7.5 Discussion

Wednesday, December 8, 2021 1:49 PM

10. $\lim_{t\to 1} \dfrac{t^3-1}{4t^3-t-3}$ $\quad \frac{0}{0}$

$\lim_{t\to 1} \dfrac{3t^2}{12t^2-1}$

$\dfrac{3}{12-1} = \dfrac{3}{11}$

7.5

$\lim_{x\to 0}$ { }

$\frac{0}{0}, \frac{\infty}{\infty}$ ↓ L'Hôpital

$\infty \cdot 0, \infty - \infty$ تبسيط / توحيد المقامات

$1^{\infty}, 0^{0}, \infty^{0}$ ↓ $\ln f(x) = e$

21. $\lim_{x\to 0} \dfrac{x^2}{\ln(\sec x)}$ $\quad \frac{0}{\ln 1} = \frac{0}{0}$

$\lim_{x\to 0} \dfrac{2x}{\frac{\sec x \tan x}{\sec x}} = \lim_{x\to 0} \dfrac{2x}{\tan x}$ $\quad \frac{0}{0}$

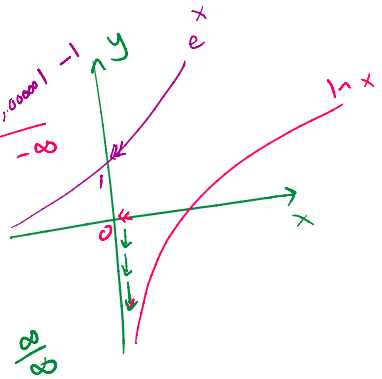
$= \lim_{x\to 0} \dfrac{2}{\sec^2 x} = \dfrac{2}{1^2} = 2$

34. $\lim_{x\to 0^+} \dfrac{\ln(e^x-1)}{\ln x}$

$\ln(e^{0^+}-1) = \ln(\text{small}+) \to -\infty$

$\ln 0^+ = \ln(\text{small}+) \to -\infty$

$\frac{-\infty}{-\infty} = \frac{\infty}{\infty}$



$= \lim_{x\to 0^+} \dfrac{\frac{e^x}{e^x-1}}{\frac{1}{x}}$

$= \lim_{x\to 0^+} \dfrac{xe^x}{e^x-1}$ $\quad \frac{0}{1-1} = \frac{0}{0}$

$= \lim_{x\to 0^+} \dfrac{xe^x+e^x}{e^x} = \lim_{x\to 0^+} \dfrac{e^x(x+1)}{e^x} = \lim_{x\to 0^+} (x+1) = 0+1 = 1$

$= \lim_{x \to 0^+}$ $\frac{x \cdot 1}{e^x}$ $= \lim_{x \to 0^+}$ $\frac{x}{e^x}$ $x \to 0^+$

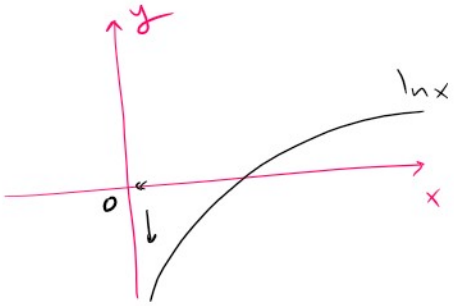
38. $\lim_{x \to 0^+} (\ln x - \ln \sin x)$

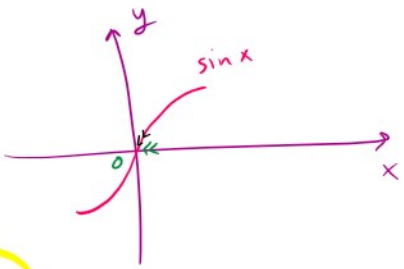
$-\infty - \ln 0^+$

$-\infty - -\infty$

$-\infty + \infty \neq 0$

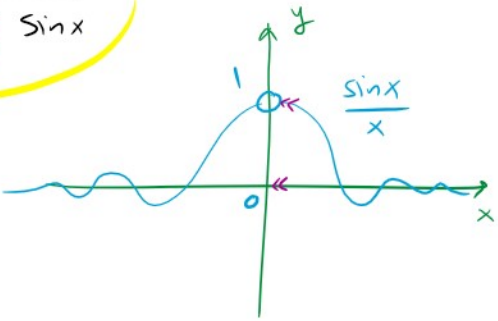
$\infty - \infty$





$\lim_{x \to 0^+} \ln \frac{x}{\sin x} = \ln \lim_{x \to 0^+} \frac{x}{\sin x}$

cont. on +



$= \ln \lim_{x \to 0^+} \frac{1}{\frac{\sin x}{x}}$

$= \ln \frac{1}{1} = \ln 1 = 0$

52. $\lim_{x \to 1^+} x^{1/(x-1)}$ → $f(x)$

$1^{\frac{1}{\text{small}+}} = 1^{\infty} = 1$ → power

$f(x) = e^{\ln f(x)}$

$\ln x^{\frac{1}{x-1}}$

$\lim_{x \to 1^+} e^{\ln x^{\frac{1}{x-1}}} = \lim_{x \to 1^+} e^{\frac{1}{x-1} \ln x}$ cont.

$\lim_{x \to 1^+} \frac{\ln x}{x-1}$ $\frac{0}{0}$

$e^{\ln 7} = 7$

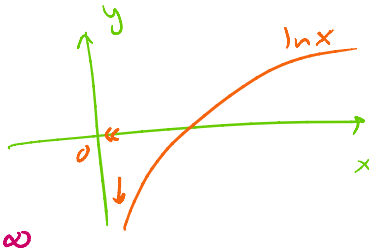
$e^{\ln f} = f$

$= e$

$= e^{\lim_{x \to 1^+} \frac{\frac{1}{x}}{1}} = e^{\lim_{x \to 1^+} \frac{1}{x}} = e^{1} = e$

$$\lim_{x \to 0^+} \sin x \cdot \ln x$$

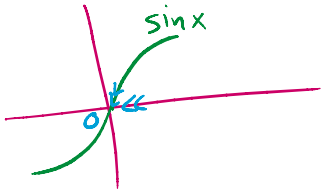
$(0) \cdot (-\infty)$ not power



$$\lim_{x \to 0^+} \frac{\ln x}{\frac{1}{\sin x}} = \lim_{x \to 0^+} \frac{\ln x}{\csc x} \quad \frac{-\infty}{\infty}$$

$\frac{1}{\text{small +}}$

$$= \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\csc x \cot x}$$



$$= \lim_{x \to 0^+} \frac{\frac{1}{x}}{\frac{1}{\sin x} \frac{\cos x}{\sin x}}$$

$$= \lim_{x \to 0^+} \frac{\sin x \tan x}{x} \quad \frac{0}{0}$$

$$= \left(\lim_{x \to 0^+} \frac{\sin x}{x}\right)\left(\lim_{x \to 0^+} \tan x\right)$$

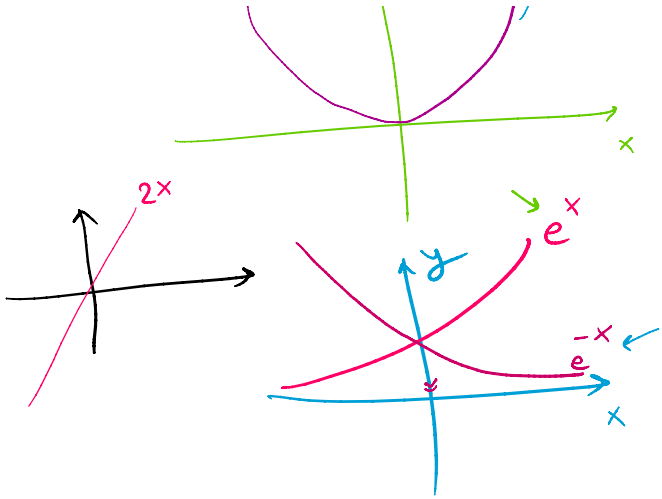
$= (1)(0)$

$= 0$

46. $\lim_{x \to \infty} x^2 e^{-x} = (\infty)\cdot(0)$

$x \to \infty$ $= (\infty)(0)$

$$\lim_{x \to \infty} \frac{x^2}{e^x} \qquad \frac{\infty}{\infty}$$



$$\lim_{x \to \infty} \frac{2x}{e^x} \qquad \frac{\infty}{\infty}$$

$$\lim_{x \to \infty} \frac{2}{e^x} = 2 \lim_{x \to \infty} e^{-x} = (2)(0) = 0$$

$\frac{2}{e^x} \to 0$

49. $\lim_{\theta \to 0} \dfrac{\theta - \sin\theta\cos\theta}{\tan\theta - \theta}$ $\qquad \frac{0-0}{0-0} = \frac{0}{0}$

$$\lim_{\theta \to 0} \frac{1 - (-\sin\theta \sin\theta + \cos\theta \cos\theta)}{\sec^2\theta - 1}$$

$1 - \cos^2\theta = \sin^2\theta$

$$\lim_{\theta \to 0} \frac{1 + \sin^2\theta - \cos^2\theta}{\sec^2\theta - 1}$$

$$\lim_{\theta \to 0} \frac{\sin^2\theta + \sin^2\theta}{\tan^2\theta} = \lim_{\theta \to 0} \frac{2\sin^2\theta}{\tan^2\theta}$$

$\lim \dfrac{\sin^2\theta}{\tan^2\theta}$

$$\lim_{\theta \to 0} 2\sin^2\theta \cot^2\theta$$

$$\lim_{\theta \to 0} 2\sin^2\theta \frac{\cos^2\theta}{\sin^2\theta}$$

$$2 \lim_{\theta \to 0} \cos^2\theta = 2(1^2) = 2$$