Engineering Thermodynamics

First Law of Thermodynamics

Dr. Hakam Shehadeh

4.1 INTERNAL ENERGY

- It is the heat energy stored in a gas.
- If a certain amount of heat is supplied to a gas the result is that temperature of gas may increase or volume of gas may increase thereby doing some external work or both temperature and volume may increase ; but it will be decided by the conditions under which the gas is supplied heat.
- *If during heating of the gas the temperature increases its internal energy will also increase.*

INTERNAL ENERGY

- *Joule's law of internal energy* states that internal energy of a perfect gas is a *function of temperature only.*
- In other words, internal energy of a gas is dependent on the temperature change only and is not affected by the change in pressure and volume.
- We do not know how to find the absolute quantity of internal energy in any substance ; however, what is needed in engineering is the change of internal energy (ΔU).

4.2 LAW OF CONSERVATION OF ENERGY

- In the early part of nineteenth century the scientists developed the concept of energy and hypothesis that it can be neither created nor destroyed; this came to be known as the *law of the conservation of energy*.
- The first law of thermodynamics is merely one statement of this general law/principle with particular reference to heat energy and mechanical energy *i.e.,* work.

4.3 FIRST LAW OF THERMODYNAMICS

- It is observed that when a system is made to undergo a complete cycle then net work is done *on* or *by* the system.
- Consider a cycle in which net work is done by the system. Since energy cannot be created, this mechanical energy must have been supplied from some source of energy. Now the system has been returned to its initial state: Therefore, its *intrinsic* energy is unchanged, and hence the mechanical energy has not been provided by the system itself.
- The only other energy involved in the cycle is the heat which was supplied and rejected in various processes.
- Hence, by the law of conservation of energy, the net work done by the system is equal to the net heat supplied to the system.

FIRST LAW OF THERMODYNAMICS

- The First Law of Thermodynamics can, therefore, be stated as follows:
- **"When a system undergoes a thermodynamic cycle then the net heat supplied to the system from the surroundings is equal to net work done by the system on its surroundings".**

$$\oint dQ = \oint dW$$

where $\oint$ represents the sum for a complete cycle

FIRST LAW OF THERMODYNAMICS

- The first law of Thermodynamics *cannot be proved analytically, but experimental evidence has repeatedly confirmed its validity*, and since no phenomenon has been shown to contradict it, the first law is accepted as a *law of nature.*
- It may be remarked that no restriction was imposed which limited the application of first law to reversible energy transformation.
- Hence the first law applies to reversible as well as irreversible transformations: For non-cyclic process, a more general formulation of first law of thermodynamics is required.
- A new concept which involves a term called *internal energy* fulfills this need.

FIRST LAW OF THERMODYNAMICS

The First Law of Thermodynamics may also be stated as follows :

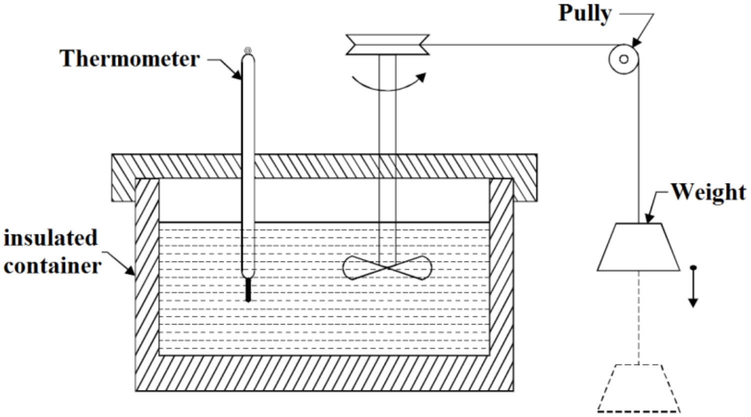
—"Heat and work are mutually convertible but since energy can neither be created nor destroyed, the total energy associated with an energy conversion remains constant".

Or

—"No machine can produce energy without corresponding expenditure of energy, *i.e.,* it is impossible to construct a perpetual motion machine of first kind".

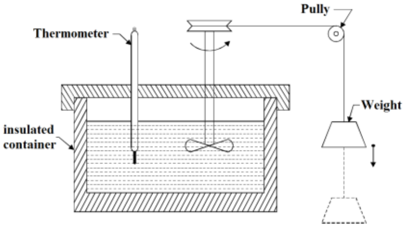
FIRST LAW OF THERMODYNAMICS

the experiment for checking first law of thermodynamics



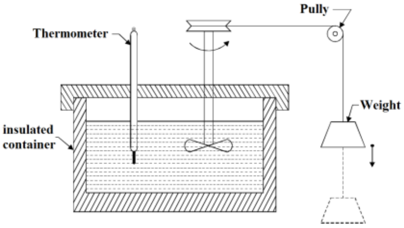
FIRST LAW OF THERMODYNAMICS

- The work input to the paddle wheel is measured by the fall of weight, while the corresponding temperature rise of liquid in the insulated container is measured by the thermometer.
- It is already known to us from experiments on heat transfer that temperature rise can also be produced by heat transfer.



FIRST LAW OF THERMODYNAMICS

- The experiments show:
    - A definite quantity of work is always required to accomplish the same temperature rise obtained with a unit amount of heat.
    - Regardless of whether the temperature of liquid is raised by work transfer or heat transfer, the liquid can be returned by heat transfer in opposite direction to the identical state from which it started.



FIRST LAW OF THERMODYNAMICS

- The previous results lead to the inference that *work and heat* are different forms of something more general, which is called *energy.*
- It can be stated as an invariable experience that whenever a physical system passes through a complete cycle the algebraic sum of the work transfers during the cycle $\oint dW$ bears a definite ratio to the algebraic sum of the heat transfers during the cycle, $\oint dQ$.
- This may be expressed by the equation,

$$\oint dW = J \oint dQ$$

➢ where J is the proportionality constant and is known as *Mechanical Equivalent of heat*. In S.I. units its value is unity, *i.e.*, 1 Nm/J.

4.4 APPLICATION OF FIRST LAW TO A PROCESS

- When a process is executed by a system, the *change in stored energy of the system is numerically equal to the net heat interactions minus the net work interaction during the process.*

$$\therefore \qquad E_2 - E_1 = Q - W$$

$$\therefore \qquad \Delta E = Q - W \qquad [\text{or } Q = \Delta E + W]$$

$$\text{or} \qquad \int_1^2 d(Q - W) = \Delta E = E_2 - E_1$$

where E represents the *total internal energy.*

APPLICATION OF FIRST LAW TO A PROCESS

- If the electric, magnetic and chemical energies are *absent* and changes in potential and kinetic energy for a *closed system* are neglected, the above equation can be written as

$$\int_1^2 d(Q - W) = \Delta U = U_2 - U_1$$

$$\therefore \qquad Q - W = \Delta U = U_2 - U_1$$

APPLICATION OF FIRST LAW TO A PROCESS

$$\int_1^2 d(Q - W) = \Delta U = U_2 - U_1$$

$$\therefore \qquad Q - W = \Delta U = U_2 - U_1$$

- Generally, when heat is added to a system its temperature rises and external work is performed due to increase in volume of the system.
- *The rise in temperature is an indication of increase of internal energy*.
- Heat *added* to the system will be considered as *positive* and the heat *removed* or *rejected*, from the system, as *negative.*

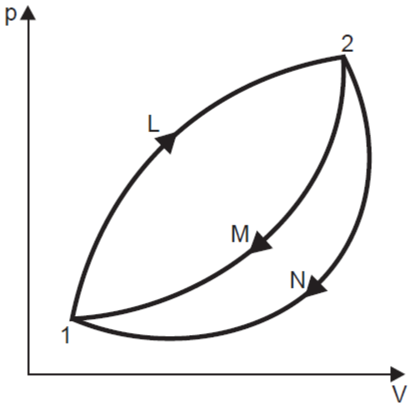
4.5 ENERGY—A PROPERTY OF SYSTEM

- Consider a system which changes its state from state 1 to state 2 by following the path *L*, and returns from state 2 to state 1 by following the path *M*. So the system undergoes a cycle.
- Writing the first law for path *L*

$$Q_L = \Delta E_L + W_L$$

- Writing the first law for path *M*

$$Q_M = \Delta E_M + W_M$$



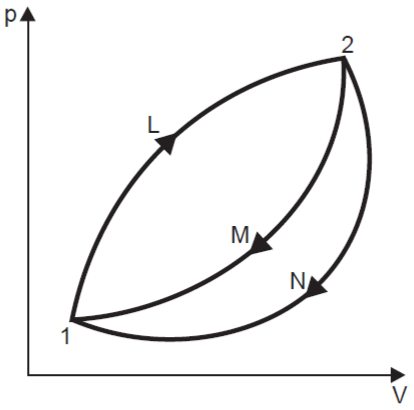
ENERGY—A PROPERTY OF SYSTEM

- The processes *L* and *M* together constitute a cycle, for which

$$\oint dW = \oint dQ$$

$$W_L + W_M = Q_L + Q_M$$

$$\text{or} \quad Q_L - W_L = W_M - Q_M$$



From the previous equations, it yields

$$\Delta E_L = - \Delta E_M$$

ENERGY—A PROPERTY OF SYSTEM

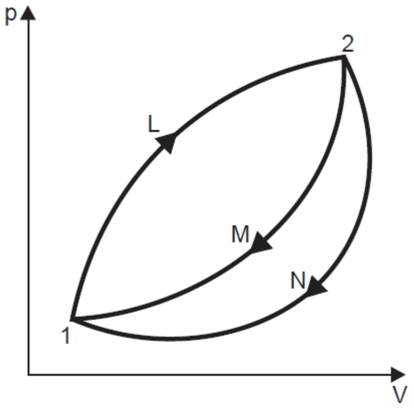
- Similarly, had the system returned from state 2 to state 1 by following the path *N* instead of path *M*

$$\Delta E_L = - \Delta E_N$$

- From equation $\Delta E_L = - \Delta E_M$

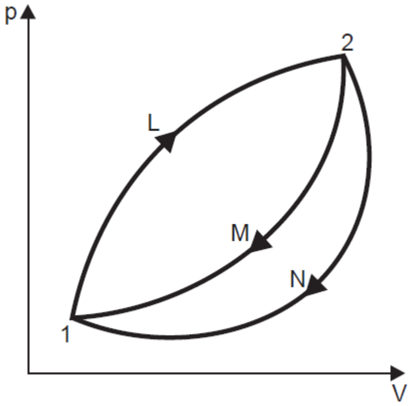
  and $\Delta E_L = - \Delta E_N$

$$\boxed{\Delta E_M = \Delta E_N}$$



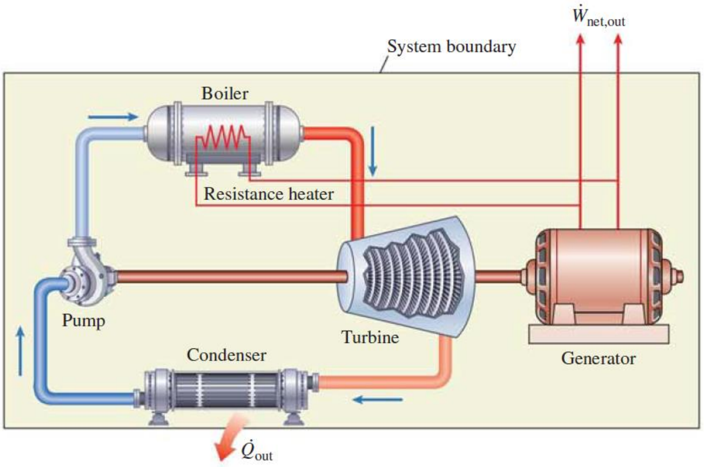
ENERGY—A PROPERTY OF SYSTEM

- Thus, it is seen that the *change in energy between two states of a system is the same, whatever path the system may follow in undergoing that change of state.*
- If some arbitrary value of energy is assigned to state 2, the value of energy at state 1 is fixed independent of the path the system follows.
- Therefore, *energy has a definite value for every state of the system.* Hence, it is *a point function and a property of the system.*



4.6

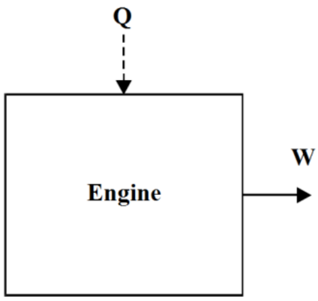
PERPETUAL MOTION MACHINE OF THE FIRST KIND—PMM 1



A perpetual-motion machine that violates the first law of thermodynamics (PMM1).

PERPETUAL MOTION MACHINE OF THE FIRST KIND—PMM 1

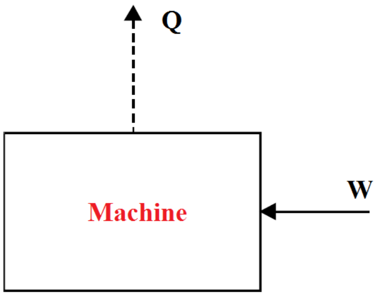
- The first law of thermodynamics states the general principle of the conservation of energy.
- Energy is neither created nor destroyed, but only gets transformed from one form to another.
- *There can be no machine which would continuously supply mechanical work without some form of energy disappearing simultaneously*.
- Such a *fictitious machine* is called a ***perpetual motion machine of the first kind,*** or in brief, PMM 1.
- A PMM 1 is thus ***impossible.***



A PMM 1

PERPETUAL MOTION MACHINE OF THE FIRST KIND—PMM 1

- The converse of the previous statement is also true, *i.e.*, there can be no machine which would continuously consume work without some other form of energy appearing simultaneously.



The Converse of PMM 1

4.7 ENERGY OF AN ISOLATED SYSTEM

- *An isolated system is one in which there is no interaction of the system with the surroundings.*
- For an isolated system,

$$dQ = 0,\ dW = 0$$

- The first law of thermodynamics gives

$$dE = 0$$

or

$$E = \text{constant}$$

- *The energy of an isolated system is always constant.*

4.8 THE PERFECT GAS

4.8.1. The Characteristic Equation of State

- At temperatures that are considerably in excess of critical temperature of a fluid, and also at very low pressure, the vapour of fluid tends to obey the equation

$$\frac{pv}{T} = \text{constant} = R$$

- In practice, no gas obeys this law rigidly, but many gases tend towards it. An imaginary ideal gas which obeys this law is called a *perfect gas*, and the equation $\frac{pv}{T} = R$, is called the *characteristic equation of a state of a perfect gas*. The constant R is called the *gas constant*.
- Each perfect gas has a different gas constant.

THE PERFECT GAS

The Characteristic Equation of State

- Units of R are Nm/kg K or kJ/kg K.
- Usually, the characteristic equation is written as

$$pv = RT$$

or for m kg, occupying V m^3

$$pV = mRT$$

— The characteristic equation in *another form*, can be derived by using **kilogram-mole** as a unit.

$$pV = mRT$$

$$pV = nMRT$$

$$MR = \frac{pV}{nT}$$

THE PERFECT GAS

The Characteristic Equation of State

- The *kilogram-mole* is defined as a quantity of a gas equivalent to M kg of the gas, where M is the molecular weight of the gas (*e.g.,* since the molecular weight of oxygen is 32, then 1 kg mole of oxygen is equivalent to 32 kg of oxygen).
- As per definition of the kilogram-mole, for m kg of a gas, we have

$$m = nM$$

where n = number of moles.

❑**Note.** Since the standard of mass is the kg, kilogram-mole will be written simply as mole.

THE PERFECT GAS

The Characteristic Equation of State

- According to *Avogadro's hypothesis* the volume of 1 mole of any gas is the same as the volume of 1 mole of any other gas, when the gases are at the same temperature and pressure.
- Therefore, $\frac{V}{n}$ is the same for all gases at the same value of *p* and *T*.
- That is the quantity $\frac{pV}{nT}$ is a *constant* for all gases.
- This constant is called *universal gas constant*, and is given the symbol, R_0.

$$MR = R_o = \frac{pV}{nT}$$

$$pV = nR_oT$$

Since $MR = R_0$, then

$$R = \frac{R_o}{M}$$

THE PERFECT GAS

The Characteristic Equation of State

- It has been found experimentally that the volume of 1 mole of any perfect gas at 1 bar and 0°C is approximately 22.71 m^3.
- Therefore

$$R_0 = \frac{pV}{nT} = \frac{1 \times 10^5 \times 22.71}{1 \times 273.15}$$

$$= 8314.3 \ \text{Nm/mole K}$$

- Using $R = \frac{R_o}{M}$, the gas constant for any gas can be found when the molecular weight is known.
- **Example.** For oxygen which has a molecular weight of 32, the gas constant

$$R = \frac{R_0}{M} = \frac{8314}{32} = 259.8 \ \text{Nm/kg K}.$$

THE PERFECT GAS

4.8.2. Specific Heats

- The **specific heat** of a solid or liquid is usually defined as the *heat required to raise unit mass through one degree temperature rise.*
- For small quantities, we have

$$\boldsymbol{dQ} = mc\boldsymbol{dT}$$

where m = mass,

c = specific heat, and

dT = temperature rise.

THE PERFECT GAS

Specific Heats

- For a **gas** there are an infinite number of ways in which heat may be added between any two temperatures, and hence a *gas could have an infinite number of specific heats.*
- However, only twomspecific heats for gases are defined.

Specific heat at ***constant volume***, $\boldsymbol{c_v}$ and,

Specific heat at ***constant pressure***, $\boldsymbol{c_p}$.

- We have

$dQ = m\, c_p\, dT$ For a reversible non-flow process at *constant pressure*

and, $dQ = m\, c_v\, dT$ For a reversible non-flow process at *constant volume*

THE PERFECT GAS

Specific Heats

- The values of c_p and c_v, for a perfect gas, are constant for any one gas at all pressures and temperatures.
- Hence, integrating eqns. → $dQ = m\,c_p\,dT$, $dQ = m\,c_v\,dT$ we have
- Flow of heat in a reversible constant pressure process

$$= mc_p\,(T_2 - T_1)$$

- Flow of heat in a reversible constant volume process

$$= mc_v\,(T_2 - T_1)$$

- In case of *real gases,* c_p and c_v *vary with temperature,* but a suitable *average value* may be used for most practical purposes.

THE PERFECT GAS

4.8.3. Joule's Law

"*The internal energy of a perfect gas is a function of the absolute temperature only*"

i.e., $u = f(T)$

- To evaluate this function let 1 kg of a perfect gas be heated at constant volume.
- According to non-flow energy equation,

$$dQ = du + dW$$

$dW = 0$, since volume remains constant

$$\therefore \quad dQ = du$$

- At constant volume for a perfect gas, from eqn. ($dQ = m\,c_v\,dT$), for 1 kg

$$dQ = c_v\,dT$$

$$\therefore \quad dQ = du = c_v\,dT$$

and integrating $u = c_v\,T + K$, K being constant.

THE PERFECT GAS

Joule's Law

- According to Joule's law $u = f(T)$, which means that internal energy varies linearly with absolute temperature. Internal energy can be made zero at any arbitrary reference temperature.
- For a perfect gas it can be assumed that $u = 0$ when $T = 0$, hence constant K is zero.

i.e., Internal energy, $u = c_v T$ for a perfect gas

or For mass m, of a perfect gas

Internal energy, $U = mc_v T$

- For a perfect gas, in any process between states 1 and 2, we have from Eqn. ($U = mc_v T$)
- Gain in internal energy,

$$U_2 - U_1 = m\, c_v\, (T_2 - T_1)$$

- Eqn. ($U_2 - U_1 = m\, c_v\, (T_2 - T_1)$) gives the gains of internal energy for a perfect gas between two states *for any process, reversible* or *irreversible.*

THE PERFECT GAS

4.8.4. Relationship Between Two Specific Heats

- Consider a perfect gas being heated at constant pressure from T_1 to T_2.
- According to non-flow equation,

$$Q = (U_2 - U_1) + W$$

- Also for a perfect gas,

$$U_2 - U_1 = mc_v\, (T_2 - T_1)$$
$$Q = mc_v\, (T_2 - T_1) + W$$

- In a constant pressure process, the work done by the fluid,

$$\begin{aligned} W &= p(V_2 - V_1) \\ &= mR(T_2 - T_1) \end{aligned} \quad \left[\begin{array}{l} \because \quad p_1V_1 = mRT_1 \\ \quad\;\; p_2V_2 = mRT_2 \\ \quad\;\; p_1 = p_2 = p \text{ in this case} \end{array}\right]$$

THE PERFECT GAS

Relationship Between Two Specific Heats

- On substituting

$$Q = mc_v (T_2 - T_1) + mR (T_2 - T_1) = m(c_v + R) (T_2 - T_1)$$

- But for a constant pressure process,

$$Q = mc_p (T_2 - T_1)$$

- By equating the two expressions, we have

$$m(c_v + R)(T_2 - T_1) = mc_p(T_2 - T_1)$$

∴ $$c_v + R = c_p$$

or $$c_p - c_v = R$$

THE PERFECT GAS

Relationship Between Two Specific Heats

- Dividing both sides by c_v, we get

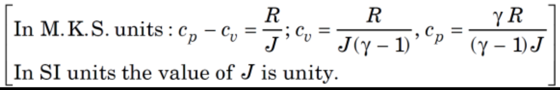
$$\frac{c_p}{c_v} - 1 = \frac{R}{c_v}$$

∴ $$c_v = \frac{R}{\gamma - 1}$$

(where $\gamma = c_p/c_v$)

- Similarly, dividing both sides by c_p, we get

$$c_P = \frac{\gamma R}{\gamma - 1}$$



$$\left[\begin{array}{l} \text{In M.K.S. units}: c_p - c_v = \dfrac{R}{J};\ c_v = \dfrac{R}{J(\gamma - 1)},\ c_p = \dfrac{\gamma R}{(\gamma - 1)J} \\ \text{In SI units the value of } J \text{ is unity.} \end{array}\right]$$

THE PERFECT GAS

4.8.5. Enthalpy

— One of the fundamental quantities which occur invariably in thermodynamics is the sum of internal energy (u) and pressure volume product (pv). This sum is called **Enthalpy** (h).

i.e., $h = u + pv$

— The enthalpy of a fluid is the property of the fluid, since it consists of the sum of a property and the product of the two properties.

- Since enthalpy is a property like internal energy, pressure, specific volume and temperature, it can be introduced into any problem whether the process is a flow or a non-flow process.
- The total enthalpy of mass, m, of a fluid can be

$$H = U + pV, \text{ where } H = mh.$$

THE PERFECT GAS

Enthalpy

- For a **perfect gas,** Referring equation ($h = u + pv$),

$$\begin{aligned} h &= u + pv \\ &= c_vT + RT && [\,pv = RT] \\ &= (c_v + R)T \\ &= c_pT && [\,c_p = c_v + R] \end{aligned}$$

i.e., $h = c_pT$

and $H = mc_pT$.

(Note that, since it has been assumed that $u = 0$ at $T = 0$, then $h = 0$ at $T = 0$).

THE PERFECT GAS

4.8.6. Ratio of Specific Heats

- The ratio of specific heat at constant pressure to the specific heat at constant volume is given the symbol γ (gamma).

i.e., $\gamma = c_p/c_v$

- Since $c_p = c_v + R$, it is clear that c_p must be *greater* than c_v for any perfect gas. It follows, therefore, that the ratio, $\gamma = c_p/c_v$ is *always greater than unity.*

THE PERFECT GAS

Ratio of Specific Heats

- In general, the approximate values of γ are as follows :
  - For *monoatomic* gases such as *argon, helium* = 1.6.
  - For *diatomic* gases such as *carbon monoxide, hydrogen, nitrogen* and *oxygen* = 1.4.
  - For triatomic gases such as *carbon dioxide* and *Sulphur dioxide* = 1.3.
  - For some hydro-carbons the value of γ is quite low.
  - [*e.g.*, for ethane $\gamma = 1.22$, and for isobutane $\gamma = 1.11$]

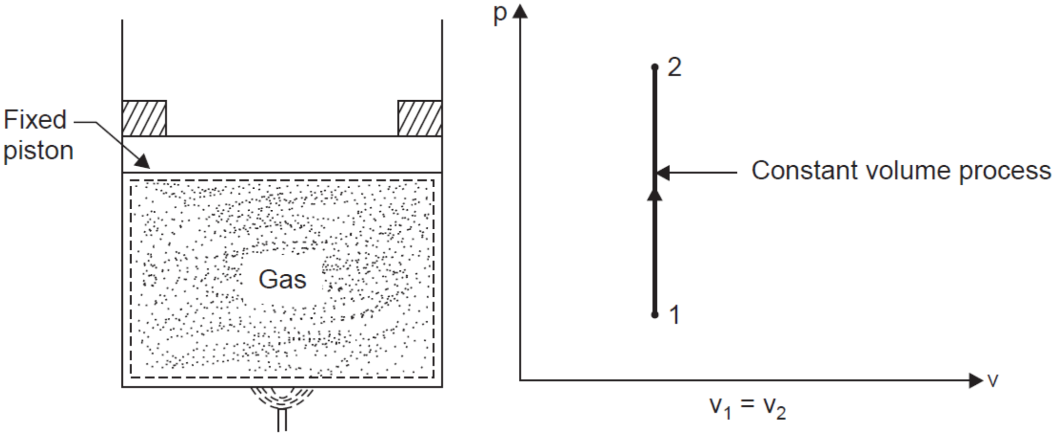
4.9 APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

1. Reversible Constant Volume (or Isochoric) Process (v = constant) :

- In a constant volume process the working substance is contained in a rigid vessel, hence the boundaries of the system are immovable and no work can be done on or by the system, other than paddle-wheel work input.
- It will be assumed that '*constant volume*' implies zero work unless stated otherwise.
- Next fig shows the system and states before and after the heat addition at constant volume.

APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

1. Reversible Constant Volume (or Isochoric) Process (v = constant) :



(a) (b)

APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

1. Reversible Constant Volume (or Isochoric) Process (v = constant) :

- Considering mass of the working substance *unity* and applying first law of thermodynamics to the process

$$Q = (u_2 - u_1) + W$$

The work done, $W = \int_1^2 pdv = 0 \; as \; dv = 0$

$$\therefore Q = (u_2 - u_1) = c_v(T_2 - T_1)$$

where c_v = Specific heat at constant volume.

- For mass, ***m***, of working substance

$$Q = U_2 - U_1 = mc_v(T_2 - T_1)$$

$$[\, mu = U]$$

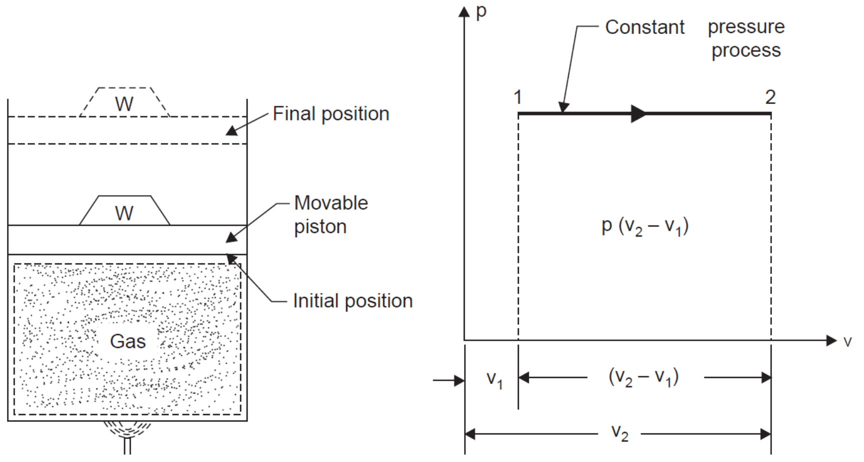
APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

2. Reversible Constant Pressure (or Isobaric) Process (p = constant).

- It can be seen from last fig part (*b*) that when the boundary of the system is *inflexible* as in a constant volume process, then the pressure rises when heat is supplied.
- Hence for a **constant pressure process**, the boundary must move against an external resistance as heat is supplied ; for instance a gas in a cylinder behind a piston can be made to undergo a constant pressure process.
- Since the *piston is pushed through a certain distance* by the force exerted by the gas, then the work is done by the gas on its surroundings.

APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

2. Reversible Constant Pressure (or Isobaric) Process (p = constant).



(a) (b)

APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

2. Reversible Constant Pressure (or Isobaric) Process (p = constant).

- Considering unit mass of working substance and applying first law of thermodynamics to the process

$$Q = (u_2 - u_1) + W$$

The work done, $W = \int_1^2 p dv = p(v_2 - v_1)$

$$\therefore Q = (u_2 - u_1) + p(v_2 - v_1) = u_2 - u_1 + pv_2 - pv_1$$

$$= (u_2 + pv_2) - (u_1 + pv_1) = h_2 - h_1 \qquad [\, h = u + pv]$$

or $\quad Q = h_2 - h_1 = c_p (T_2 - T_1)$

where h = Enthalpy (specific), and cp = Specific heat at constant pressure.

- For mass, m, of working substance

$$Q = H_2 - H_1 = mc_p (T_2 - T_1) \qquad [\, mh = H]$$

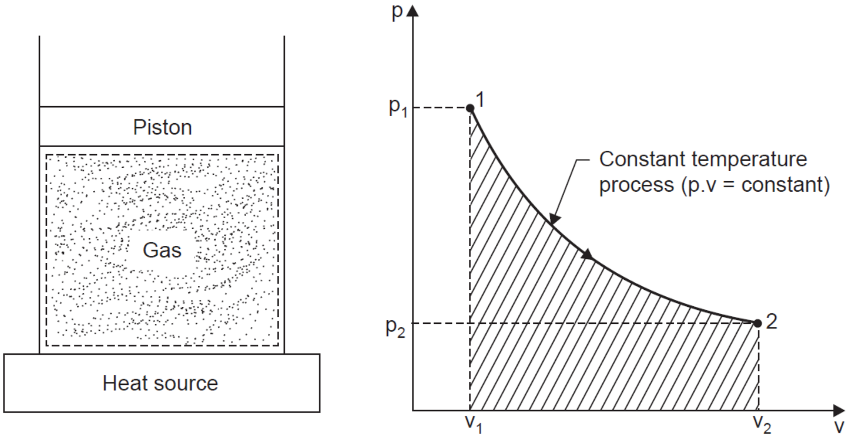
APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

3. Reversible Temperature (or Isothermal) Process (pv = constant, T = constant) :

- A *process at a constant temperature is called an isothermal process.*
- When a working substance in a cylinder behind a piston expands from a high pressure to a low pressure there is a tendency for the temperature to fall.
- In an isothermal expansion heat must be added continuously in order to keep the temperature at the initial value.
- Similarly in an isothermal compression heat must be removed from the working substance continuously during the process.
- Next fig. shows the system and states before and after the heat addition at constant temperature.

APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

3. Reversible Temperature (or Isothermal) Process (pv = constant, T = constant) :



(a)

(b)

APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

3. Reversible Temperature (or Isothermal) Process (pv = constant, T = constant) :

- Considering unit mass of working substance and applying first law of thermodynamics to the process

$$Q = (u_2 - u_1) + W$$

$$= c_v (T_2 - T_1) + W$$

$$= 0 + W \qquad [\, T_2 = T_1]$$

- The work done, $W = \int_1^2 p dv$
- In this case pv = constant or $p = \frac{C}{v}$ (where C = constant)

$\therefore$ $$\mathrm{W} = \int_{v_1}^{v_2} C \frac{dv}{v} = C[log_e v]_{v_1}^{v_2} = C \, log_e \frac{v_2}{v_1}$$

APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

3. Reversible Temperature (or Isothermal) Process (pv = constant, T = constant) :

- The constant C can either be written as p_1v_1 or as p_2v_2, since

$$p_1v_1 = p_2v_2 = \text{constant, } C$$

i.e., $W = p_1v_1 \, log_e \frac{v_2}{v_1}$ per unit mass of working substance

or $W = p_2v_2 \, log_e \frac{v_2}{v_1}$ per unit mass of working substance

$$\therefore Q = W = p_1v_1 \, log_e \frac{v_2}{v_1}$$

- For mass, m, of the working substance

$$Q = p_1V_1 \, log_e \frac{V_2}{V_1}$$

Or $$Q = p_1V_1 \, log_e \frac{p_1}{p_2} \qquad \left[\frac{v_2}{v_1} = \frac{p_1}{p_2}\right]$$

APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

4. Reversible Adiabatic Process (pv^{γ} = constant) :

- An **adiabatic process** *is one in which no heat is transferred to or from the fluid during the process.*
- Such a process can be reversible or irreversible.
- The reversible adiabatic non-flow process will be considered in this section.
- Considering unit mass of working substance and applying first law to the process

$$Q = (u2 - u1) + W$$

$$O = (u2 - u1) + W$$

or $$W = (u1 - u2)$$ for any adiabatic process

APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

4. Reversible Adiabatic Process (pv^{γ} = constant) :

- $W = (u1 - u2)$ is true for an adiabatic process whether the process is reversible or not.
- In an adiabatic expansion, the work done W by the fluid is at the expense of a reduction in the internal energy of the fluid.
- Similarly in an adiabatic compression process all the work done on the fluid goes to increase the internal energy of the fluid.
- *For an adiabatic process to take place, perfect thermal insulation for the system must be available.*

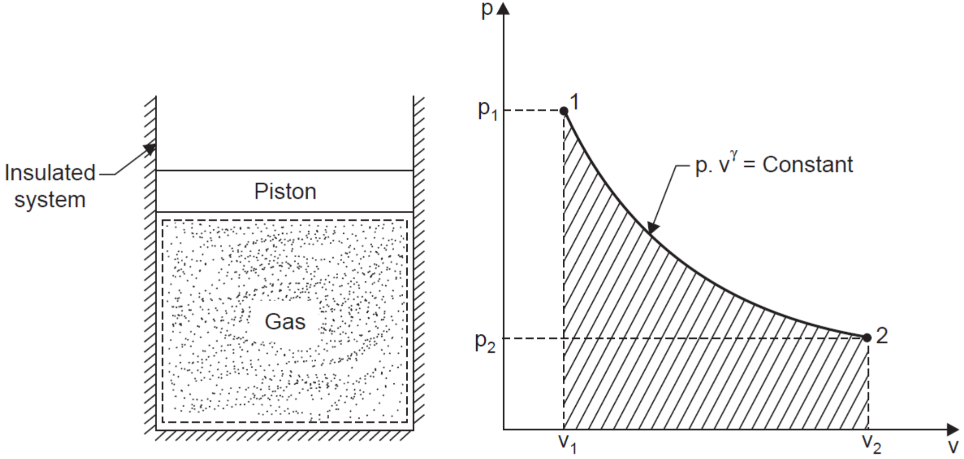
APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

4. Reversible Adiabatic Process (pv^{γ} = constant) :

- **To derive the law pv^{γ} = constant :**

APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

4. Reversible Adiabatic Process (pv^{γ} = constant) :



(a) (b)

APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

4. Reversible Adiabatic Process (pv^{γ} = constant) :

- **Expression for work W:**
- The work done is given by the shaded area, and this area can be evaluated by integration.

$$W = \int_1^2 p dv$$

- Therefore, since pv^{γ} = constant, C, then

$$W = \int_{v_1}^{v_2} C \frac{dv}{v^{\gamma}} = C \left| \frac{v^{-\gamma+1}}{-\gamma+1} \right|_{v_1}^{v_2}$$

$$= C \left(\frac{v_2^{-\gamma+1} - v_1^{-\gamma+1}}{1-\gamma} \right) = C \left(\frac{v_1^{-\gamma+1} - v_2^{-\gamma+1}}{\gamma - 1} \right)$$

APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

4. Reversible Adiabatic Process (pv^{γ} = constant) :

- **Expression for work W:**
- The constant in the last equation can be written as $p_1 v_1^{\gamma}$ or as $p_2 v_2^{\gamma}$. Hence,

$$W = \frac{p_1 v_1^{\gamma} v_1^{-\gamma+1} - p_2 v_2^{\gamma} v_2^{-\gamma+1}}{\gamma - 1} = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1}$$

$$W = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1}$$

$$W = \frac{R(T_1 - T_2)}{\gamma - 1}$$

APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

4. Reversible Adiabatic Process (pv^{γ} = constant) :

- **Relationship between T and v, and T and p:**

By using equation $pv = RT$, the relationship between T and v, and T and p, may by derived as follows :

i.e., $$pv = RT$$

$$\therefore \quad p = \frac{RT}{v}$$

Putting this value in the equation pv^{γ} = constant

$$\frac{RT}{v} \cdot v^{\gamma} = \text{constant}$$

i.e., $$Tv^{\gamma-1} = \text{constant} \qquad \ldots(4.34)$$

Also $v = \frac{RT}{p}$; hence substituting in equation pv^{γ} = constant

$$p\left(\frac{RT}{p}\right)^{\gamma} = \text{constant}$$

$$\therefore \quad \frac{T^{\gamma}}{p^{\gamma-1}} = \text{constant}$$

or $$\frac{T}{(p)^{\frac{\gamma-1}{\gamma}}} = \text{constant} \qquad \ldots(4.35)$$

Therefore, for a reversible adiabatic process for a perfect gas between states 1 and 2, we can write :

From Eqn. (4.31), pv^{γ} = constant

$$p_1 v_1^{\gamma} = p_2 v_2^{\gamma} \quad \text{or} \quad \frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} \qquad \text{...(4.36)}$$

From Eqn. (4.34), $Tv^{\gamma-1}$ = constant

$$T_1 v_1^{\gamma-1} = T_2 v_2^{\gamma-1} \quad \text{or} \quad \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} \qquad \text{...(4.37)}$$

From Eqn. (4.35),

$$\frac{T_1}{(p_1)^{\frac{\gamma-1}{\gamma}}} = \frac{T_2}{(p_2)^{\frac{\gamma-1}{\gamma}}} \quad \text{or} \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \qquad \text{...(4.38)}$$

From eqn. (4.30), the work done in an adiabatic process per kg of gas is given by $W = (u_1 - u_2)$. The gain in internal energy of a perfect gas is given by equation :

$W = (u_1 - u_2)$ ...(4.30)
for any adiabatic process

$$u_2 - u_1 = c_v (T_2 - T_1) \qquad \text{(for 1 kg)}$$

$$\therefore \quad W = c_v (T_1 - T_2)$$

Also, we know that

$$c_v = \frac{R}{\gamma - 1}$$

Hence substituting, we get

$$W = \frac{R (T_1 - T_2)}{\gamma - 1}$$

Using equation, $pv = RT$

$$W = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1}$$

This is the same expression obtained before as eqn. (4.32).

$$\boxed{W = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1} \qquad \text{...(4.32)}}$$

APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

5. Polytropic Reversible Process (pv^n = constant) :

- It is found that many processes in practice approximate to a reversible law of form pv^n = constant, where n is a constant.
- Both vapours and perfect gases obey this type of law closely in many non-flow processes.
- Such processes are *internally reversible*.

APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

5. Polytropic Reversible Process (pv^n = constant) :

- We know that for any reversible process, $W = \int p\,dv$
- For a process in pv^n = constant, we have

$$p = \frac{C}{v^n} \text{ , where } C \text{ is a constant}$$

$$\therefore \quad W = C\int_{v_1}^{v_2}\frac{dv}{v^n} = C\left|\frac{v^{-n+1}}{-n+1}\right| = C\left(\frac{v_2^{-n+1} - v_1^{-n+1}}{-n+1}\right)$$

$$i.e., \quad W = C\left(\frac{v_1^{-n+1} - v_2^{-n+1}}{n-1}\right) = \frac{p_1v_1^{n}v_1^{-n+1} - p_2v_2^{n}v_2^{-n+1}}{n-1}$$

(since the constant C, can be written as **$p_1v_1^n$** or as **$p_2v_2^n$**)

APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

5. Polytropic Reversible Process (pv^n = constant) :

i.e., Work done, $W = \dfrac{p_1v_1 - p_2v_2}{n-1}$ ...(4.39)

or $W = \dfrac{R(T_1 - T_2)}{n-1}$ ...(4.40)

Eqn. (4.39) is true for any working substance undergoing a reversible polytropic process. It follows also that for any polytropic process, we can write

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^n \quad ...(4.41)$$

The following relations can be derived (following the same procedure as was done under reversible adiabatic process)

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1} \quad ...(4.42)$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \quad ...(4.43)$$

APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

5. Polytropic Reversible Process (pv^n = constant) :

- **Heat transfer during polytropic process (for perfect gas pv = RT) :**

Using non-flow energy equation, the heat flow/transfer during the process can be found,

$$i.e., \quad Q = (u_2 - u_1) + W$$

$$= c_v(T_2 - T_1) + \frac{R(T_1 - T_2)}{n-1}$$

$$i.e., \quad Q = \frac{R(T_1 - T_2)}{n-1} - c_v\,(T_1 - T_2)$$

$$\text{Also} \quad c_v = \frac{R}{(\gamma - 1)}$$

APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

5. Polytropic Reversible Process (pv^n = constant) :

- **Heat transfer during polytropic process (for perfect gas pv = RT) :**

On substituting,

$$Q = \frac{R}{n-1}(T_1 - T_2) - \frac{R}{(\gamma - 1)}(T_1 - T_2)$$

$$i.e., \quad Q = R(T_1 - T_2)\left(\frac{1}{n-1} - \frac{1}{\gamma - 1}\right)$$

$$= \frac{R\,(T_1 - T_2)(\gamma - 1 - n + 1)}{(\gamma - 1)(n-1)} = \frac{R(T_1 - T_2)(\gamma - n)}{(\gamma - 1)(n-1)}$$

$$\therefore \quad Q = \frac{(\gamma - n)}{(\gamma - 1)} \frac{R(T_1 - T_2)}{(n-1)}$$

$$\text{or} \quad Q = \left(\frac{\gamma - n}{\gamma - 1}\right) W \qquad \left[\because W = \frac{R(T_1 - T_2)}{(n-1)}\right]$$

APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

5. Polytropic Reversible Process (pv^n = constant) :

- **Heat transfer during polytropic process (for perfect gas pv = RT) :**

In a polytropic process, the *index n depends only on the heat and work quantities* during the process. The various processes considered earlier are special cases of polytropic process for a perfect gas. For example,

When $n = 0$ $\quad pv^\circ = \text{constant}\ i.e.,\ p = \text{constant}$

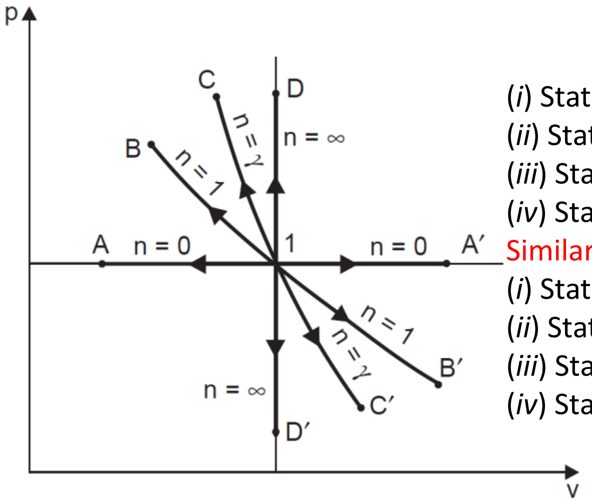
When $n = \infty$ $\quad pv^\infty = \text{constant}$

or $p^{1/\infty}\ v = \text{constant},\ i.e.,\ v = \text{constant}$

When $n = 1$ $\quad pv = \text{constant},\ i.e.,\ T = \text{constant}$

[since $(pv)/T$ = constant for a perfect gas]

When $n = \gamma$ $\quad pv^\gamma = \text{constant},\ i.e.,$ reversible adiabatic



(*i*) State 1 to state *A* is *constant pressure cooling* ($n = 0$).
(*ii*) State 1 to state *B* is *isothermal compression* ($n = 1$).
(*iii*) State 1 to *C* is *reversible adiabatic compression* ($n = \gamma$).
(*iv*) State 1 to state *D* is *constant volume heating* ($n = \infty$).

Similarly,

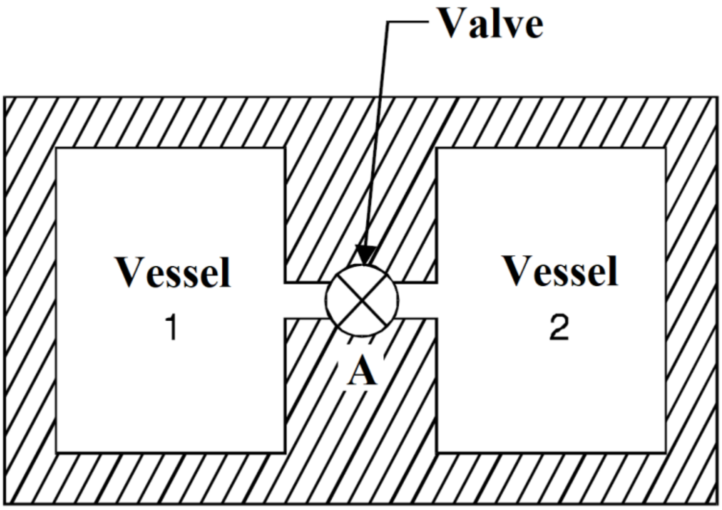
(*i*) State 1 to state *A*′ is *constant pressure heating* ($n = 0$).
(*ii*) State 1 to state *B*′ is *isothermal expansion* ($n = 1$).
(*iii*) State 1 to *C*′ is *reversible adiabatic expansion* ($n = \gamma$).
(*iv*) State 1 to state *D*′ is *constant volume cooling* ($n = \infty$).

It may be noted that, since γ is always greater than unity, than process 1 to *C* must lie between processes 1 to *B* and 1 to *D* ; similarly, process 1 to *C*′ must lie between processes 1 to *B*′ and 1 to *D*′.

APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

6. Free Expansion

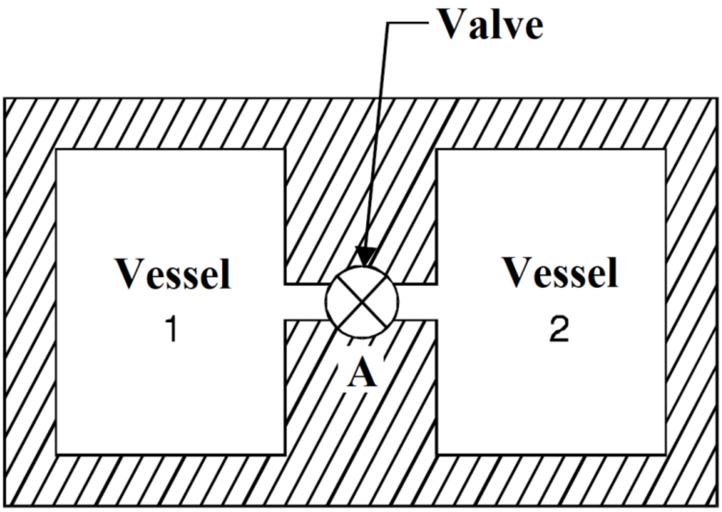
- Consider two vessels 1 and 2 interconnected by a short pipe with a valve *A*, and perfectly thermally insulated.
- Initially let the vessel 1 be filled with a fluid at a certain pressure, and let 2 be completely evacuated.



APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

6. Free Expansion

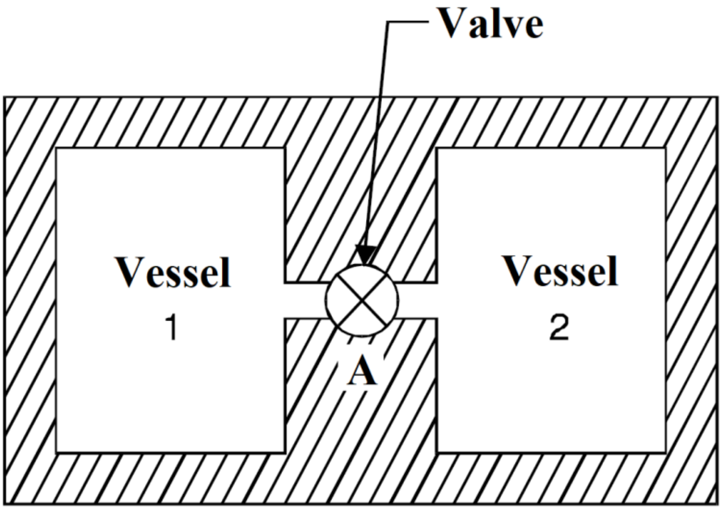
- When the valve *A* is opened the fluid in 1 will expand rapidly to fill both vessels 1 and 2.
- The pressure finally will be lower than the initial pressure in vessel 1.



APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

6. Free Expansion

- This is known as *free or **unresisted expansion***.
- The process is ***highly irreversible*** ; since the fluid is eddying continuously during the process.



APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

6. Free Expansion

- Now applying first law of thermodynamics (or nonflow energy equation) between the initial and final states,

$$Q = (u_2 - u_1) + W$$

- In this process, no work is done on or by the fluid, since the boundary of the system does not move. No heat flows to or from the fluid since the system is well lagged.
- The process is therefore, *adiabatic but irreversible*.

i.e., $u_2 - u_1 = 0$ or $u_2 = u_1$

APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

6. Free Expansion

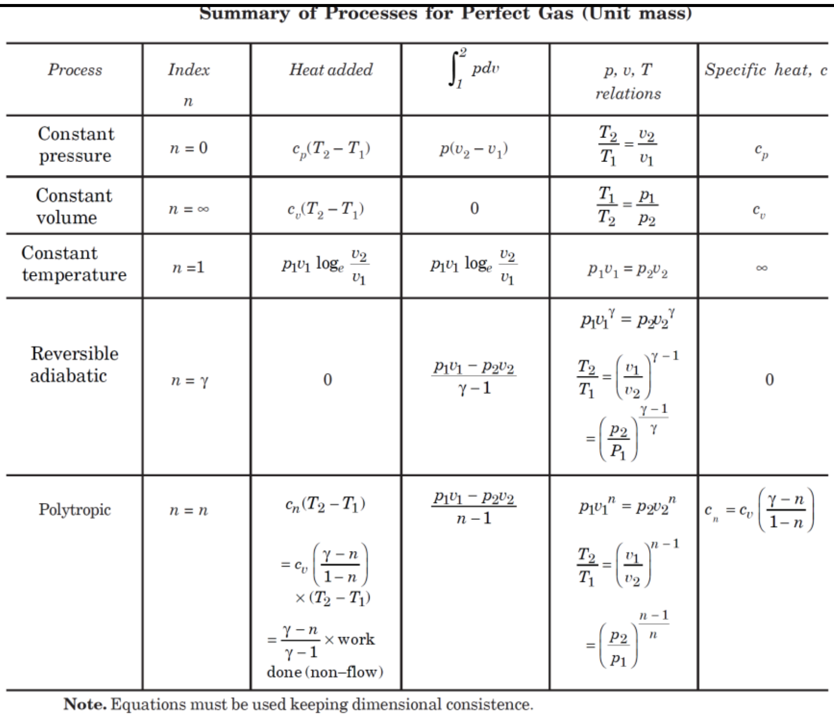
- In a free expansion, therefore, the internal energy initially equals the internal energy finally.
- For a perfect gas,

$$u = c_v T$$

∴ For a free expansion of a perfect gas,

$$c_v T_1 = c_v T_2 \qquad i.e., \quad T_1 = T_2$$

- That is, for a perfect gas undergoing a free expansion, the initial temperature is equal to the final temperature.



Summary of Processes for Perfect Gas (Unit mass)

Process	Index n	Heat added	$\int_1^2 p\,dv$	p, v, T relations	Specific heat, c
Constant pressure	$n = 0$	$c_p(T_2 - T_1)$	$p(v_2 - v_1)$	$\frac{T_2}{T_1} = \frac{v_2}{v_1}$	c_p
Constant volume	$n = \infty$	$c_v(T_2 - T_1)$	0	$\frac{T_1}{T_2} = \frac{p_1}{p_2}$	c_v
Constant temperature	$n = 1$	$p_1 v_1 \log_e \frac{v_2}{v_1}$	$p_1 v_1 \log_e \frac{v_2}{v_1}$	$p_1 v_1 = p_2 v_2$	∞
Reversible adiabatic	$n = \gamma$	0	$\frac{p_1 v_1 - p_2 v_2}{\gamma - 1}$	$p_1 v_1^{\gamma} = p_2 v_2^{\gamma}$ $\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1}$ $= \left(\frac{p_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}}$	0
Polytropic	$n = n$	$c_n(T_2 - T_1)$ $= c_v \left(\frac{\gamma - n}{1 - n}\right) \times (T_2 - T_1)$ $= \frac{\gamma - n}{\gamma - 1} \times$ work done (non–flow)	$\frac{p_1 v_1 - p_2 v_2}{n - 1}$	$p_1 v_1^{n} = p_2 v_2^{n}$ $\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n - 1}$ $= \left(\frac{p_2}{p_1}\right)^{\frac{n - 1}{n}}$	$c_n = c_v \left(\frac{\gamma - n}{1 - n}\right)$

Note. Equations must be used keeping dimensional consistence.

EXAMPLES FOR PART 1