1.7) Orthogonal Signals

Orthogonality → It is the property that allows transmission of more than one signal over a common channel with successful detection

Orthogonal signals → The signals are orthogonal if they are mutually independent

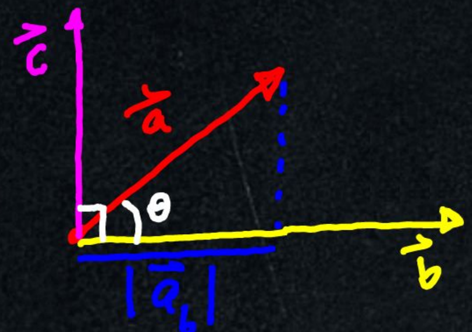
Orthogonal vectors :-

$|\vec{a}_b| = |\vec{a}| \cos\theta$

$|\vec{a}_b| \, |\vec{b}| \neq 0$

$(|\vec{a}| \cos\theta)(|\vec{b}|) \neq 0$

$|\vec{a}| |\vec{b}| \cos\theta \neq 0 \Rightarrow \boxed{\vec{a} \cdot \vec{b} \neq 0}$



However, $\vec{b} \cdot \vec{c} = 0$

$\Rightarrow \vec{b}$ + $\vec{c}$ are orthogonal vectors signal space "inner product"

Condition for orthogonal signals

$$\int_{-\infty}^{\infty} x_1(t)\, x_2(t)\, dt = 0$$ Non perodic signal

$$\int_{0}^{T} x_1(t)\, x_2(t)\, dt = 0$$ Perodic signal

properties

1) Two harmonics of different frequencies are always orthogonal

$$x_1(t) = \sin(n\omega_0 t + \phi_1) \qquad n \neq m$$

$$x_2(t) = \sin(m\omega_0 t + \phi_2)$$

$$\int_0^T x_1(t)\, x_2(t)\, dt = \int_0^T \sin(n\omega_0 t + \phi_1)\sin(m\omega_0 t + \phi_2)\, dt = 0$$

2) Sin and cos functions with the same phase and the same frequency are orthogonal

$$\int_0^T \sin(n\omega_0 t + \phi)\cos(n\omega_0 t + \phi)\, dt = 0$$

3) DC value and sin function are orthogonal

$$\int_0^T f(k)\left(\sin(n\omega_0 t+\phi)\right)dt = 0$$

4) $x_1(t)$ and $x_2(t)$ are orthogonal

Average power of $x_1(t) = P_{x_1}$	Total energy of $x_1(t) = E_{x_1}$
Average power of $x_2(t) = P_{x_2}$	Total energy of $x_2(t) = E_{x_2}$

$\Rightarrow$ Average power of $y(t) = P_y = P_{x_1} + P_{x_2}$

where $y(t) = x_1(t) + x_2(t)$

$\Rightarrow$ Total energy of $y(t) = E_y = E_{x_1} + E_{x_2}$

EX :- Calculate the average power of y(t)

periodic signal

$$y(t) = \underbrace{2\sin(3\omega_0 t + 45^\circ)}_{x_1(t)} + \underbrace{4\sin(4\omega_0 t + 35^\circ)}_{x_2(t)}$$

$$P_y = \frac{(2)^2}{2} + \frac{(4)^2}{2}$$

$x_1(t) + x_2(t)$ are $\underline{0}$

$$P_y = \boxed{10}\ W$$

EX :- Calculate the average power and total energy of the following signals

1) $x(t) = 2\cos(2\omega_0 t + 45^\circ) + 3\sin(2\omega_0 t + 45^\circ)$

$$P_x = \frac{(2)^2}{2} + \frac{(3)^2}{2} = 6.5\ W \qquad E_x = E_1 + E_2 = \infty$$

2) $x(t) = 2 + 4\sin(3\omega_0 t + 45^\circ)$

$$P_x = (2)^2 + (4)^2/2 = 12\ W \qquad E_y = E_1 + E_2 = \infty$$

Ex :- Calculate the average power and RMS value of $x(t) = 2\sin(3t) + 3\cos(3t + \frac{\pi}{3})$

In general $A_1 \sin(\omega t + \alpha) + A_2 \sin(\omega t + \beta) = A_o \sin(\omega t + \phi)$

you can show it using phasor

$$A_o = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\alpha - \beta)}$$

$\cos x = \sin(x + \frac{\pi}{2})$

$$x(t) = 2\sin(3t) + 3\sin(3t + \frac{\pi}{3} + \frac{\pi}{2})$$

$$x(t) = 2\sin(3t) + 3\sin(3t + \frac{5\pi}{6}) = A_o \sin(3t + \phi)$$

$$A_o = \sqrt{(2)^2 + (3)^2 + 2(2)(3)\cos(0 - \frac{5\pi}{6})} = \sqrt{2.607}$$

$$P = \frac{A_o^2}{2} = \boxed{1.303}\ W \qquad RMS = \sqrt{P} = \boxed{1.141}$$