Ch.8 Techniques of Integration:-

Some integral Formulas:-

$\boxed{1}$ $\int a \, dx = ax + c$

② $\int a x^n dx = \frac{a x^{n+1}}{n+1} + c, \; n \neq -1$

$\boxed{3}$ $\int \frac{a}{x} dx = a \ln|x| + c$

$\boxed{4}$ $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$

$\boxed{5}$ $\int e^{ax} = \frac{e^{ax}}{a} + c$

$\boxed{6}$ $\int a^x dx = \frac{a^x}{\ln a} + c$

$\boxed{7}$ $\int \sin(ax) \, dx = \frac{-1}{a} \cos(ax) + c$

$\boxed{8}$ $\int \cos(ax) \, dx = \frac{1}{a} \sin(ax) + c$

$\boxed{9}$ $\int \tan x \, dx = \ln|\sec x| + c$

$\boxed{10}$ $\int \cot x \, dx = \ln|\sin x| + c$

$\boxed{11}$ $\int \sec x \, dx = \ln|\sec x + \tan x| + c$

$\boxed{12}$ $\int \csc x \, dx = -\ln|\csc x + \cot x| + c = \ln|\csc x - \cot x| + c$

$\boxed{13}$ $\int \sec^2 x \, dx = \tan x + c$

$\boxed{14}$ $\int \csc^2 x \, dx = -\cot x + c$

$\boxed{15}$ $\int \sec x \tan x \, dx = \sec x + c$

$\boxed{16}$ $\int \csc x \cot x \, dx = -\csc x + C$

$\boxed{17}$ $\int \cosh(ax)\, dx = \frac{\sinh(ax)}{a} + c$

$\boxed{18}$ $\int \sinh(ax)\, dx = \frac{\cosh(ax)}{a} + C$

$\boxed{19}$ $\int \text{sech}^2 x \, dx = \tanh x + C$

$\boxed{20}$ $\int \text{csch}^2 x \, dx = -\coth x + C$

$\boxed{21}$ $\int \text{sech}\, x \tanh x \, dx = -\text{sech}\, x + C$

$\boxed{22}$ $\int \text{csch}\, x \coth x \, dx = -\text{csch}\, x + C$

$\boxed{23}$ $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C$

$\boxed{24}$ $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$

$\boxed{25}$ $\int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \sec^{-1} \left|\frac{x}{a}\right| + C$

7.5. Note: Indeterminate Form

There are seven indeterminate forms that include 0, 1, ∞

$\lim f(x)$ →

- → $\frac{0}{0}$, $\frac{\infty}{\infty}$ — l'Hopital Rule
- → $0 \cdot \infty$, $\infty - \infty$ — Indeterminate product / difference — اعادة صياغة
  - Exp: $\lim\limits_{x \to 0^+} x(\ln x)^2 = \lim \dfrac{(\ln x)^2}{\frac{1}{x}}$
  - $\lim\limits_{x \to 0^+} (\ln x - \ln \sin x) = \lim\limits_{x \to 0^+} \ln\left(\dfrac{x}{\sin x}\right)$
  - $\lim\limits_{x \to \frac{\pi}{2}^-} \left(\frac{\pi}{2} - x\right) \tan x = \lim\limits_{x \to \frac{\pi}{2}^-} \dfrac{\left(\frac{\pi}{2} - x\right)}{\cot x}$
- → 1^{∞}, 0^0, ∞^0 — $y = e^{\ln(f(x))}$

7.6 Inverse Trigonometric Fonction

Note

Important properties of Inverse Trig.

- $\sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}$, $|x| \leq 1$
- $\tan^{-1}(x) + \cot^{-1}(x) = \frac{\pi}{2}$, $\forall x$
- $\sec^{-1}(x) + \csc^{-1}(x) = \frac{\pi}{2}$, $|x| \geq 1$

$\sin^{-1}(-x) = -\sin^{-1}(x)$

$\tan^{-1}(-x) = -\tan^{-1}(x)$

$\csc^{-1}(-x) = -\csc^{-1}(x)$

$\cos^{-1}(-x) = \pi - \cos^{-1}(x)$

$\sec^{-1}(-x) = \pi - \sec^{-1}(x)$

$\cot^{-1}(-x) = \pi - \cot^{-1}(x)$

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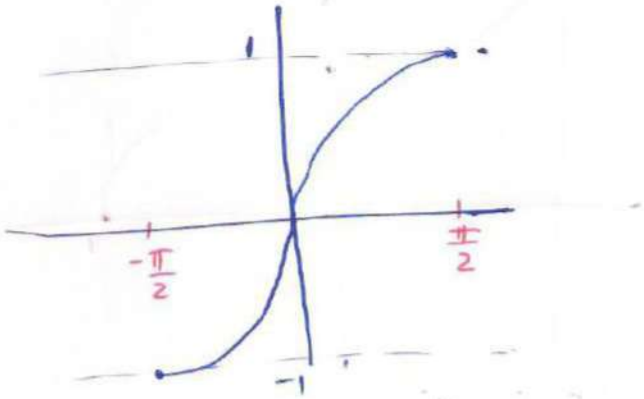
* $\sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)$, $x \geq 1$

* $\csc^{-1}(x) = \sin^{-1}\left(\frac{1}{x}\right)$, $x \geq 1$

* $\cot^{-1}(x) = \tan^{-1}\left(\frac{1}{x}\right)$, $x > 0$

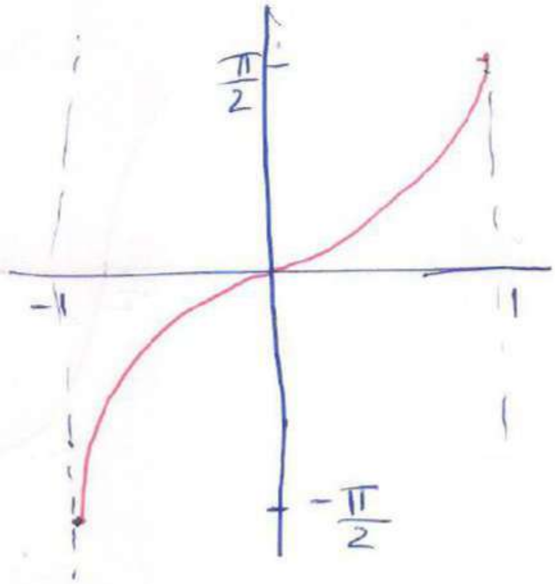
7.6 Inverse Trigonometric fonction

[I] $y = \sin x$



$y = \sin x \,,\quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$y = \sin^{-1}(x) = \text{arc} \sin(x)$



Some properties of $y = \sin^{-1}(x)$

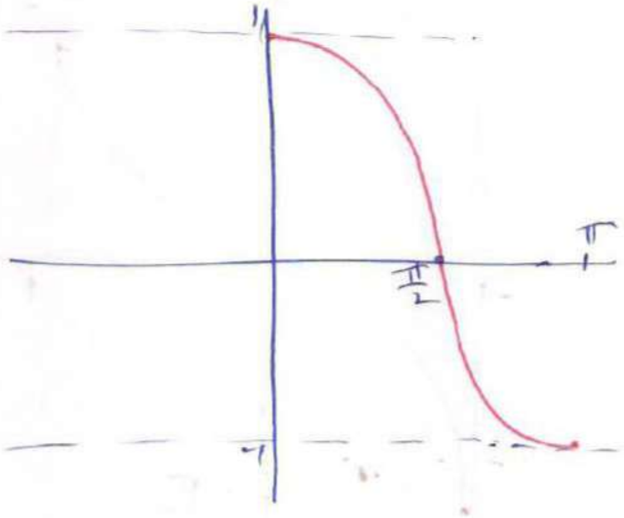
① $\text{Dom}\left(\sin^{-1}(x)\right) = [-1, 1]$, Range $\left(\sin^{-1}(x)\right) = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

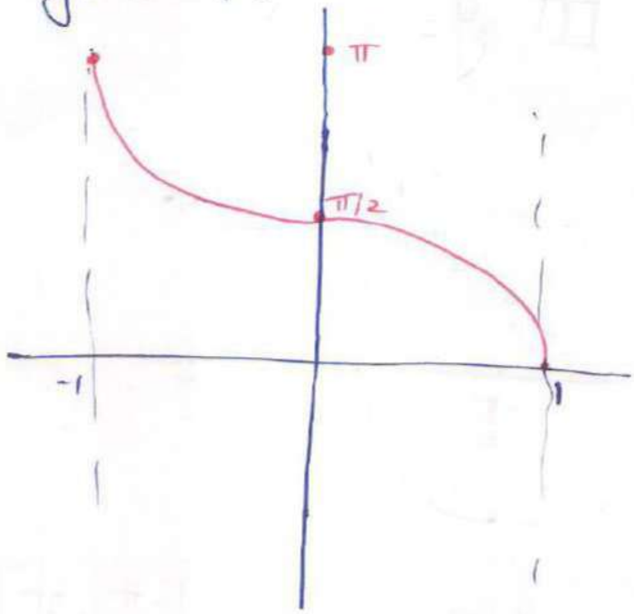
② $y = \sin^{-1}(x)$ is equivalent to $\sin y = x$, if $x \in \text{Dom}\left(\sin^{-1}(x)\right)$

③ $\sin^{-1}(\sin x) = x$ if $x \in \text{Dom}(\sin x)$
$x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

④ $\lim_{x \to 1^-} \sin^{-1}(x) = \frac{\pi}{2}$ $\qquad \lim_{x \to -1^+} \sin^{-1}(x) = -\frac{\pi}{2}$

$\boxed{2}$ $y = \cos x$ $\qquad\qquad$ $y = \cos^{-1}(x) = \arccos x$





$y = \cos x \;,\; x \in [0, \pi]$ $\qquad\qquad$ $y = \cos^{-1}(x) \;,\; x \in [-1, 1]$

Some properties $y = \cos^{-1}(x)$

① $D_{om}(\cos^{-1} x) = [-1, 1]$, Range $(\cos^{-1}(x)) = [0, \pi]$

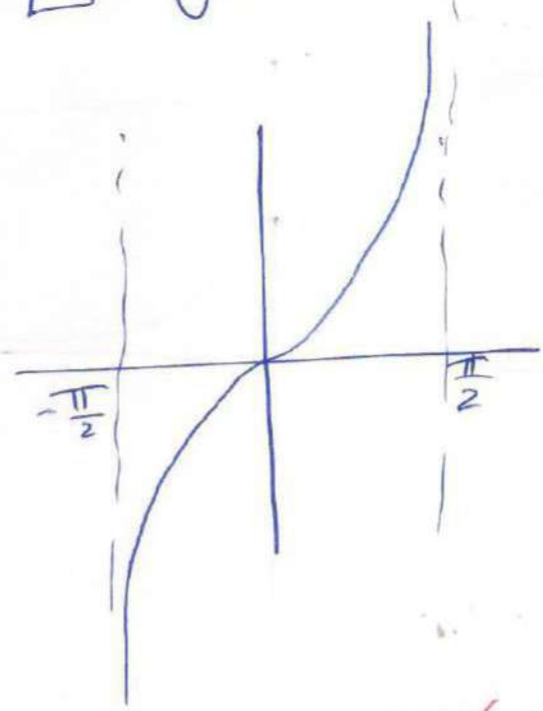
② $y = \cos^{-1} x$ is equivalent to $\cos y = x$, if $x \in D_{om}(\cos^{-1} x)$

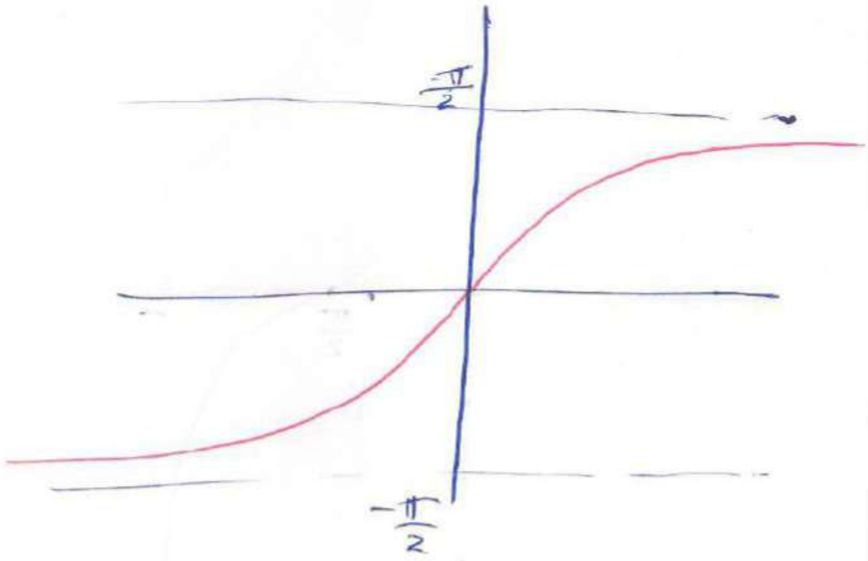
③ $\cos^{-1}(\cos x) = x$, if $x \in [0, \pi]$

$\cos(\cos^{-1} x) = x$, if $x \in [-1, 1]$

④ $\lim_{x \to -1^+} \cos^{-1}(x) = \pi$ $\qquad\qquad$ $\lim_{x \to 1^-} \cos^{-1}(x) = 0$

$\boxed{3}$ $y = \tan x$ $\qquad\qquad$ $y = \tan^{-1} x = \arctan x$





$y = \tan x \quad , \quad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Some properties of $y = \tan^{-1}(x)$

① $\text{Dom}\left(\tan^{-1} x\right) = (-\infty, \infty)$ $\qquad$ $\text{Range}\left(\tan^{-1} x\right) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

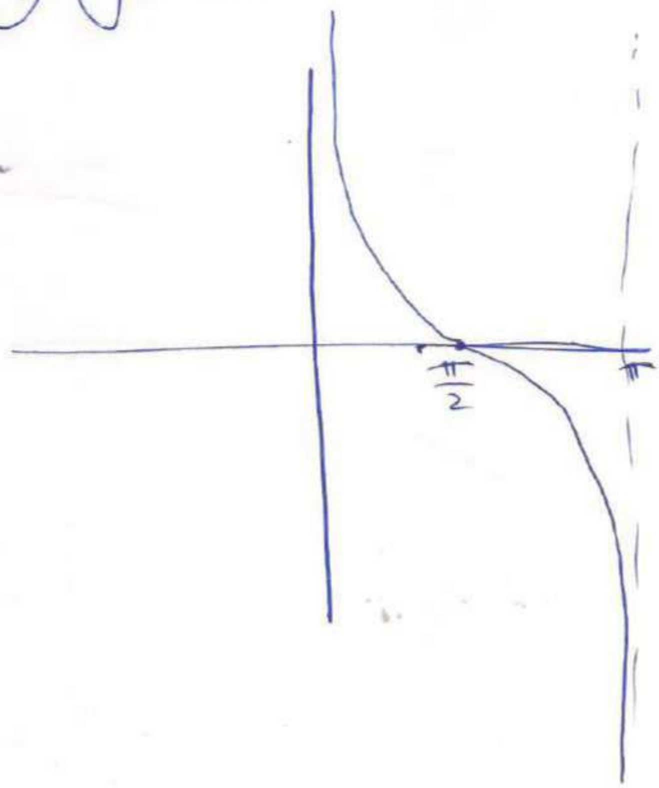
② $y = \tan^{-1} x$ is equivalent to $\tan y = x$, if $x \in (-\infty, \infty)$

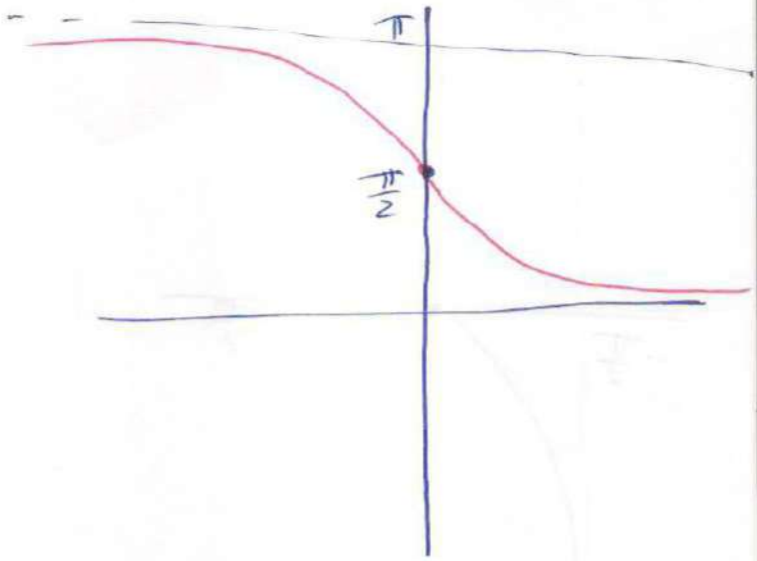
③ $\tan^{-1}(\tan x) = x$, if $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\tan\left(\tan^{-1}(x)\right) = x$, if $x \in (-\infty, \infty)$

④ $\lim_{x \to \infty} \tan^{-1} x = \frac{\pi}{2}$ $\qquad$ $\lim_{x \to -\infty} \tan^{-1} x = -\frac{\pi}{2}$

(4) $y = \cot x$ $\qquad\qquad\qquad$ $y = \cot^{-1}(x) = \text{arc}\cot x$





$y = \cot x\,, \quad x \in (0, \pi)$ $\qquad\qquad$ $y = \cot^{-1}(x)\,, \quad x \in (-\infty, \infty)$

Some properties of $y = \cot^{-1}(x)$

(1) $\text{Dom}(\cot^{-1} x) = (-\infty, \infty)$ $\qquad$ $\text{Range}(\cot^{-1} x) = (0, \pi)$

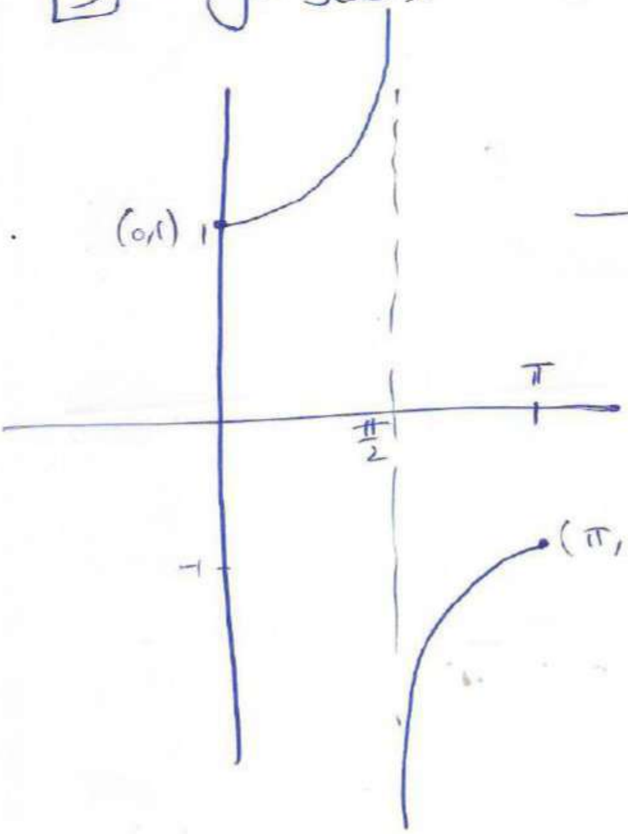
(2) $y = \cot^{-1} x$ is equivalent to $\cot y = x$, if $x \in (-\infty, \infty)$

(3) $\cot(\cot^{-1} x) = x$, if $x \in (-\infty, \infty)$

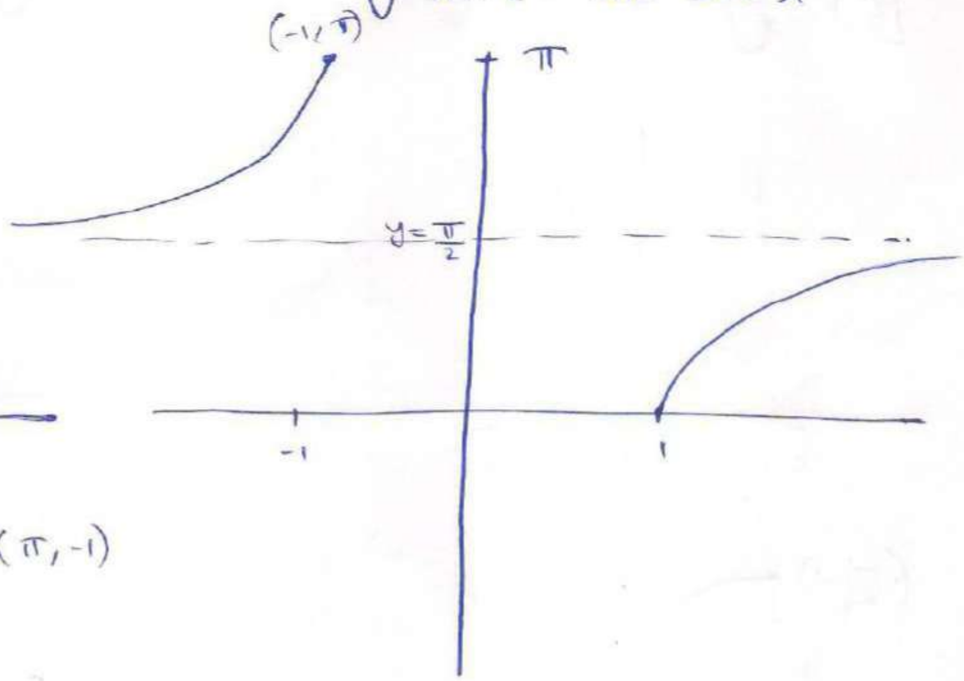
$\cot^{-1}(\cot x) = x$, if $x \in (0, \pi)$

(4) $\lim\limits_{x \to \infty} \cot^{-1} x = 0$ $\qquad$ $\lim\limits_{x \to -\infty} \cot^{-1} x = \pi$

$\boxed{5}$ $y = \sec x$



$y = \sec^{-1} x = \text{arc } \sec x$



$y = \sec x, \quad x \in [0, \pi] \setminus \{\frac{\pi}{2}\}$

$y = \sec^{-1} x, \quad x \in (-\infty, -1] \cup [1, \infty)$

Some properties of $y = \sec^{-1} x$

① $\text{Dom}(\sec^{-1} x) = (-\infty, -1] \cup [1, \infty)$

$\text{Range}(\sec^{-1} x) = [0, \pi] \setminus \{\frac{\pi}{2}\}$.

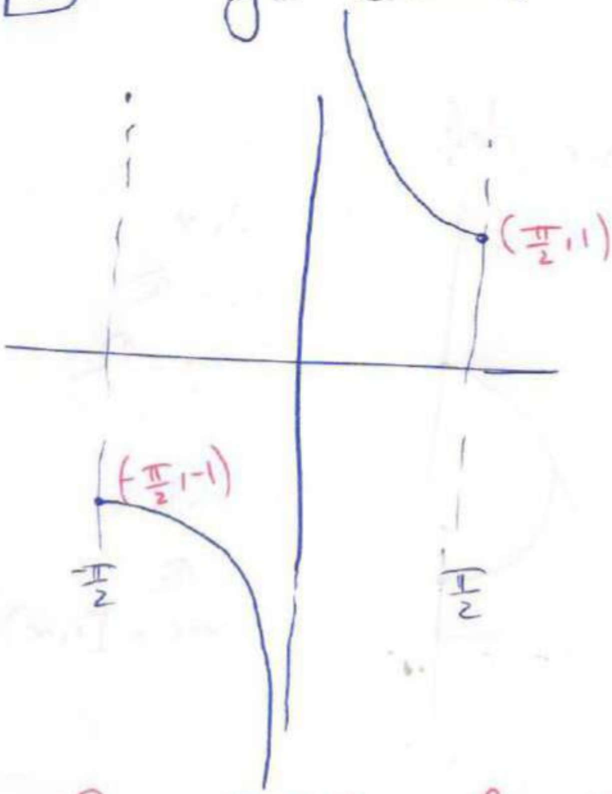
② $y = \sec^{-1} x$ is equivalent to $\sec y = x$, if $x \in \text{Dom}(\sec^{-1} x)$

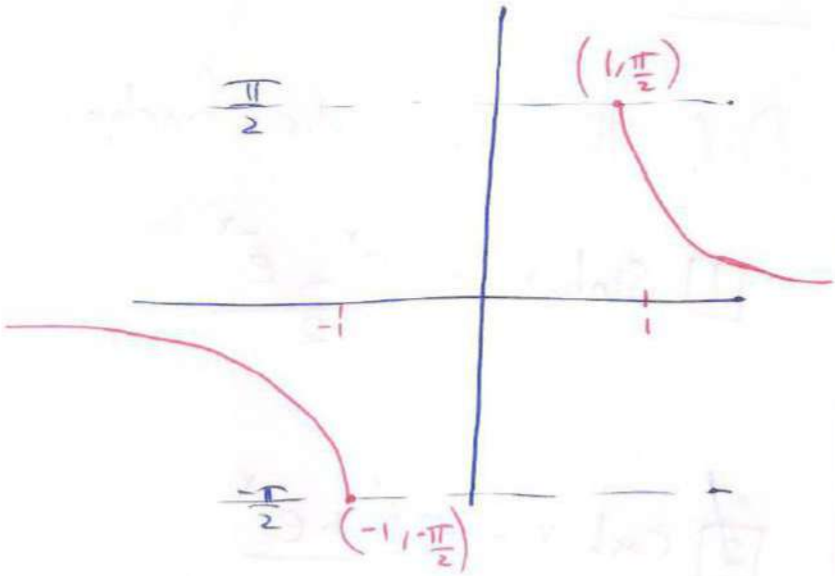
③ $\sec^{-1}(\sec x) = x$, if $x \in [0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$

$\sec(\sec^{-1} x) = x$, if $x \in (-\infty, -1] \cup [1, \infty)$

④ $\lim_{x \to \infty} \sec^{-1} x = \lim_{x \to -\infty} \sec^{-1} x = \frac{\pi}{2}$

$\boxed{6}$ $y = \csc x$ $\qquad$ $y = \csc^{-1} x = \text{arc}\csc(x)$





Some properties of $y = \csc^{-1} x$

① $\text{Dom}(\csc^{-1} x) = (-\infty, -1] \cup [1, \infty)$

Range $(\csc^{-1} x) = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \setminus \{0\}$

② $y = \csc^{-1} x$ is equivalent to $\csc y = x$, if $x \in \text{Dom}(\csc^{-1} x)$

③ $\csc^{-1}(\csc x) = x$, if $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \setminus \{0\}$

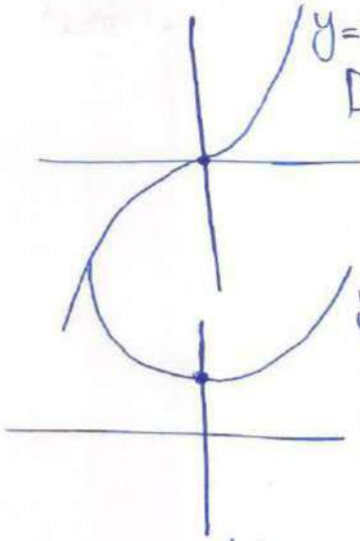
$\csc(\csc^{-1} x) = x$, if $x \in (-\infty, -1] \cup [1, \infty)$

④ $\lim\limits_{x \to \infty} \csc^{-1} x = \lim\limits_{x \to -\infty} \csc^{-1} x = 0$

7.7 Hyperbolic Funchons

Def: The hyperbolic funchons are defined as :-

[1] $\sinh x = \dfrac{e^x - e^{-x}}{2}$



$y = \sinh x$

Dom $= \mathbb{R}$

Range $= \mathbb{R}$

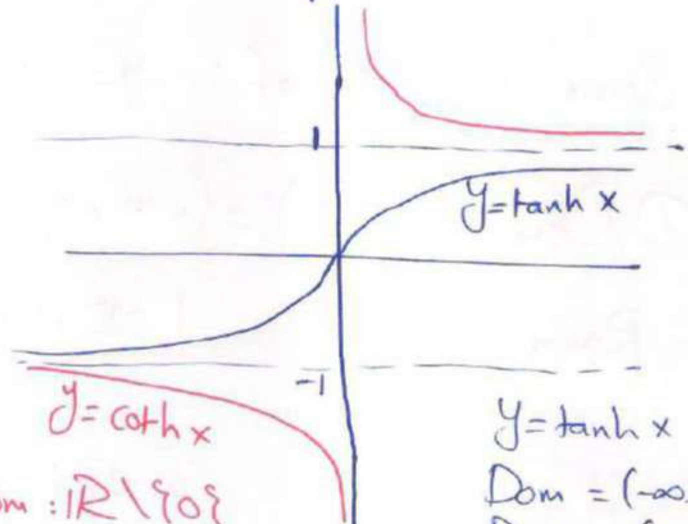
[2] $\cosh x = \dfrac{e^x + e^{-x}}{2}$

$y = \cosh x$

$D = \mathbb{R}$

Range $= [1, \infty)$

[3] $\tanh x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$



[4] $\coth x = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

Dom : $\mathbb{R} \setminus \{0\}$

Range : $(-\infty, -1) \cup (1, \infty)$

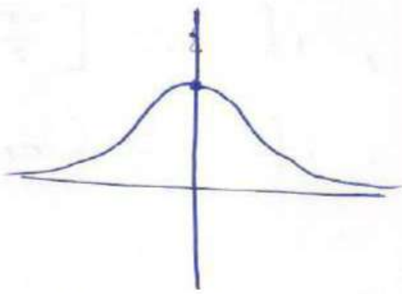
$y = \tanh x$

Dom $= (-\infty, \infty)$

Range $= (-1, 1)$

[5] $\operatorname{sech} x = \dfrac{2}{e^x + e^{-x}}$

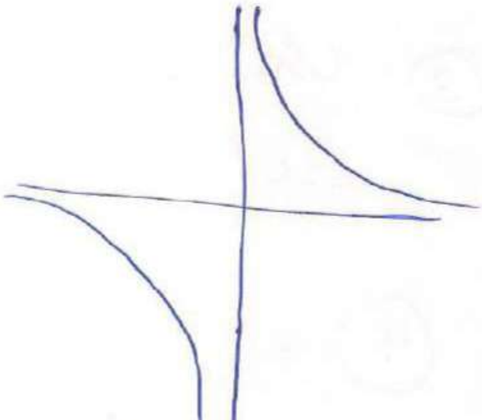
[6] $\operatorname{csch} x = \dfrac{2}{e^x - e^{-x}}$



$y = \operatorname{sech} x$

Dom $= (-\infty, \infty)$

Range $= (0, 1]$



$y = \operatorname{csch} x$

$D = \mathbb{R} \setminus \{0\}$

Range $= \mathbb{R} \setminus \{0\}$