10.4

Wednesday, March 24, 2021 11:28 AM

DCT

$d_n \le a_n \le c_n$

$\sum d_n$ div

$\sum c_n$ conv.

$\sum a_n$ div

$\sum a_n$ conv.

$\sum_{n=1}^{\infty} a_n$?? → conv. ✓ / Div. ✓

div: $\sum d_n$ given

conv: $\sum c_n$ given

$\left.\begin{array}{l} a_n > 0 \\ d_n > 0 \\ c_n > 0 \end{array}\right\} \forall$ larg n

Exp Check for Conv./Div

(1) $\displaystyle\sum_{n=1}^{\infty} \frac{3+\sin n}{n^2}$ → a_n

$\sum a_n$

$a_n > 0$ ✓

div

$\displaystyle \le \sum_{n=1}^{\infty} \frac{3+\sin n}{n^2} \le \sum_{n=1}^{\infty} \frac{4}{n^2}$ → conv. p-series $p=2$

$\sin n$ → 1

conv.

p-series Test $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ conv if $p>1$

div if $p \le 1$

Hence, $\displaystyle\sum_{n=1}^{\infty} \frac{3+\sin n}{n^2}$ conv. by DCT

(2) $\displaystyle\sum^{\infty}$ $7n$ div by n^{th} term test since

② $\displaystyle\sum_{n=1}^{\infty} \frac{7n}{5n+1}$ div by n^{th} term test since

$\displaystyle\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{7n}{5n+1} = \frac{7}{5} \neq 0$

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③ $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{3n+1}\right)^n \leq \frac{1}{2}$

$\Rightarrow$

$\text{div } \boxed{X} \leq \displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{3n+1}\right)^n \leq \sum_{n=1}^{\infty} \left(\frac{n}{3n}\right)^n = \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$

conv

Convergent geometric

$= \dfrac{a}{1-r} = \dfrac{\frac{1}{3}}{1-\frac{1}{3}}$

$= \frac{1}{3} / \frac{2}{3} = \frac{1}{2}$

Hence, $\displaystyle\sum \left(\frac{n}{3n+1}\right)^n$ Conv. by DCT ✓

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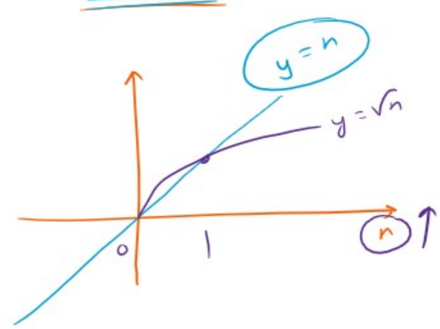
④ $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n+\sqrt{n}}$ Use DCT

a_n

$n = 1, 2, 3, \ldots$

$n > \sqrt{n}$ for large n



conv

div

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{n} \leq \sum \frac{3}{n+\sqrt{n}} \leq \boxed{X}$

$\Downarrow$

div - p series $p=1$

or div Harmonic Series

by DCT

$n + n > n + \sqrt{n}$

$3n > 2n > n + \sqrt{n}$

$3n > n + \sqrt{n}$

$n > \dfrac{n+\sqrt{n}}{3} \rightarrow \dfrac{1}{a_n}$

or div ✓ div by DCT

Hence, $\sum \frac{3}{n+\sqrt{n}}$

$n > \frac{n+\sqrt{n}}{3}$

$\sum \frac{1}{n} < \sum \frac{3}{n+\sqrt{n}}$

LCT
limit

$\sum_{n=1}^{\infty} a_n$??

Find b_n s.t $\sum b_n$ is known

$a_n > 0,\ b_n > 0$

✓ • $\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0$ (c finite) then both ($\sum a_n$ and $\sum b_n$) are conv. or both div

• $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ and $\sum b_n$ conv then $\sum a_n$ conv.

• $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$ and $\sum b_n$ div then $\sum a_n$ div.

Exp check for Conv/Div. LCT

(46) ① $\sum_{n=1}^{\infty} \tan \frac{1}{n} \to a_n$

find $b_n = \frac{1}{n}$ s.t $\sum_{n=1}^{\infty} \frac{1}{n}$ div H. Series

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\tan \frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{\frac{\sin \frac{1}{n}}{\cos \frac{1}{n}}}{\frac{1}{n}}$$

$\sin \frac{1}{n}$ ... $u = \frac{1}{n}$

$n \to \infty$

$$= \lim_{n\to\infty} \frac{\sin \frac{1}{n}}{\frac{1}{n}} \lim_{n\to\infty} \cos \frac{1}{n} \qquad u = \frac{1}{n}$$

$$= \lim_{u\to 0^+} \frac{\sin u}{u} \lim_{u\to 0^+} \cos u$$

$$= (1) \cos 0 = (1)(1) = 1 > 0$$

Hence, $\sum \tan \frac{1}{n}$ div by LCT

(36) (2)

$$\sum_{n=1}^{\infty} \frac{n+2^n}{n^2\, 2^n} \to a_n$$

Find $b_n = \frac{1}{n^2}$ s.t

$\sum \frac{1}{n^2}$ conv. p-series

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{n+2^n}{\cancel{n^2}\, 2^n}}{\frac{1}{\cancel{n^2}}} = \lim_{n\to\infty} \frac{n+2^n}{2^n} \quad \frac{\infty}{\infty}$$

$$= \lim_{n\to\infty} \frac{1+2^n \ln 2}{2^n \ln 2} \quad \frac{\infty}{\infty} = \lim_{n\to\infty} \frac{2^n (\ln 2)^2}{2^n (\ln 2)^2} = 1 > 0$$

$$\frac{A+B}{C} = \frac{A}{C} + \frac{B}{C}$$

Hence, $\sum \frac{n+2^n}{n^2 2^n}$ conv. by LCT

$$\sum_{n=1}^{\infty} \frac{\overset{A}{n} + \overset{B}{2^n}}{\underbrace{n^2\, 2^n}_{C}} = \sum_{n=1}^{\infty} \left(\frac{1}{n\, 2^n} + \frac{1}{n^2} \right) \le \sum \frac{1}{2^n} + \sum \frac{1}{n^2}$$

conv.

conv. geom + conv. p-serL

Hence, $\sum \frac{n+2^n}{n^2 2^n}$ conv. by DCT

conv. geom series + conv. p-serie

conv

LCT

(3) $\sum_{n=1}^{\infty} \frac{n+2}{n^3+n^2+5} \to a_n$

Find $b_n = \frac{1}{n^2}$

$\sum \frac{1}{n^2}$ conv. ✓

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{n+2}{n^3+n^2+5}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{(n+2)}{n^3+n^2+5} \cdot \frac{n^2}{1} = 1 > 0$$

Hence, $\sum \frac{n+2}{n^3+n^2+5}$ conv. by LCT.

LCT

(10) $\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{n^2+2}} \to a_n$

Find $b_n = \frac{1}{n^{\frac{1}{2}}}$ s.t

✓ $\sum \frac{1}{\sqrt{n}}$ known div - pserie

$\frac{\frac{1}{n}}{\frac{n^{\frac{1}{2}}}{n}} = \frac{1}{\sqrt{n}}$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\sqrt{\frac{n+1}{n^2+2}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{\sqrt{n+1}}{\sqrt{n^2+2}} \cdot \frac{\sqrt{n}}{1}$$

$$= \lim_{n\to\infty} \sqrt{\frac{n^2+n}{n^2+2}} = \sqrt{\lim_{n\to\infty} \frac{n^2+n}{n^2+2}} = \sqrt{1} = 1 > 0$$

cont.

Hence, $\sum \sqrt{n+1}$ div by LCT

Hence, $\sum \sqrt{\frac{n+1}{n^2+2}}$ div by LC

(22) $\sum_{n=1}^{\infty} \frac{n+1}{n^2\sqrt{n}} \rightarrow a_n$ Use LCT and DCT

LCT: Find $b_n = \frac{n^1}{n^{2.5}} = \frac{1}{n^{1.5}} = \frac{1}{n^{\frac{3}{2}}} \Rightarrow \sum \frac{1}{n^{\frac{3}{2}}}$ conv. p-series

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{n+1}{\cancel{n^2\sqrt{n}}}}{\cancel{\frac{1}{n^{\frac{3}{2}}}}} = \lim_{n\to\infty} \frac{n+1}{n^1} = 1 > 0$$

$\sum \frac{n+1}{n^2\sqrt{n}}$ conv. by LCT

DCT

$$\sum_{n=1}^{\infty} \frac{n+1}{n^2\sqrt{n}} \leq \sum \frac{n+\boxed{n}}{n^2\sqrt{n}} = \sum \frac{2n}{n^2\sqrt{n}} = 2\sum \frac{1}{n^{\frac{3}{2}}}$$

conv.

L

conv. p-series

Hence, $\sum \frac{n+1}{n^2\sqrt{n}}$ conv. by DCT.

