\ln z)^2 - \frac{(\ln z)}{2} + \frac{1}{4} \right] z^2 + C$$

u^2		e^{2u}
$2u$	$+$	$e^{2u}/2$
2	$-$	$e^{2u}/4$
0	$+$	$e^{2u}/8$

---

36 $\int \frac{(\ln x)^3}{x} \, dx$

Let $u = \ln x$ then $du = \frac{1}{x} dx$

$$\Rightarrow \int \frac{(\ln x)^3}{x} \, dx = \int u^3 \, du = \frac{u^4}{4} + C =$$

$$= \frac{(\ln x)^4}{4} + C$$

(8.1) (39) $\int x^3 \sqrt{x^2+1}\, dx$

$u = x^2+1$

by substitution

Let $u = x^2$, $dv = \underbrace{x\sqrt{x^2+1}}\, dx$

$du = 2x\,dx$ $\qquad v = \frac{1}{3}(x^2+1)^{3/2}$

$$\Rightarrow \int x^3 \sqrt{x^2+1}\, dx = \frac{x^2}{3}\sqrt{(x^2+1)^3} - \frac{2}{3}\int \underbrace{x(x^2+1)^{3/2}\, dx}_{\text{Again substitution}}$$

$$= \frac{x^2}{3}\sqrt{(x^2+1)^3} - \frac{2}{3}\cdot\frac{1}{5}(x^2+1)^{5/2} + C_1$$

$$= \frac{x^2}{3}(x^2+1)^{3/2} - \frac{2}{15}(x^2+1)^{5/2} + C_1$$

---

(46) $\int \sqrt{x}\, e^{\sqrt{x}}\, dx$

Let $u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}}\, dx \Rightarrow 2u\,du = dx$

$$\int \sqrt{x}\, e^{\sqrt{x}}\, dx = \int 2u^2 e^u\, du.$$

$$= 2u^2e^u - 4ue^u + 4e^u + C_1$$

$$= \left[2(\sqrt{x})^2 - 4\sqrt{x} + 4\right] e^{\sqrt{x}} + C_1$$

$2u^2$	(+)	e^u
$4u$	(−)	e^u
4	(+)	e^u
0		e^u

(8.4) 50 $\int_0^{\frac{1}{\sqrt{2}}} 2x \sin^{-1}(x^2)\, dx$

Let $u = \sin^{-1}(x^2)$, $dv = 2x\, dx$

$du = \frac{2x}{\sqrt{1-x^4}}\, dx$, $v = x^2$

$$\int_0^{\frac{1}{\sqrt{2}}} 2x \sin^{-1}(x^2)\, dx = x^2 \sin^{-1}(x^2)\Big]_0^{\frac{1}{\sqrt{2}}} - \underbrace{\int_0^{\frac{1}{\sqrt{2}}} \frac{2x^3}{\sqrt{1-x^4}}\, dx}_{*}$$

For $*$: Let $u = 1-x^4$, then $du = -4x^3\, dx$

then $\int_0^{\frac{1}{\sqrt{2}}} \frac{2x^3\, dx}{\sqrt{1-x^4}} = -\frac{1}{2}\int \frac{du}{\sqrt{u}} = -\sqrt{u} = -\sqrt{1-x^4}\Big]_0^{\frac{1}{\sqrt{2}}}$

$= -\left(\sqrt{1-\frac{1}{4}} - \sqrt{1}\right) = -\left(\frac{\sqrt{3}}{2} - 1\right)$

then Finally :

$$\int_0^{\frac{1}{\sqrt{2}}} 2x \sin^{-1}(x^2)\, dx = \frac{1}{2}\sin^{-1}\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2} - 1\right)$$

$$= \frac{1}{2}\frac{\pi}{6} + \frac{\sqrt{3}}{2} - 1$$

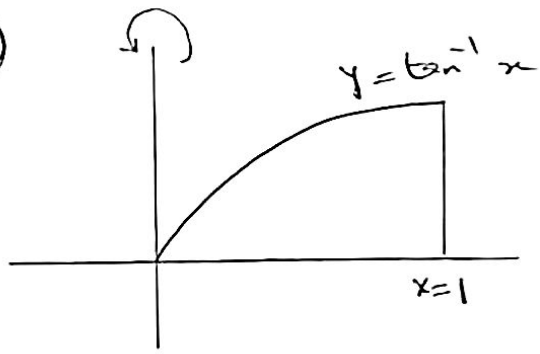
$$= \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$$

(8.1) 58 Consider the region bounded by the graphs

$y = \tan^{-1} x$, $y = 0$ & $x = 1$

a) Find the area of the region.

$$A = \int_0^1 \tan^{-1} x \, dx \qquad (\text{back to } 11)$$



$$= x \tan^{-1} x \Big]_0^1 - \frac{1}{2} \ln(1+x^2) \Big]_0^1$$

$$= \tan^{-1} 1 - \frac{1}{2} \ln 2 = \frac{\pi}{4} - \frac{1}{2} \ln 2.$$

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b) Find the volume of the solid formed by revolving this region about the y-axis

$$V = \int_0^1 2\pi \, x (\tan^{-1} x) \, dx \qquad (\text{shell method})$$

Let $u = \tan^{-1} x$, $dv = x \, dx$

$du = \frac{1}{1+x^2} dx$, $v = \frac{x^2}{2}$

$$\Rightarrow V = \frac{x^2}{2} \tan^{-1} x \Big]_0^1 - \frac{1}{2} \int \frac{x^2}{1+x^2} dx$$

$$\begin{array}{r|l} & 1 \\ \hline x^2+1 & x^2 \\ & -x^2 \mp 1 \\ \hline & -1 \end{array}$$

$$= \frac{x^2}{2} \tan^{-1} x \Big]_0^1 - \frac{1}{2} \left[\int_0^1 \left(1 - \frac{1}{1+x^2}\right) dx \right] = \boxed{\frac{\pi(\pi-2)}{2}}$$