# 1.3 Matrix Arithmetic

In this section, we introduce the standard notations used for matrices and vectors and define arithmetic operations (addition, subtraction, and multiplication) with matrices. We will also introduce two additional operations: *scalar multiplication* and *transposition*. We will see how to represent linear systems as equations involving matrices and vectors and then derive a theorem characterizing when a linear system is consistent.

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# Matrices

A **matrix** is a rectangular array of numbers. The numbers in the array are called the **entries** in the matrix.

# Size of a matrix

The **size** of a matrix is described in terms of the **number of rows** (horizontal lines) and the **number of columns** (vertical lines) it contains.

# Notations

We will use **capital letters** to denote matrices and **lowercase letters** to denote numerical quantities. The entry that occurs in row $i$ and column $j$ of a matrix $A$ will be denoted by $a_{ij}$. Thus, a general $m \times n$ matrix might be written as

$$A = [a_{ij}]_{m\times n} = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

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## Row Vector

A matrix with only one row is called a <u>row vector</u> (or a row matrix).

## Column Vector

A matrix with only one column is called a <u>column vector</u> (or a column matrix).

## Notations

It is common practice to denote row and column vectors by **<u>boldface lowercase letters</u>** rather than capital letters. For such matrices, double subscripting of the entries is unnecessary.

Thus, a general $1 \times n$ row vector $\mathbf{a}$ would be written as

$$\mathbf{a} = [a_1 \quad a_2 \quad \cdots \quad a_n]$$

and a general $m \times 1$ column vector $\mathbf{b}$ would be written as

$$\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

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If $A$ is an $m \times n$ matrix, then the row vectors of $A$ are given by

$$\vec{\mathbf{a}}_i = (a_{i1}, a_{i2}, \ldots, a_{in}) \qquad i = 1, \ldots, m$$

ınd the column vectors are given by

$$\mathbf{a}_j = \begin{pmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{mj} \end{pmatrix} \qquad j = 1, \ldots, n$$

**EXAMPLE 1** If

$$A = \begin{pmatrix} 3 & 2 & 5 \\ -1 & 8 & 4 \end{pmatrix}$$

then

$$\mathbf{a}_1 = \begin{pmatrix} 3 \\ -1 \end{pmatrix}, \quad \mathbf{a}_2 = \begin{pmatrix} 2 \\ 8 \end{pmatrix}, \quad \mathbf{a}_3 = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

and

$$\vec{\mathbf{a}}_1 = (3, 2, 5), \quad \vec{\mathbf{a}}_2 = (-1, 8, 4)$$

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The matrix $A$ can be represented in terms of either its column vectors or its row vectors:

$$A = (\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n) \qquad \text{or} \qquad A = \begin{pmatrix} \vec{\mathbf{a}}_1 \\ \vec{\mathbf{a}}_2 \\ \vdots \\ \vec{\mathbf{a}}_m \end{pmatrix}$$

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## Equality

For two matrices to be equal, they must have the same dimensions and their corresponding entries must agree.

**Definition**

Two $m \times n$ matrices $A$ and $B$ are said to be **equal** if $a_{ij} = b_{ij}$ for each $i$ and $j$.

**Example.** Find the values of $x$, $y$, $z$ and $w$ such that

$$\begin{bmatrix} 0 & y \\ z & 1 \end{bmatrix} = \begin{bmatrix} x-1 & 4 \\ 2 & w+1 \end{bmatrix}$$

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## Scalar Multiplication

If $A$ is a matrix and $\alpha$ is a scalar, then $\alpha A$ is the matrix formed by multiplying each of the entries of $A$ by $\alpha$.

**Definition**

> If $A$ is an $m \times n$ matrix and $\alpha$ is a scalar, then $\alpha A$ is the $m \times n$ matrix whose $(i, j)$ entry is $\alpha a_{ij}$.

For example, if

$$A = \begin{bmatrix} 4 & 8 & 2 \\ 6 & 8 & 10 \end{bmatrix}$$

then

$$\frac{1}{2}A = \begin{bmatrix} 2 & 4 & 1 \\ 3 & 4 & 5 \end{bmatrix} \quad \text{and} \quad 3A = \begin{bmatrix} 12 & 24 & 6 \\ 18 & 24 & 30 \end{bmatrix}$$

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## Matrix Addition

Two matrices with the same dimensions can be added by adding their corresponding entries.

**Definition**

If $A = (a_{ij})$ and $B = (b_{ij})$ are both $m \times n$ matrices, then the **sum** $A + B$ is the $m \times n$ matrix whose $(i, j)$ entry is $a_{ij} + b_{ij}$ for each ordered pair $(i, j)$.

For example,

$$\begin{pmatrix} 3 & 2 & 1 \\ 4 & 5 & 6 \end{pmatrix} + \begin{pmatrix} 2 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 5 & 4 & 3 \\ 5 & 7 & 9 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 1 \\ 8 \end{pmatrix} + \begin{pmatrix} -8 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \\ 10 \end{pmatrix}$$

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If we define $A - B$ to be $A + (-1)B$, then it turns out that $A - B$ is formed by subtracting the corresponding entry of $B$ from each entry of $A$. Thus,

$$\begin{aligned}
\begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix} &= \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix} + (-1)\begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix} \\
&= \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} -4 & -5 \\ -2 & -3 \end{bmatrix} \\
&= \begin{bmatrix} 2-4 & 4-5 \\ 3-2 & 1-3 \end{bmatrix} \\
&= \begin{bmatrix} -2 & -1 \\ 1 & -2 \end{bmatrix}
\end{aligned}$$

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If $O$ represents the matrix, with the same dimensions as $A$, whose entries are all 0, then

$$A + O = O + A = A$$

We will refer to $O$ as the *zero matrix*. It acts as an additive identity on the set of all $m \times n$ matrices. Furthermore, each $m \times n$ matrix $A$ has an additive inverse. Indeed,

$$A + (-1)A = O = (-1)A + A$$

It is customary to denote the additive inverse by $-A$. Thus,

$$-A = (-1)A$$

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# Matrix Multiplication

**Definition**

If $A = (a_{ij})$ is an $m \times n$ matrix and $B = (b_{ij})$ is an $n \times r$ matrix, then the product $AB = C = (c_{ij})$ is the $m \times r$ matrix whose entries are defined by

$$c_{ij} = \vec{\mathbf{a}}_i \mathbf{b}_j = \sum_{k=1}^{n} a_{ik} b_{kj}$$



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## EXAMPLE 2

$$A = \begin{bmatrix} 4 & 2 & 1 \\ 5 & 3 & 7 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$A\mathbf{x} = \begin{bmatrix} 4x_1 + 2x_2 + x_3 \\ 5x_1 + 3x_2 + 7x_3 \end{bmatrix}$$

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## EXAMPLE 3

$$A = \begin{bmatrix} -3 & 1 \\ 2 & 5 \\ 4 & 2 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$A\mathbf{x} = \begin{bmatrix} -3 \cdot 2 + 1 \cdot 4 \\ 2 \cdot 2 + 5 \cdot 4 \\ 4 \cdot 2 + 2 \cdot 4 \end{bmatrix} = \begin{bmatrix} -2 \\ 24 \\ 16 \end{bmatrix}$$

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**EXAMPLE 8** If

$$A = \begin{bmatrix} 3 & -2 \\ 2 & 4 \\ 1 & -3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -2 & 1 & 3 \\ 4 & 1 & 6 \end{bmatrix}$$

then

$$AB = \begin{bmatrix} 3 & -2 \\ \mathbf{2} & \mathbf{4} \\ 1 & -3 \end{bmatrix} \begin{bmatrix} -2 & 1 & \mathbf{3} \\ 4 & 1 & \mathbf{6} \end{bmatrix}$$

$$= \begin{bmatrix} 3\cdot(-2) - 2\cdot 4 & 3\cdot 1 - 2\cdot 1 & 3\cdot 3 - 2\cdot 6 \\ 2\cdot(-2) + 4\cdot 4 & 2\cdot 1 + 4\cdot 1 & \mathbf{2\cdot 3 + 4\cdot 6} \\ 1\cdot(-2) - 3\cdot 4 & 1\cdot 1 - 3\cdot 1 & 1\cdot 3 - 3\cdot 6 \end{bmatrix}$$

$$= \begin{bmatrix} -14 & 1 & -3 \\ 12 & 6 & \mathbf{30} \\ -14 & -2 & -15 \end{bmatrix}$$

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**EXAMPLE 9** If

$$A = \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 2 \\ 4 & 5 \\ 3 & 6 \end{pmatrix}$$

then it is impossible to multiply $A$ times $B$, since the number of columns of $A$ does not equal the number of rows of $B$. However, it is possible to multiply $B$ times $A$.

$$BA = \begin{pmatrix} 1 & 2 \\ 4 & 5 \\ 3 & 6 \end{pmatrix} \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 8 \\ 17 & 26 \\ 15 & 24 \end{pmatrix}$$

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If $A$ and $B$ are both $n \times n$ matrices, then $AB$ and $BA$ will also be $n \times n$ matrices, but, in general, they will not be equal. *Multiplication of matrices is not commutative*.

**EXAMPLE 10** If

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$$

then

$$AB = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 3 \\ 0 & 0 \end{pmatrix}$$

and

$$BA = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$$

Hence $AB \neq BA$. ■

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# Matrix Form of a Linear System

Matrix multiplication has an important application to systems of linear equations. Consider a system of $m$ linear equations in $n$ unknowns:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ \vdots \qquad \vdots \qquad\qquad \vdots \quad &= \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned} \qquad (3)$$

Since two matrices are equal if and only if their corresponding entries are equal, we can replace the $m$ equations in this system by the single matrix equation

$$\begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \qquad \vdots \qquad\qquad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

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The $m \times 1$ matrix on the left side of this equation can be written as a product to give

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}}_{\mathbf{b}}$$

If we designate these matrices by $A$, $\mathbf{x}$, and $\mathbf{b}$, respectively, then we can replace the original system of $m$ equations in $n$ unknowns by the single matrix equation

$$A\mathbf{x} = \mathbf{b}$$

The matrix $A$ in this equation is called the coefficient matrix of the system.

The augmented matrix for the system is obtained by adjoining $\mathbf{b}$ to $A$ as the last column; thus the augmented matrix is

$$[A \mid \mathbf{b}] = \left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{array}\right]$$

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**EXAMPLE 4** Write the following system of equations as a matrix equation of the form $A\mathbf{x} = \mathbf{b}$:

$$\begin{aligned} 3x_1 + 2x_2 + x_3 &= 5 \\ x_1 - 2x_2 + 5x_3 &= -2 \\ 2x_1 + x_2 - 3x_3 &= 1 \end{aligned}$$

**Solution**

$$\begin{pmatrix} 3 & 2 & 1 \\ 1 & -2 & 5 \\ 2 & 1 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix}$$

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An alternative way to represent the linear system (3) as a matrix equation is to express the product $A\mathbf{x}$ as a sum of column vectors:

$$A\mathbf{x} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{bmatrix}$$

$$= x_1 \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} + x_2 \begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{bmatrix} + \cdots + x_n \begin{bmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{bmatrix}$$

Thus, we have

$$A\mathbf{x} = x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \cdots + x_n\mathbf{a}_n$$

Using this formula, we can represent the system of equations (3) as a matrix equation of the form

$$x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \cdots + x_n\mathbf{a}_n = \mathbf{b}$$

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**EXAMPLE 5** The linear system

$$\begin{aligned} 2x_1 + 3x_2 - 2x_3 &= 5 \\ 5x_1 - 4x_2 + 2x_3 &= 6 \end{aligned}$$

can be written as a matrix equation of the form $x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \cdots + x_n\mathbf{a}_n = \mathbf{b}$

$$x_1 \begin{bmatrix} 2 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} 3 \\ -4 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

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**Definition**

If $\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n$ are vectors in $\mathbb{R}^m$ and $c_1, c_2, \ldots, c_n$ are scalars, then a sum of the form

$$c_1\mathbf{a}_1 + c_2\mathbf{a}_2 + \cdots + c_n\mathbf{a}_n$$

is said to be a **linear combination** of the vectors $\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n$.

**EXAMPLE 6**

$$\begin{pmatrix} 5 \\ 6 \end{pmatrix} = 2\begin{pmatrix} 2 \\ 5 \end{pmatrix} + 3\begin{pmatrix} 3 \\ -4 \end{pmatrix} + 4\begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

Thus, the vector $\begin{pmatrix} 5 \\ 6 \end{pmatrix}$ is a linear combination of the three column vectors of the coefficient matrix.

**Remark:**

we can represent the system of equations (3) as a matrix equation of the form

$$x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \cdots + x_n\mathbf{a}_n = \mathbf{b}$$

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## Theorem 1.3.1 Consistency Theorem for Linear Systems

*A linear system $A\mathbf{x} = \mathbf{b}$ is consistent if and only if $\mathbf{b}$ can be written as a linear combination of the column vectors of $A$.*

**Proof.**

**EXAMPLE 7** The linear system

$$\begin{aligned} x_1 + 2x_2 &= 1 \\ 2x_1 + 4x_2 &= 1 \end{aligned}$$

is inconsistent since the vector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ cannot be written as a linear combination of the column vectors $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$.

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# EXERCISES

**11.** Let $A$ be a $5 \times 3$ matrix. If

$$\mathbf{b} = \mathbf{a}_1 + \mathbf{a}_2 = \mathbf{a}_2 + \mathbf{a}_3$$

then what can you conclude about the number of solutions of the linear system $A\mathbf{x} = \mathbf{b}$? Explain.

**12.** Let $A$ be a $3 \times 4$ matrix. If

$$\mathbf{b} = \mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3 + \mathbf{a}_4$$

then what can you conclude about the number of solutions to the linear system $A\mathbf{x} = \mathbf{b}$? Explain.

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## The Transpose of a Matrix

Given an $m \times n$ matrix $A$, it is often useful to form a new $n \times m$ matrix whose columns are the rows of $A$.

**Definition**

The **transpose** of an $m \times n$ matrix $A$ is the $n \times m$ matrix $B$ defined by

$$b_{ji} = a_{ij} \tag{8}$$

for $j = 1, \ldots, n$ and $i = 1, \ldots, m$. The transpose of $A$ is denoted by $A^T$.

**EXAMPLE 11**

**(a)** If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, then $A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$.

**(b)** If $B = \begin{bmatrix} -3 & 2 & 1 \\ 4 & 3 & 2 \\ 1 & 2 & 5 \end{bmatrix}$, then $B^T = \begin{bmatrix} -3 & 4 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 5 \end{bmatrix}$.

**(c)** If $C = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$, then $C^T = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$. ■

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# Properties of Transpose

If $A$ and $B$ are matrices (such that the stated operations can be performed) and $c$ is a scalar. Then

1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$
3. $(A - B)^T = A^T - B^T$
4. $(cA)^T = c\,A^T$
5. $(AB)^T = B^T A^T$

**Definition** An $n \times n$ matrix $A$ is said to be **symmetric** if $A^T = A$.

The following are some examples of symmetric matrices:

$$\begin{bmatrix} 1 & 0 \\ 0 & -4 \end{bmatrix} \qquad \begin{bmatrix} 2 & 3 & 4 \\ 3 & 1 & 5 \\ 4 & 5 & 3 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & -2 \\ 2 & -2 & -3 \end{bmatrix}$$

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# EXERCISES

**10.** Let $A$ and $B$ be symmetric $n \times n$ matrices. For each of the following, determine whether the given matrix must be symmetric or could be nonsymmetric:

**(a)** $C = A + B$ **(b)** $D = A^2$

**(c)** $E = AB$ **(d)** $F = ABA$

**(e)** $G = AB + BA$ **(f)** $H = AB - BA$

**16.** A matrix $A$ is said to be *skew symmetric* if $A^T = -A$. Show that if a matrix is skew symmetric, then its diagonal entries must all be 0.

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# Trace of Matrices

If $A$ is a square matrix, then the trace of $A$, denoted by $\operatorname{tr}(A)$, is defined to be the sum of the entries on the main diagonal of $A$.

The trace of $A$ is undefined if $A$ is not a square matrix.

## Properties of Trace

If $A$ and $B$ are matrices and $c$ is a scalar. Then

1. $\operatorname{tr}(A^T) = \operatorname{tr}(A)$
2. $\operatorname{tr}(A + B) = \operatorname{tr}(A) + \operatorname{tr}(B)$
3. $\operatorname{tr}(A - B) = \operatorname{tr}(A) - \operatorname{tr}(B)$
4. $\operatorname{tr}(cA) = c\operatorname{tr}(A)$

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