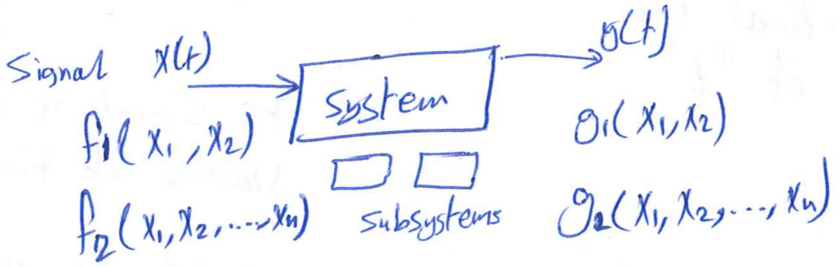
Signals and Systems

Chapter 1 :- Signal and System modeling Concept

1- Introduction :-

- a signal is a function of time that represent a physical variable.
  one or more indepent variable
- a system is a combination and interconnection of several components to perform a desired task.



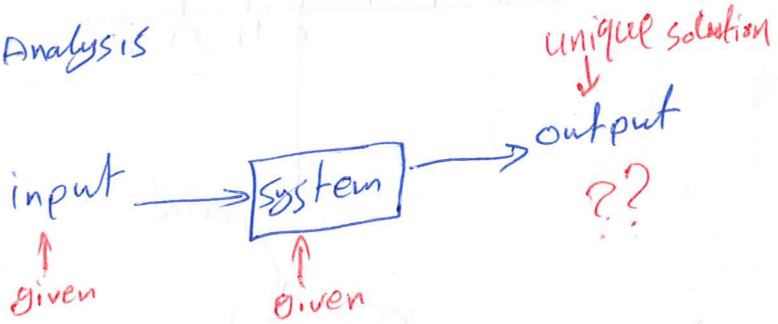
Signals

- Voltage
- Current
- audio
- image
- velocity
- position

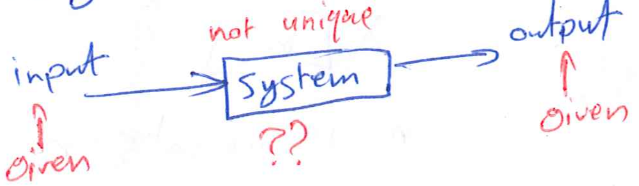
Systems

- electric Circuit
- motor
- PC
- University

- Analysis



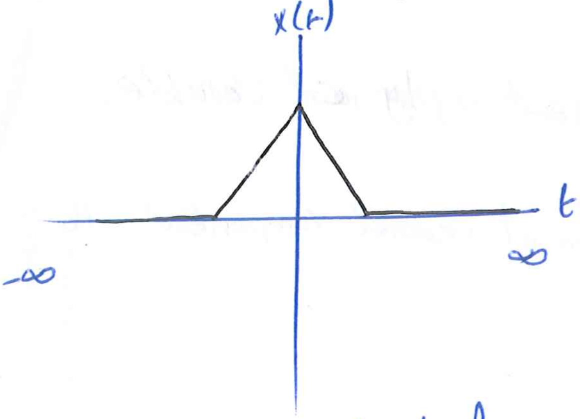
- design



Signal $\xrightarrow{\text{will concedor}}$ one independent Variable (Time)

$X(t)$

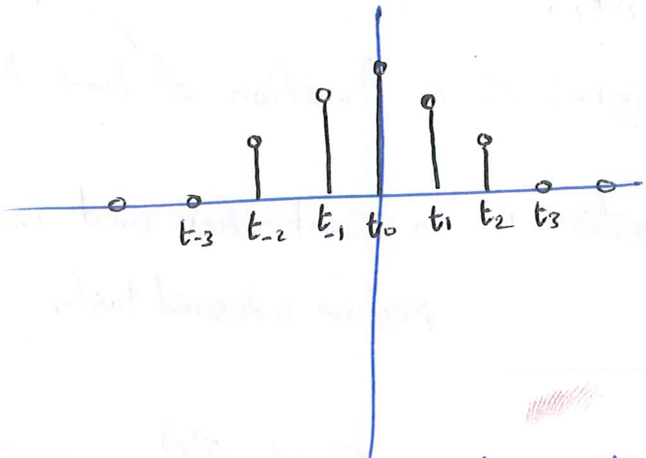
Continuous time Signal (CT)



the signal is defined for every value of "t"

$X[n]$

discrete time signal (DT)

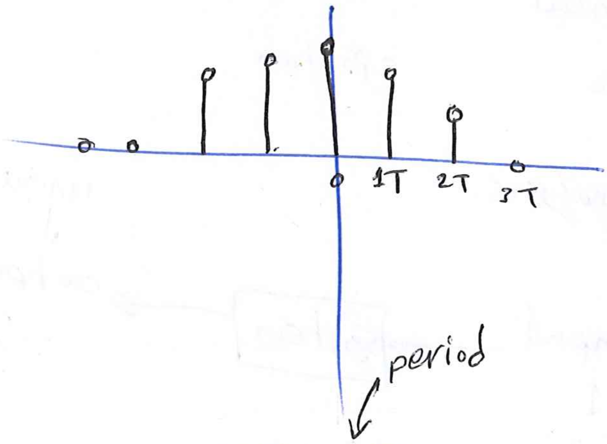


The Signal is exist for a dircrete Values of time.

Could be uniformly Sampled and could be not

$t_2 - t_1 = t_{n+1} - t_n \quad \forall n$

Uniformly Sampled



period

$X[nT]$

$X[n]$

n is an integer

If most (if not all) of the physical "phenomenon" are Continuous, then why do we need to study discrete time signals or systems ??

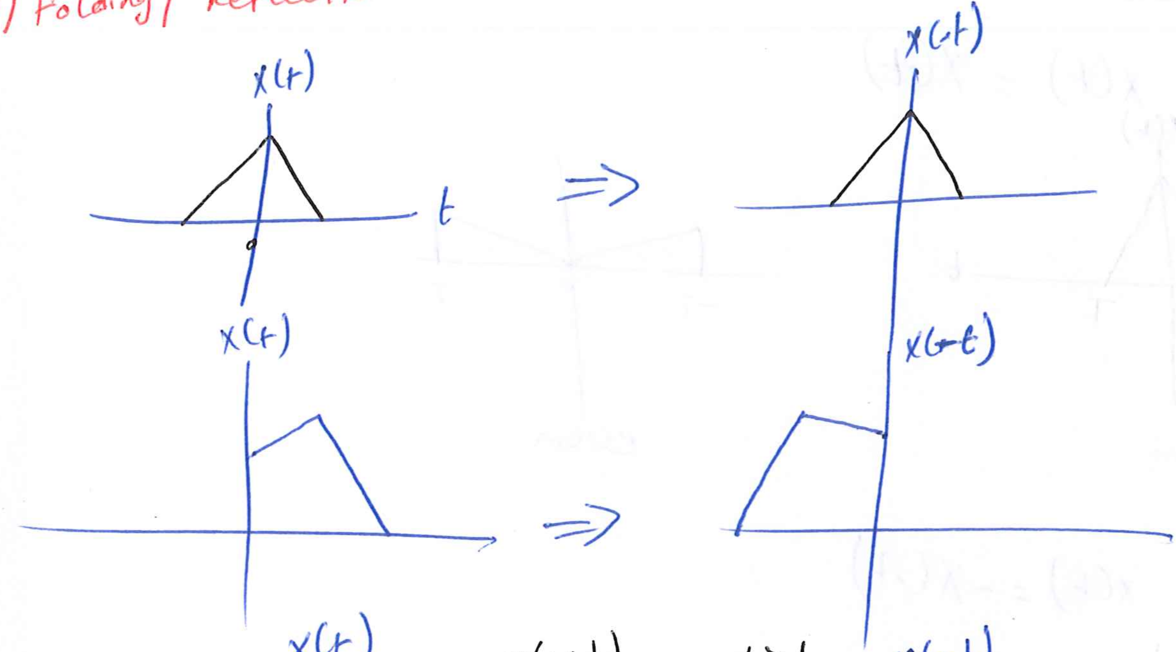
most continuous systems can be analysed more comfortabbly "accurately" with discretization because of the revolution of Itegrated Circuits " computers "

Single Variable Signal

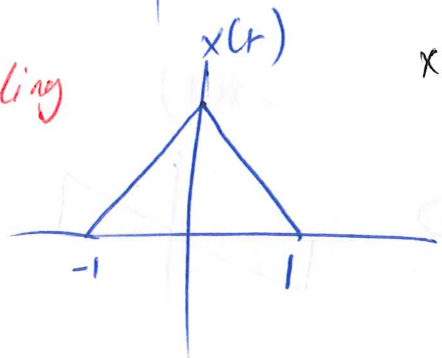
- our concern → Deterministic
  - CT, DT
  - periodic, aperiodic
  - even, odd, neither
  - energy, power, neither
- Random

2- Signals and their transportations

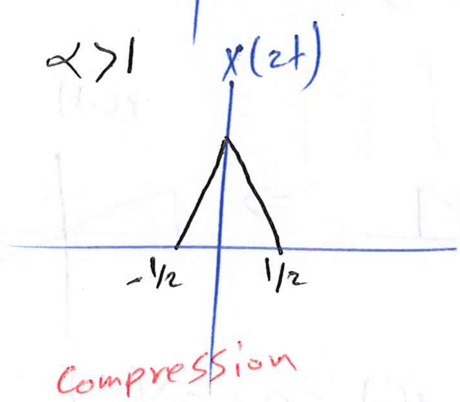
a) folding / Reflection



b) Scaling



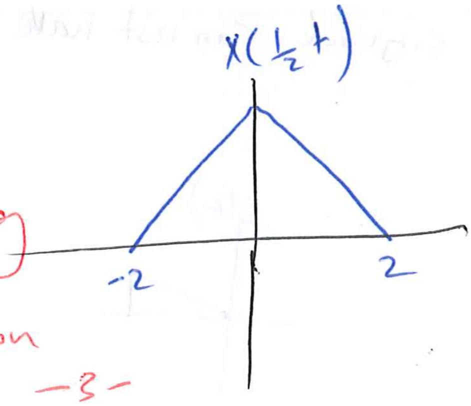
$x(\alpha t)$ $\alpha > 1$



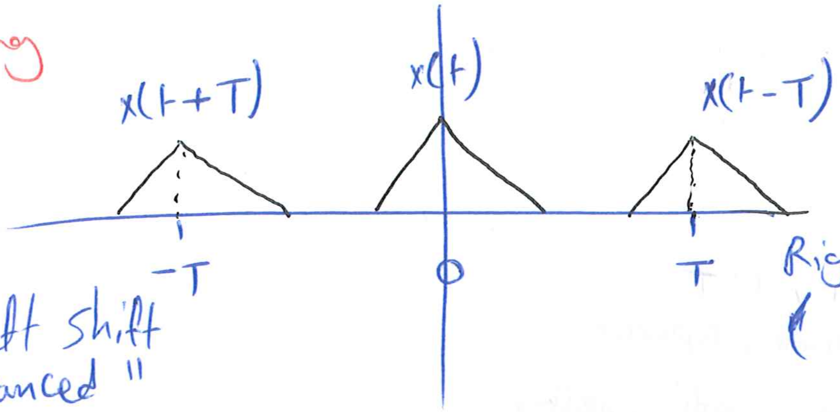
Compression

$\alpha < 1$

Stretching or expansion



c) Shifting



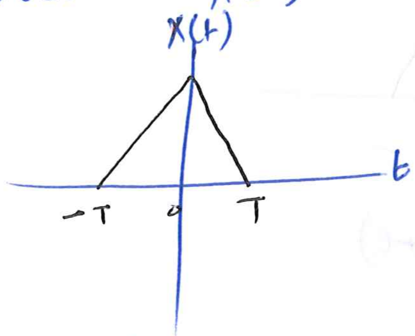
Right shift (delayed)

left shift "advanced" not realistic

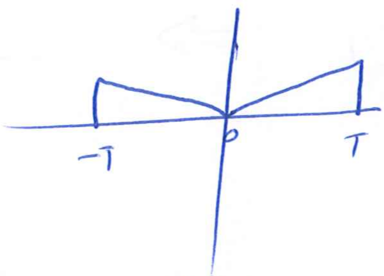
3 - Classifications of signals

a) Even and Odd

- Even $x(t) = x(-t)$

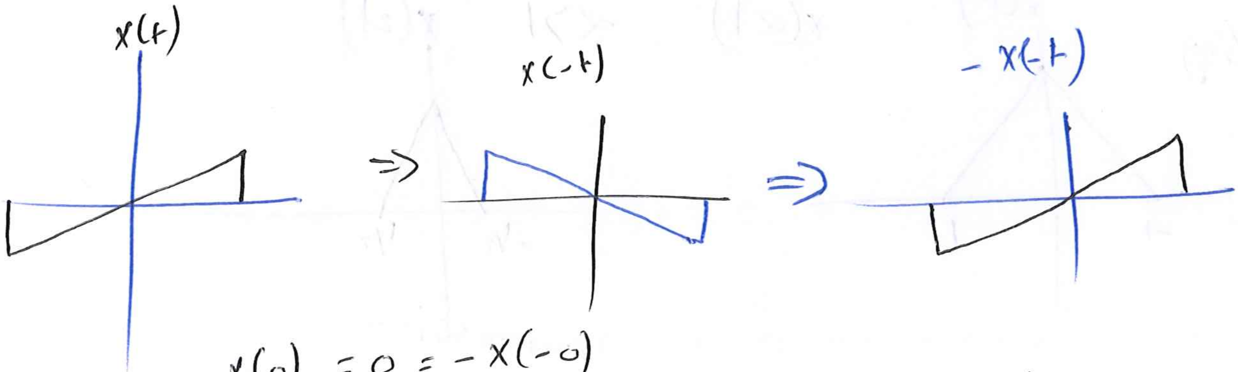


even



even

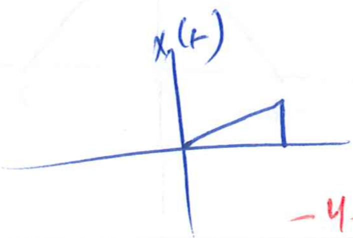
- Odd $x(t) = -x(-t)$



$x(0) = 0 = -x(-0)$

odd signal can not have a non zero at $t=0$

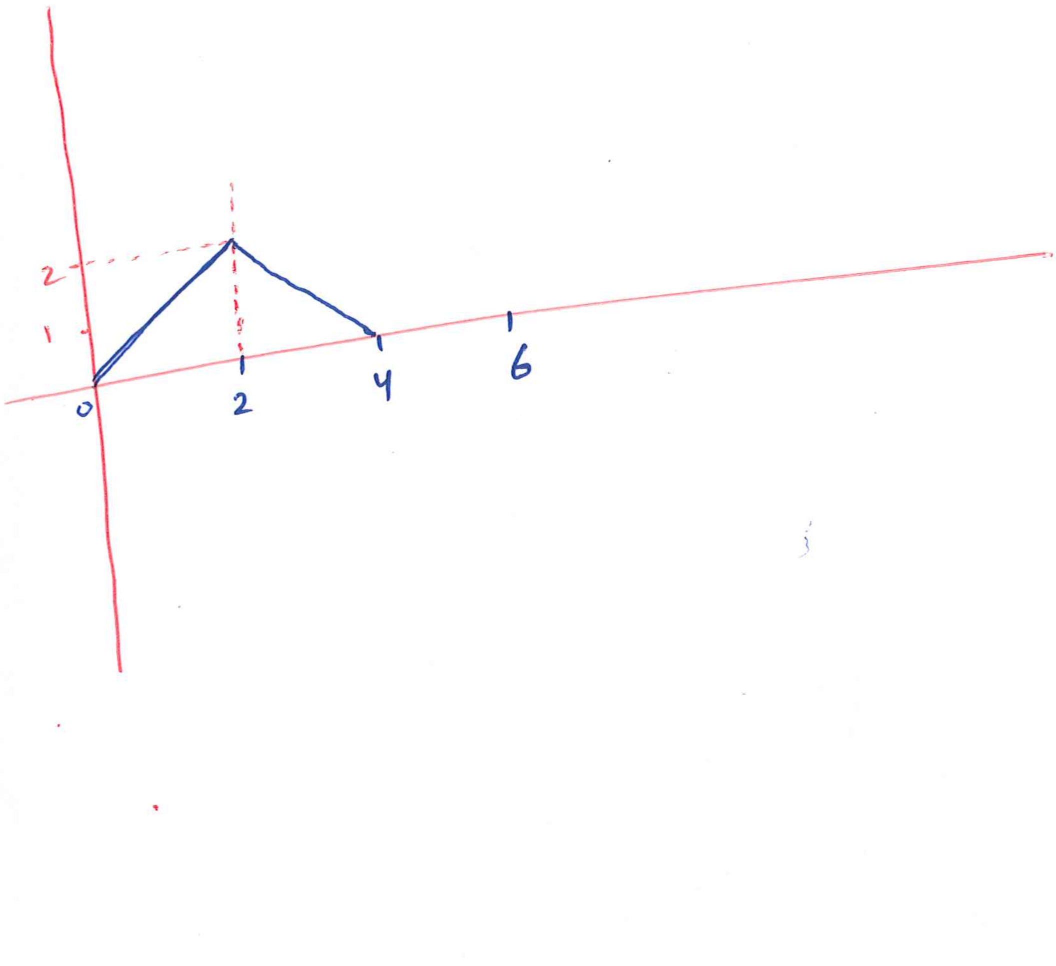
- neither



Given the following signal, is it an energy or power signal? why?

Plot the signal, if the signal is limited, it is an energy signal, if not it is power signal

a) $x(t) = r(t) - r(t-2) + 2u(t-4) - 4u(t-6)$



General Signal $x(t) = Ev[x(t)] + od[x(t)]$

$$x(-t) = Ev[x(-t)] + od[x(-t)]$$

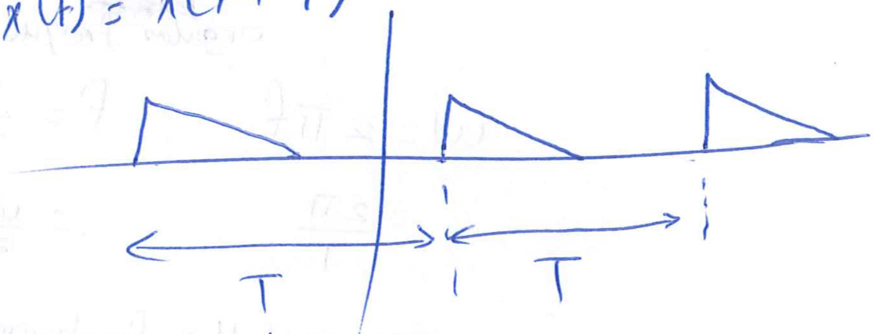
$$= Ev[x(t)] - od[x(t)]$$

$$\therefore \quad Ev[x(t)] = \frac{x(t) + x(-t)}{2}$$

$$od[x(t)] = \frac{x(t) - x(-t)}{2}$$

b) Periodic Signal $x(t) = x(t+T)$

CT:



DT: $x[n] = x[\underbrace{n + N}_{\text{integers}}]$

$x[n] = x[n+N]$

N: integer, called period if the smallest integer

$$F = \frac{1}{N}$$

CT $\quad x(t) = x(t+T)$

$x(t+T) = x(t+2T)$

$= x(t+3T)$

$= x(t+mT)$ integer

smallest m ⇒ Fundamental period (T_0)

$\frac{1}{T_0}$ = number of period of 1 second

$= f_0$ cycles/s (Hz)

Examples :-

For the following signals, check the periodicity of the signals, justify your answers -

1) $x_1(t) = A \sin(2\pi f_0 t + \theta)$

If $x(t + T_0) = x(t)$ $\therefore$ the signal is periodic

for any sinusoidal signal

$$x(t) = A \cos(\underbrace{w_0}_{\text{angular frequency}} t + \theta)$$

~~Period~~ angular frequency

$w = 2\pi f$ $\quad f = \frac{1}{T} \Rightarrow T = \frac{1}{f} = \frac{2\pi}{w}$

$= \frac{2\pi}{T}$ $\quad = \frac{w}{2\pi}$

T_0 called the fundamental period (second)

f_0 " " " frequency (Hertz)

w_0 " " " angular frequency (rad/s)

$$x_1(t + T_0) = A \sin(2\pi f_0 (t + T_0) + \theta)$$

$$= A \sin(2\pi f_0 t + 2\pi f_0 T_0 + \theta)$$

$$= A \sin(2\pi f_0 t + \theta + 2\pi f_0 T_0)$$

$$\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$$

$$x_1(t + T_0) = A[\sin(2\pi f_0 t)\cos(2\pi f_0 T_0) + \cos(2\pi f_0 t)\sin(2\pi f_0 T_0)$$

1 $\quad$ 0

$f_0 = \frac{1}{T_0} \Rightarrow \cos(2\pi \frac{1}{T_0} T_0) = \cos(2\pi) = 1$

$\sin(2\pi \frac{1}{T_0} T_0) = \sin(2\pi) = 0$

$\Rightarrow x_1(t + T_0) = A[\sin(2\pi f_0 t)$ $\quad$ $\therefore x_1(t)$ is periodic signal

2) $x_2(t) = 3\sin(15t)$

$2\pi f_0 = 15$

$f_0 = \frac{15}{2\pi} \Rightarrow T_0 = \frac{2\pi}{15}$

$x_2(t+T_0) = 3\sin(15(t+T_0))$

$= 3\sin(15t + 15T_0)$

$= 3[\sin(15t)\cos(15T_0) + \cos(15t)\sin(15T_0)]$

$= 3[\sin(15t)\underbrace{\cos(2\pi)}_{1} + \cos(15t)\underbrace{\sin(2\pi)}_{0}]$

$= 3\sin(15t) = x_2(t) \therefore$ periodic

3) $x_3(t) = A + B\cos(2\pi f_0 t)$

$x_3(t+T_0) = A + B\cos(2\pi f_0(t+T_0))$

$= A + B\cos(2\pi f_0 t + 2\pi f_0 T_0)$

$\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$

$x_3(t+T_0) = A + B[\cos(2\pi f_0 t)\cos(2\pi f_0 T_0) - \sin(2\pi f_0 t)\sin(2\pi f_0 T_0)]$

$= A + B[\cos(2\pi f_0 t)\underbrace{\cos(2\pi)}_{1} - \sin(2\pi f_0 t)\underbrace{\sin(2\pi)}_{0}]$

$= A + B\cos(2\pi f_0 t) = x_3(t) \therefore$ periodic

The sinusoidal signal is a periodic signal but the summation may be periodic and may be not.

The sum is periodic if and only if the ratio of two sinusoids respective periods can be expressed as a rational number. $\frac{f_1}{f_2} = \frac{n_1}{n_2}$ ← integers

Fundamental frequency and fundamental period

Fundamental frequency $f_0 = GCD(f_1, f_2, ---, f_n)$

" period $T_0 = LCM(T_1, T_2 ----, T_n)$

$$GCD\left(\frac{a}{b}, \frac{c}{d}\right) = \frac{GCD(a,c)}{LCM(b,d)}$$

$$GCD\left(\frac{12}{5}, \frac{20}{7}\right) = \frac{GCD(12,20)}{LCM(5,7)} = \frac{4}{35}$$

$$LCM\left(\frac{a}{b}, \frac{c}{d}\right) = \frac{LCM(a,c)}{GCD(b,d)}$$

$$LCM\left(\frac{4}{5}, \frac{3}{7}\right) = \frac{LCM(4,3)}{GCD(5,7)} = \frac{12}{1} = 12$$

Examples:-

Find the fundamental frequency of the following signals:-

1) $x_1(t) = \cos\left(\frac{10\pi}{3} t\right) + \sin\left(\frac{5\pi}{4} t\right)$

$w_1 = \frac{10}{3}\pi$ rad/s $\qquad w_2 = \frac{5}{4}\pi$ rad/s

$f_1 = \frac{5}{3}$ Hz $\qquad f_2 = \frac{5}{8}$ Hz

$T_1 = \frac{3}{5}$ s $\qquad T_1 = \frac{8}{5}$ s

$$f_0 = GCD\left(\frac{5}{3}, \frac{5}{8}\right) = \frac{GCD(5,5)}{LCM(3,8)} = \frac{5}{24}$$

$T_0 = \frac{24}{5}$

$w_0 = 2\pi f_0 = 2\pi\left(\frac{5}{24}\right) = 2\left(\frac{5\pi}{24}\right) = \frac{5\pi}{12}$

$$\frac{f_1}{f_2} = \frac{\frac{8}{3}}{\frac{5}{8}} = \frac{8}{3}$$

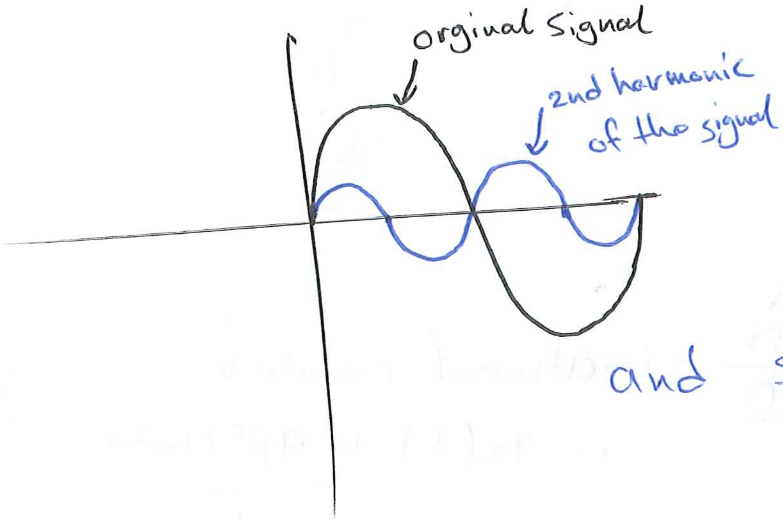
rational number

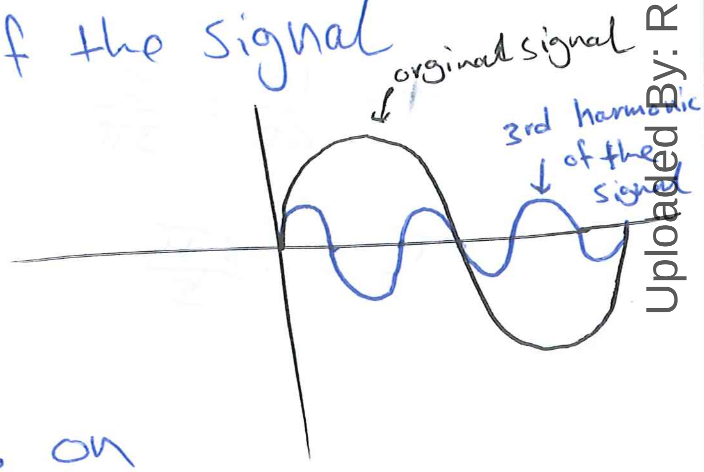
$\therefore x_1(t)$ periodic

we can rewrite $x_1(t)$ as a function of the f_0 as

$$x_1(t) = \cos\left(8 \frac{5\pi}{12} t\right) + \sin\left(3 \frac{5\pi}{12} t\right)$$

3rd and 8th harmonic of the signal





and so on

2) $x_2(t) = \sin\left(\frac{5\pi}{6} t\right) + \cos\left(\frac{3\pi}{4} t\right) + \sin\left(\frac{\pi}{3} t\right)$

$\omega_1 = \frac{5\pi}{6}$ rad/s $\omega_2 = \frac{3\pi}{4}$ rad/s $\omega_3 = \frac{\pi}{3}$ rad/s

$f_1 = \frac{5}{12}$ Hz $f_2 = \frac{3}{8}$ Hz $f_3 = \frac{1}{6}$ Hz

$T_1 = \frac{12}{5}$ s $T_2 = \frac{8}{3}$ s $T_3 = 6$ s

$$f_0 = GCD\left(\frac{5}{12}, \frac{3}{8}, \frac{1}{6}\right) = \frac{GCD(5,3,1)}{LCM(12,8,6)} = \frac{1}{24} \text{ Hz}$$

$T_0 = 24$ s

$\omega_0 = 2\pi f_0 = 2\pi \frac{1}{24} = \frac{\pi}{12}$ rad/s

$$x(t) = \sin\left(10 \frac{\pi}{12} t\right) + \cos\left(9 \frac{\pi}{12} t\right) + \sin\left(4 \frac{\pi}{12} t\right)$$

$$= \sin(10\omega_0 t) + \cos(9\omega_0 t) + \sin(4\omega_0 t)$$

4th, 9th and 10th harmonics

$\frac{f_1}{f_2}$ & $\frac{f_1}{f_3}$ are rational numbers $\therefore x_2(t)$ periodic

3- $x_3(t) = \cos\left(\frac{10}{3}t\right) + \sin\left(\frac{5\pi}{4}t\right)$

$w_1 = \frac{10}{3} = 2\pi f_1$

$w_2 = \frac{5\pi}{4} = 2\pi f_2$

$f_1 = \frac{10}{6\pi} = \frac{5}{3\pi}$

$f_2 = \frac{5}{8}$

$T_0 = \frac{3\pi}{5}$

$T_0 = \frac{8}{5}$

$\frac{f_1}{f}$ irrational number

$\therefore x_3(t)$ is $\underbrace{\text{aperiodic}}_{\text{not periodic}}$

$f_0 = GCD\left(\frac{5}{3\pi}, \frac{5}{8}\right)$

does not exist

π is an ~~irrati~~ irrational number, Can not be expressed as a ratio of two integers

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* Sum of two ~~or more~~ signals with periods T_1 and T_2 is periodic if and only if the ratio $\frac{T_1}{T_2}$ is a rational number

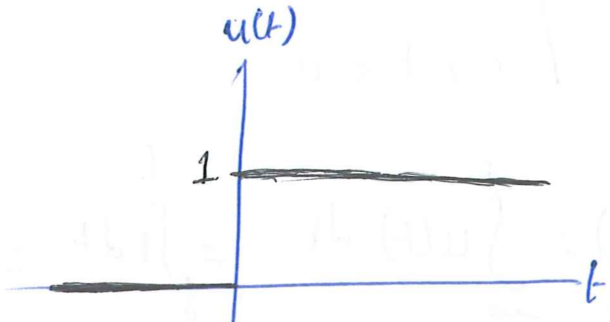
$\frac{T_1}{T_2} = \frac{n_1}{n_2}$ n_1, and n_2 are integers

Singularity functions

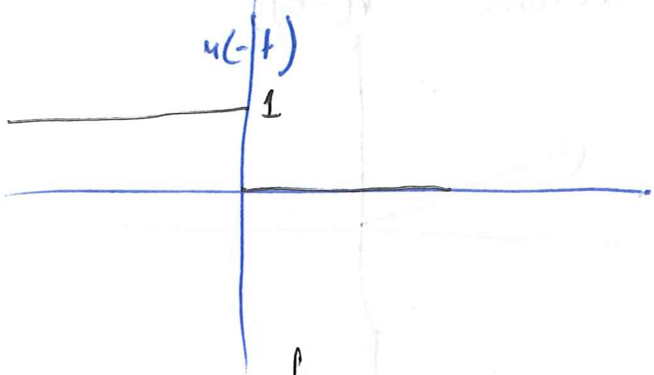
Singularity functions are discontinuous functions or their derivatives are discontinuous.

1) Unit step function

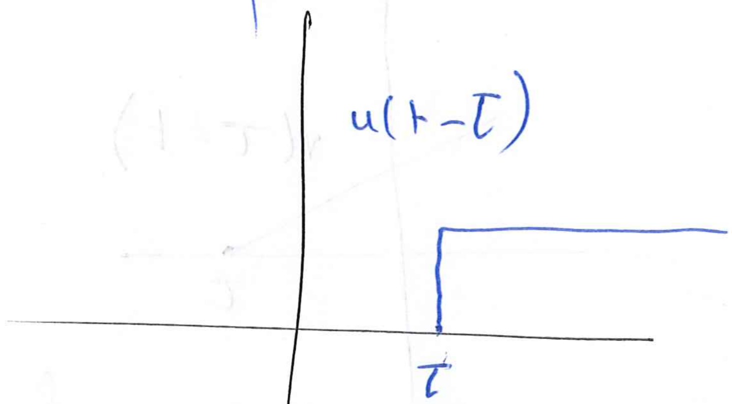
$$u(t) = \begin{cases} 0, & t<0 \\ 1, & t>0 \end{cases}$$

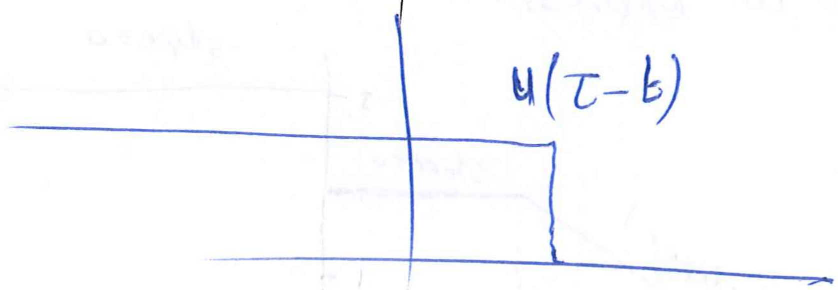


Reflection



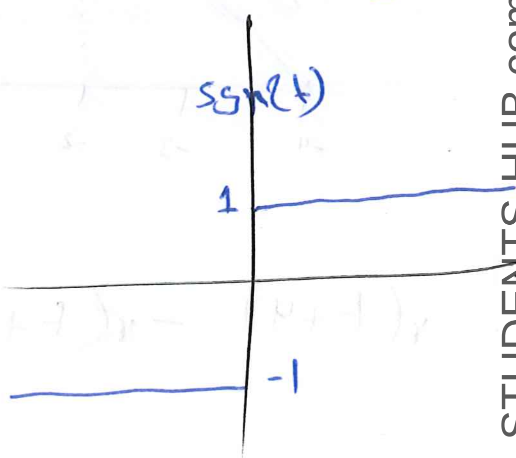
Shifting





Signum function

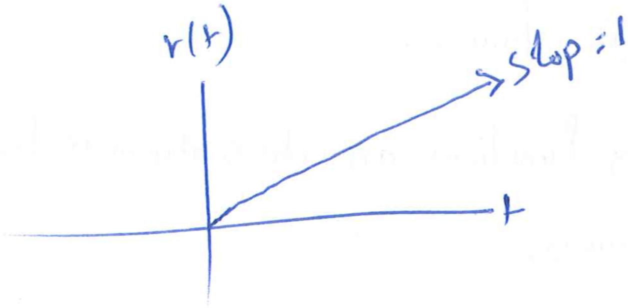
$$sgn(t) = \begin{cases} 1, & t>0 \\ 0, & t=0 \\ -1, & t<0 \end{cases}$$



$$sgn(t) = -1 + 2u(t)$$
$$= u(t) - u(-t)$$

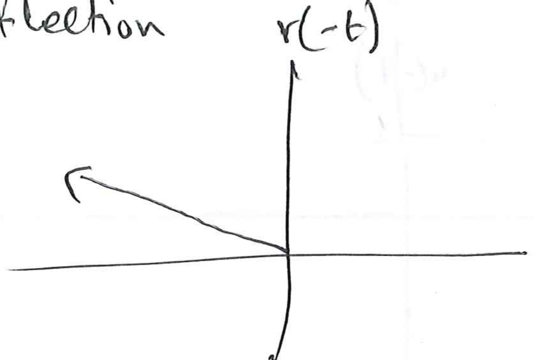
2- Ramp function

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

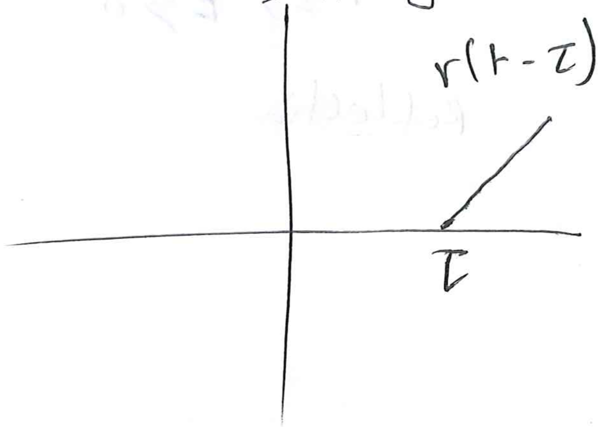


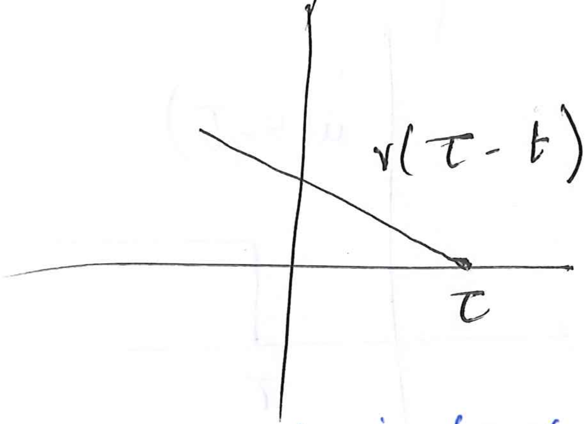
$$r(t) = \int_{-\infty}^{t} u(t)\, dt = \int_{0}^{t} 1\, dt = t\,u(t)$$

reflection

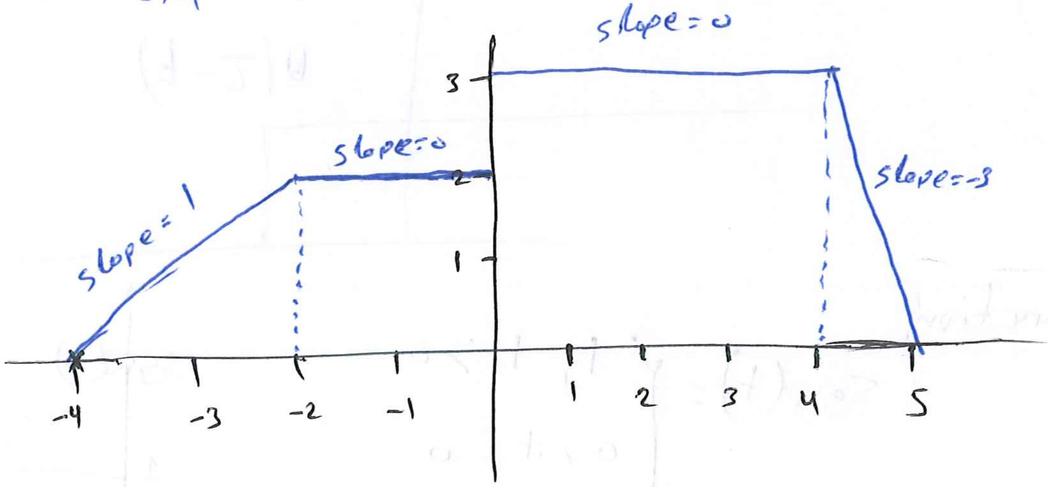


shifting



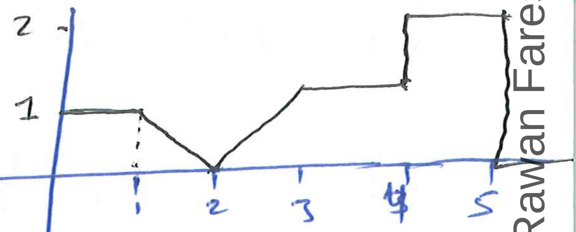


write an expression in terms of singularity functions



$$r(t+4) - r(t+2) + u(t) - 3r(t-4) + 3r(t-5)$$

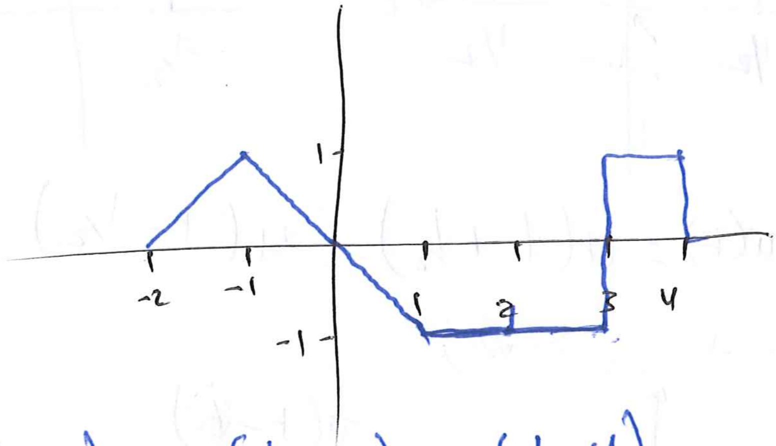
write the expression in terms of singularity functions



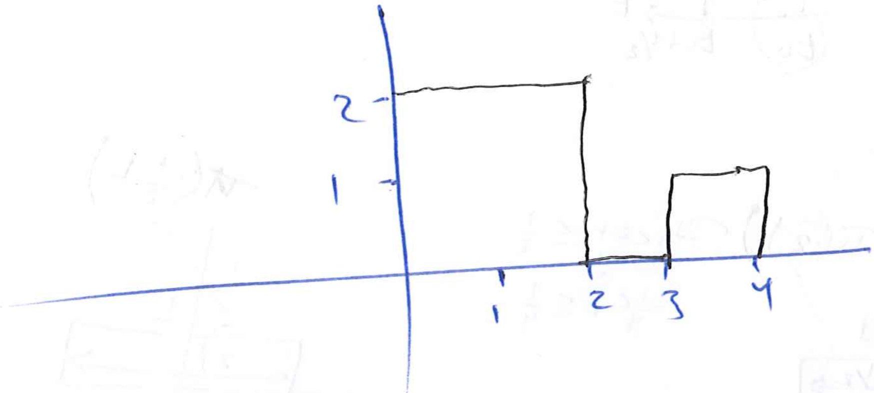
$$u(t) - r(t-1) + 2r(t-2) - r(t-3) + u(t-4) - 2u(t-5)$$

- Sketch the following Signal

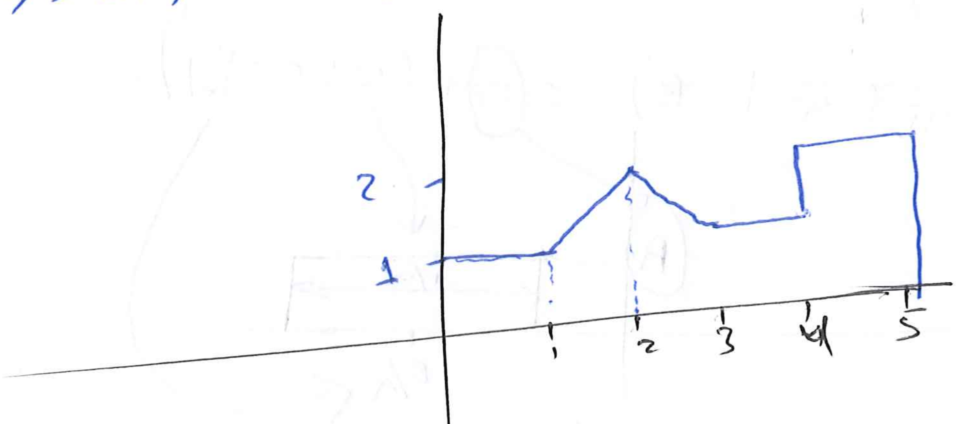
$$x_1(t) = r(t+2) - 2r(t+1) + r(t-1) + 2u(t-3) - u(t-4)$$



$$x_2(t) = 2u(t) - 2u(t-2) + u(t-3) - u(t-4)$$

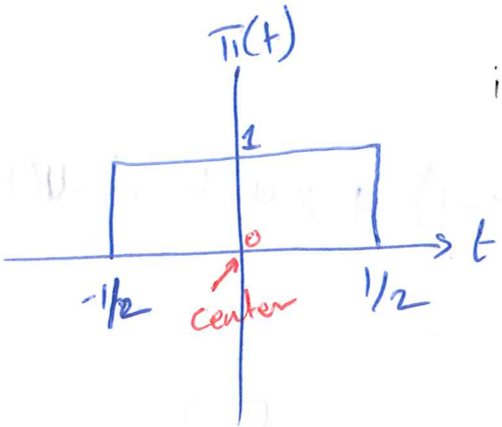


$$x_3(t) = u(t) + r(t-1) - 2r(t-2) + r(t-3) + u(t-4) - 2u(t-5)$$

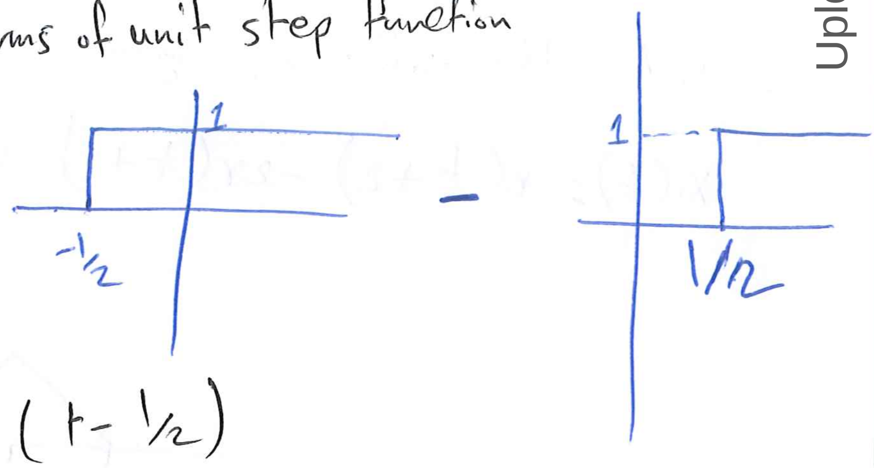


3- Unit puls function

$$\Pi(t) = \begin{cases} 1, & -\frac{1}{2} < t \le \frac{1}{2} \\ 0, & o.w \end{cases}$$

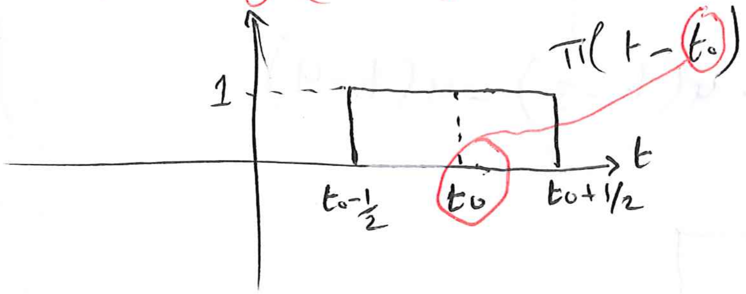


in terms of unit step function



$$\Pi(t) = u(t + 1/2) - u(t - 1/2)$$

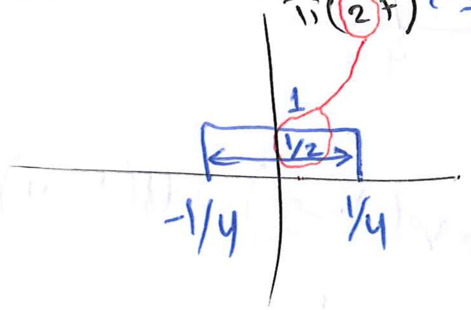
- shifting (time)



- time scaling

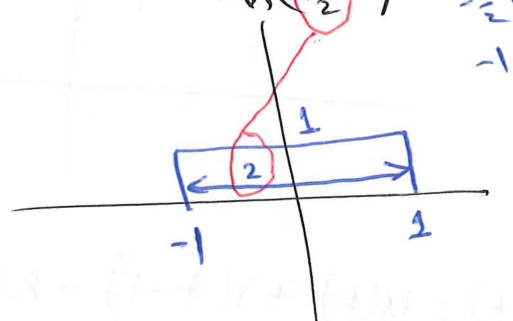
$\Pi(2t)$ $\quad -\frac{1}{2} < 2t \le \frac{1}{2}$

$-\frac{1}{4} < t \le \frac{1}{4}$

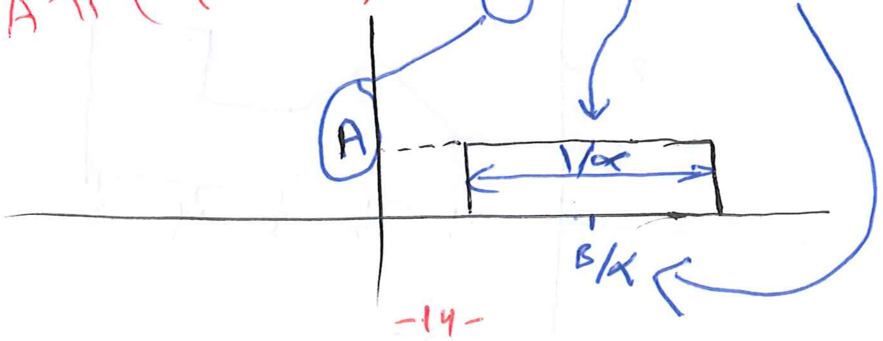


$\Pi(\frac{1}{2}t)$ $\quad -\frac{1}{2} \le \frac{1}{2}t \le \frac{1}{2}$

$-1 < t \le 1$

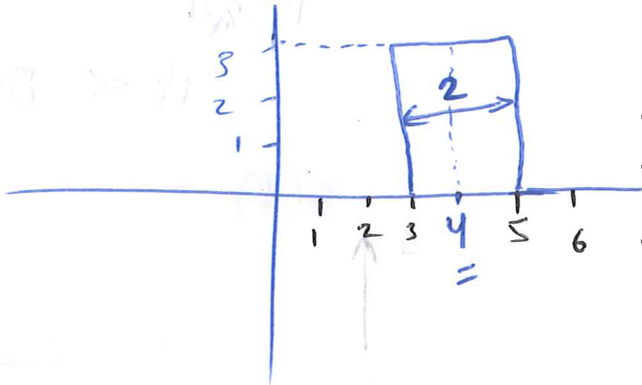


$$x(t) = A\,\Pi(\alpha t - B) = A\,\Pi\left(\alpha\left(t - \frac{B}{\alpha}\right)\right)$$

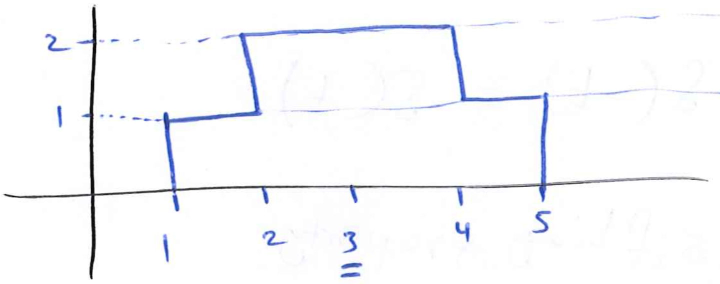


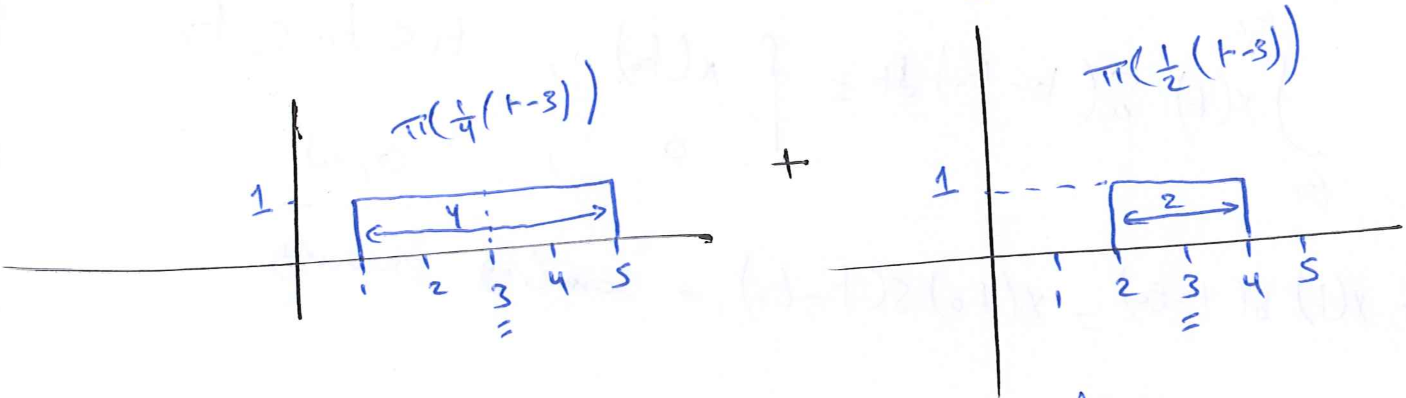
Example: sketch $x(t) = 3\,\Pi(\frac{1}{2}t - 2)$

$= 3\,\Pi(\frac{1}{2}(t-4))$



Example: express $x(t)$ in terms of pulse function





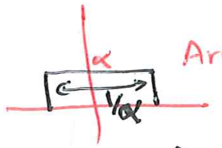
$\Pi(\frac{1}{4}(t-3))$ + $\Pi(\frac{1}{2}(t-3))$

$x(t) = \Pi(\frac{1}{4}(t-3)) + \Pi(\frac{1}{2}(t-3))$

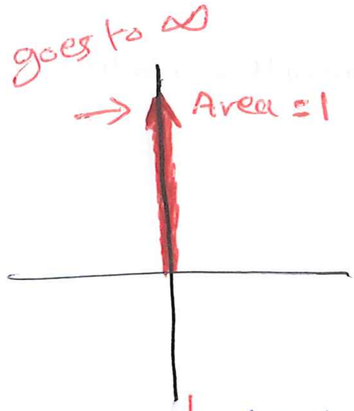
$= \Pi(\frac{t}{4} - \frac{3}{4}) + \Pi(\frac{t-3}{2})$

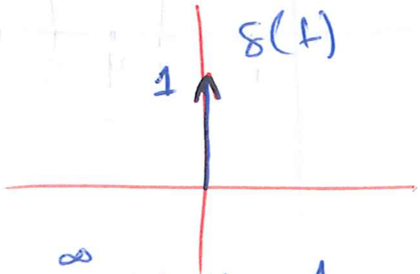
$\Pi(\frac{t-3}{4})$

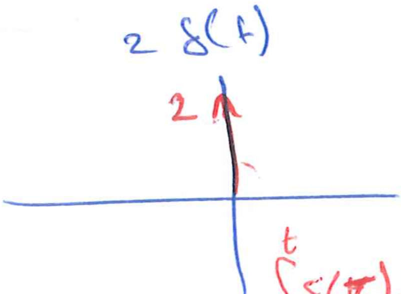
4- Unit impuls function

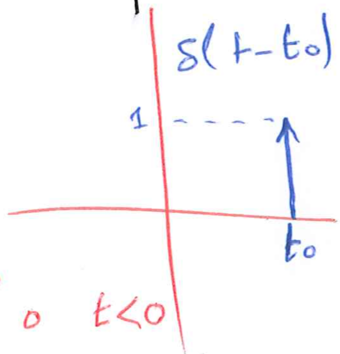


If α goes to zero ⟹









1- $\int_{-\infty}^{\infty} \delta(t)\, dt = 1$

$$\int_{-\infty}^{t} \delta(\tau)\, d\tau = \begin{cases} 0 & t<0 \\ 1 & t>0 \end{cases} = u(t)$$

2- $\delta(at) = \delta(t) / |a|$

3- $\delta(-t) = \delta(t)$

$$\frac{du(t)}{dt} = \delta(t)$$

4- sifting property

$$\int_{t_1}^{t_2} x(t)\, \delta(t-t_0)\, dt = \begin{cases} x(t_0), & t_1 < t_0 < t_2 \\ 0, & 0,\omega \end{cases}$$

5- $x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$ - sampling property

6- Derivative property

$$\int_{-\infty}^{\infty} x(t) \frac{d^n}{dt^n} \delta(t-\tau)\, dt = (-1)^n \frac{d^n}{dt^n} x(t) \Big|_{t=\tau}$$

$$x(t)\Big|_{t \to \infty} = 0 \text{ or finite}$$

Example: Evaluate the following integrals

a) $\int_5^{10} \cos(2\pi t)\, \delta(t-2)\, dt$ $\therefore = 0$

$\underline{\underline{5}}$ ~~defined~~ required from 5 - 10

$\underline{\underline{2}}$ defined only at $t=2$

b) $\int_0^5 \cos(2\pi t)\, \delta(t-2)\, dt$ $\therefore = \cos(2\pi(2)) = \cos(4\pi) = 1$

$\underline{\underline{2}}$ at $t=2$

from 0 - 5

c) $\int_{-\infty}^{\infty} [e^{-3t} + \cos(2\pi t)]\, \dot{\delta}(t)\, dt$

$= (-1)^1 \frac{d^1}{dt}[e^{-3t} + \cos(2\pi t)]$

$= (-1)[-3e^{-3t} - 2\pi \sin(2\pi t)]\Big|_{t=0}$

$= (-1)[-3e^{-3(0)} - 2\pi \sin(2\pi(0))]$

$= (-1)(-3) = 3$

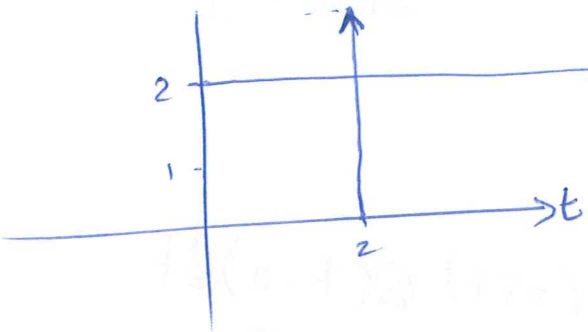
d) $\int_{-\infty}^{\infty} e^{3t}\, \ddot{\delta}(t-2)\, dt$

$= (-1)^2 \frac{d^2}{dt^2}[e^{3t}] = (-1)^2 (3)(3e^{3t})\Big|_{t=2}$

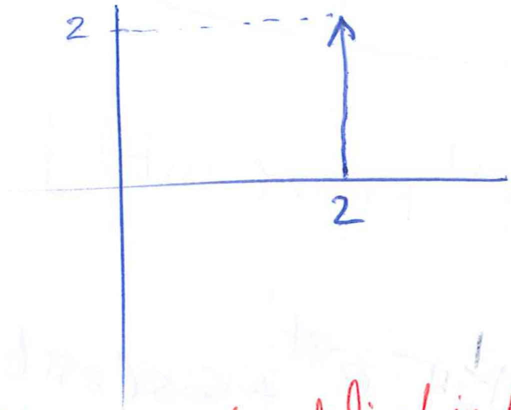
$= 9e^6$

Example: sketch the following signals:

a) $x_1(t) = 2u(t) + \delta(t-2)$



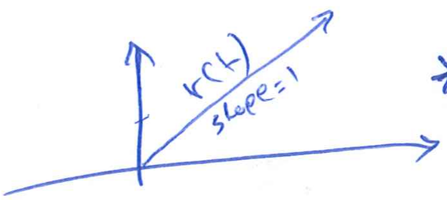
b) $x_2(t) = 2u(t)\,\delta(t-2)$



Example: plot accurately the following signals defined in terms of singularity functions:

a) $x_1(t) = \sum_{n=0}^{\infty} x_a(t-2n)$ [plot $0 \le t \le 6$]

where $x_a(t) = r(t)\,u(2-\cancel{t})$



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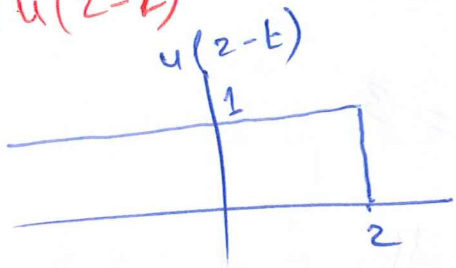
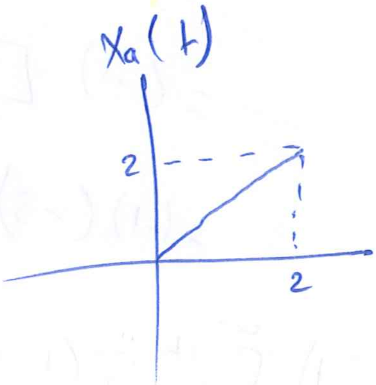
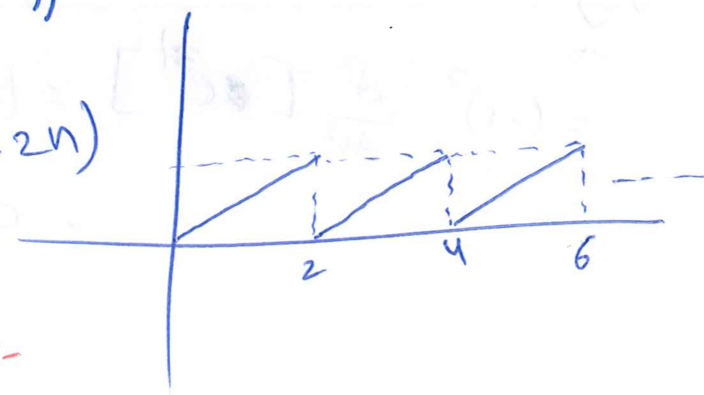




$$x_1(t) = \sum_{n=0}^{\infty} r(t-2n)\,u(2-(t-2n))$$

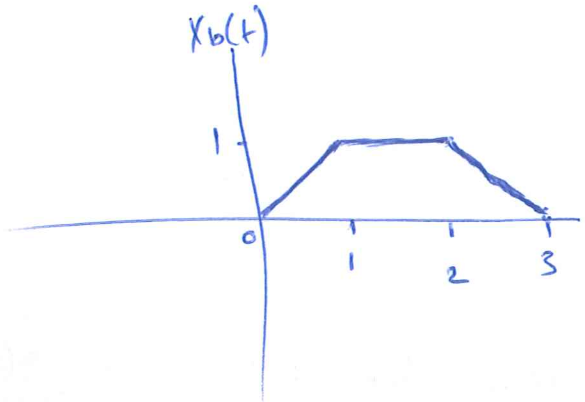
$$= \sum_{n=0}^{\infty} r(t-2n)\,u(2-t+2n)$$

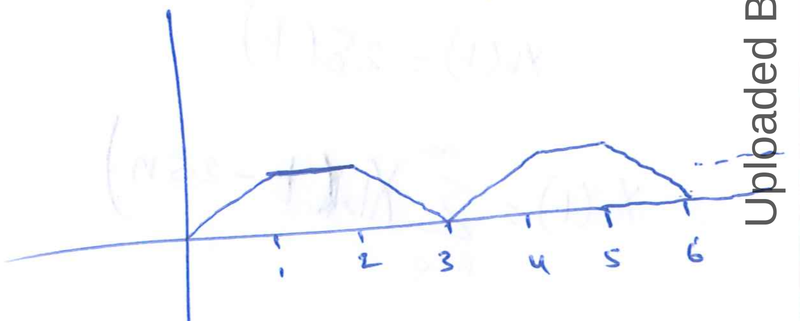


b) $x_2(t) = \sum_{n=0}^{\infty} x_b(t-3n)$ [Plot for $0 \le t \le 6$]

$x_b(t) = r(t) - r(t-1) - r(t-2) + r(t-3)$

$x_2(t) = \sum_{n=0}^{\infty} r(t-3n) - r(t-3n-1) - r(t-3n-2)$
$+ r(t-3n-3)$



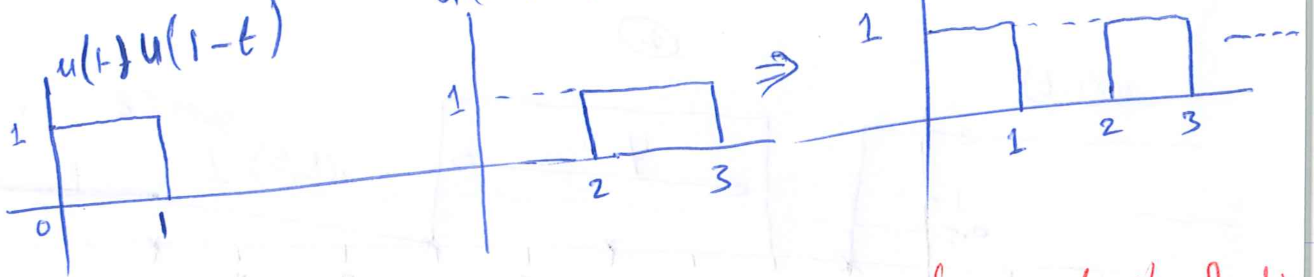


c) $x_3(t) = \sum_{n=0}^{\infty} u(t-2n)\, u(1+2n-t)$

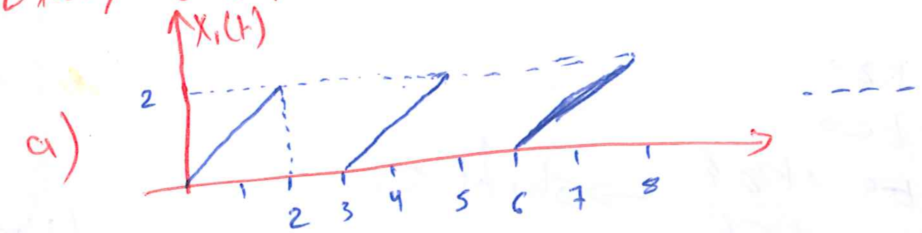
$x_3(t)$

$u(t-2)\,u(3-t)$

$u(t)\,u(1-t)$



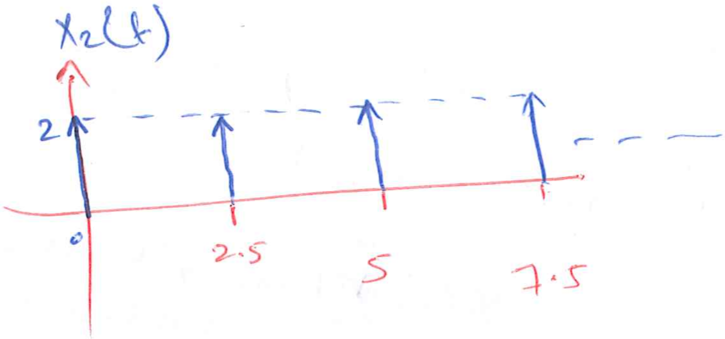
Example :- Express the Signal shown in terms of singularity function



$x_a(t) = r(t)\, u(2-t)$

$x_1(t) = \sum_{n=0}^{\infty} x_a(t-3n)$

b)

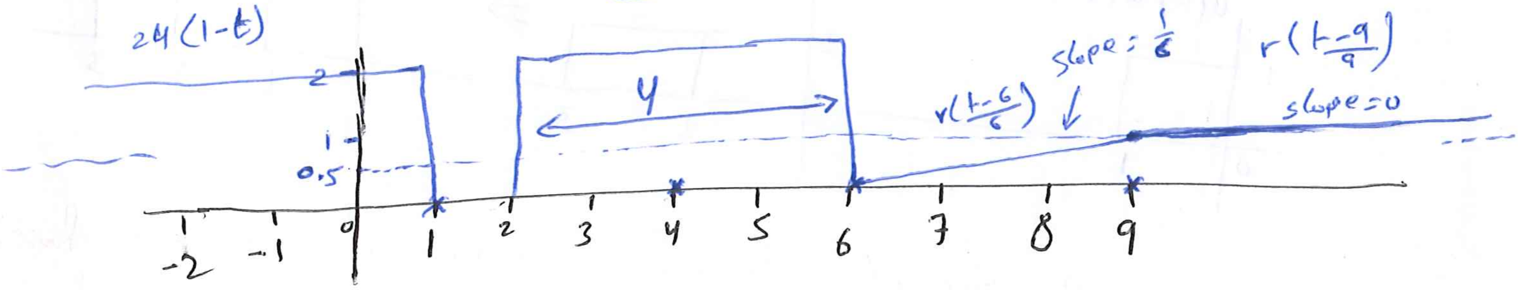


$$x_b(t) = 2\delta(t)$$

$$x_2(t) = \sum_{n=0}^{\infty} x_b(t - 2.5n)$$

Example: plot the following signal using the elementary signals

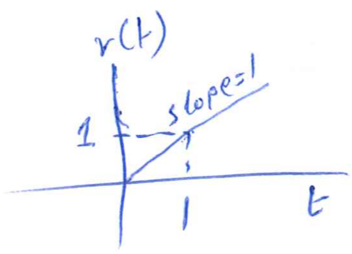
$$x(t) = \underbrace{2\Pi\left(\frac{t-4}{4}\right)}_{②} + r\left(\frac{t-6}{6}\right) - r\left(\frac{t-9}{6}\right) + \underbrace{2u(1-t)}_{①}$$

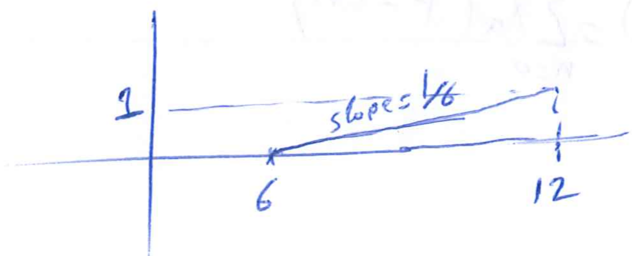


$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$r(t-6) = \begin{cases} t-6, & t \geq 6 \\ 0, & t < 6 \end{cases} \rightarrow \text{shifting}$$

$$r\left(\frac{t-6}{6}\right) = r\left(\frac{1}{6}(t-6)\right) = \begin{cases} \frac{t-6}{6} & t \geq 6 \\ 0 & t < 6 \end{cases} \Bigg\} \text{ shifting + expansion}$$



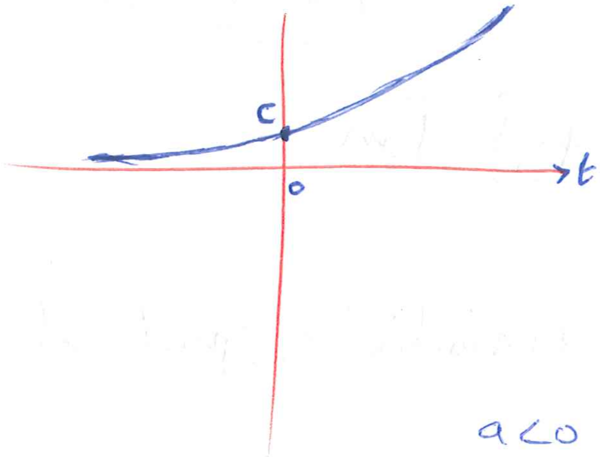


- phasor Signals and spectra :-

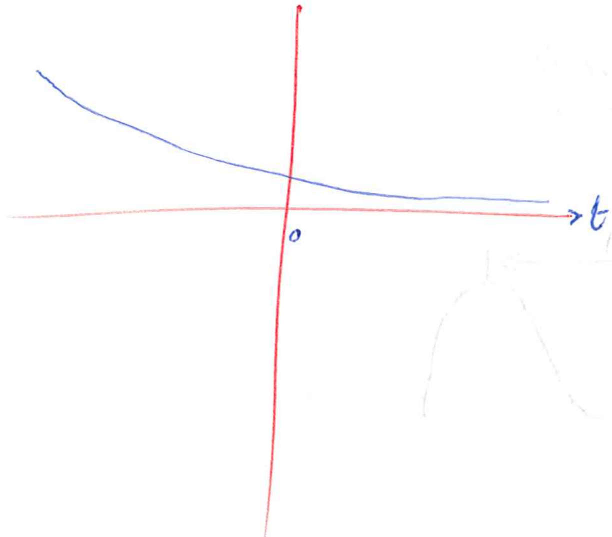
* Exponential signal

$$C e^{at}$$

1. C: real a: real & +ve ($a>0$)



2. C: real a: real & -ve ($a<0$)



3. C real a is imaginary

$a = j\omega_0 t$

$e^{j\omega_0 t} \overset{??}{=} e^{j\omega_0 (t+T)}$ is it periodic

$= e^{j\omega_0 t}\, e^{j\omega_0 T} \rightarrow$ if $= 1$ if and only if

$\omega_0 T = 2\pi m$

$T = \frac{2\pi}{\omega_0}$

special case of the complex exponential signal is sinusoidal signal (cos and sin)

$$\cos w_o t = Re\left[e^{jw_o t}\right]$$

$$\sin w_o t = Im\left[e^{jw_o t}\right]$$

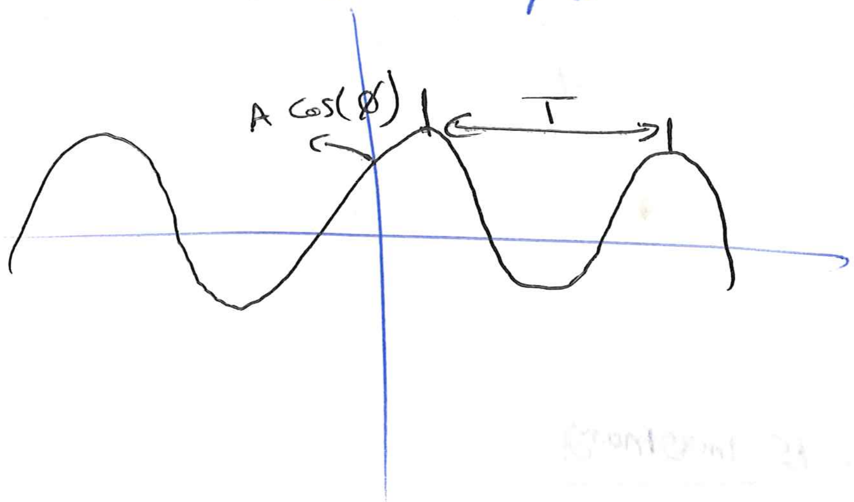
<u>Euler</u> identity

$$e^{jw_o t} = Re + j\,Im$$

both $\left.\begin{matrix}\cos w_o t \\ \sin w_o\end{matrix}\right\}$ periodic signal of $T = \frac{2\pi}{w_o}$

general sinusoidal signal

$$A \cos(w_o t + \phi)$$



$x_1(t) = e^{jw_o t} \rightarrow f_1 = \frac{w_o}{2\pi}$

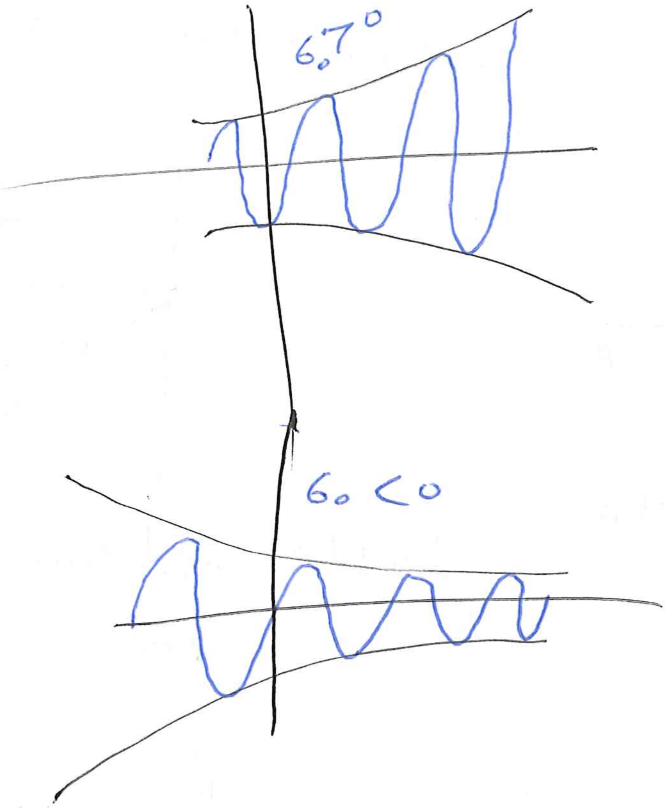
$x_2(t) = e^{jmw_o t} \rightarrow f = m\frac{w_o}{2\pi} = mf_1 \rightarrow$ the mth harmonic

$\boxed{4}$ c real $\quad a$ is complex

$$a = (\sigma_0 + j\omega_0)$$

$$c\, e^{(\sigma_0 + j\omega_0)t} = c\, e^{\sigma_0 t} e^{j\omega_0 t}$$

$$Re\left[c\, e^{\sigma_0 t} e^{j\omega_0 t}\right] = c\, e^{\sigma_0 t} \cos \omega_0 t$$



$\boxed{5}$ c Complex $\quad a$ is complex

$$a = \sigma_0 + j\omega_0$$

$$c = c_1 + j c_2$$

$$= |c| e^{j\theta} \qquad \theta = \tan^{-1} \frac{c_2}{c_1}$$

$$x(t) = c\, e^{at} = |c|\, e^{\sigma_0 t} e^{j(\omega_0 t + \theta)}$$

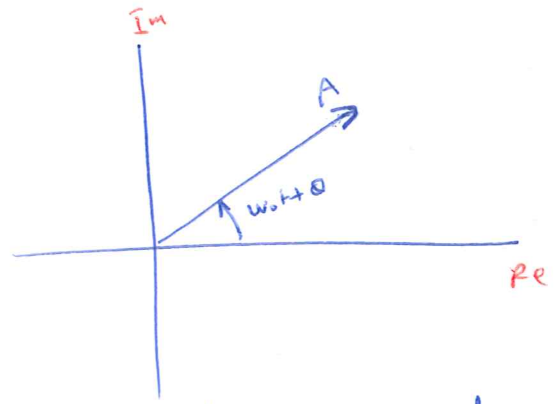
$$x(t) = A e^{j(\omega_0 t + \theta)}$$

$$= A\cos(\omega_0 t + \theta) + jA\sin(\omega_0 t + \theta)$$

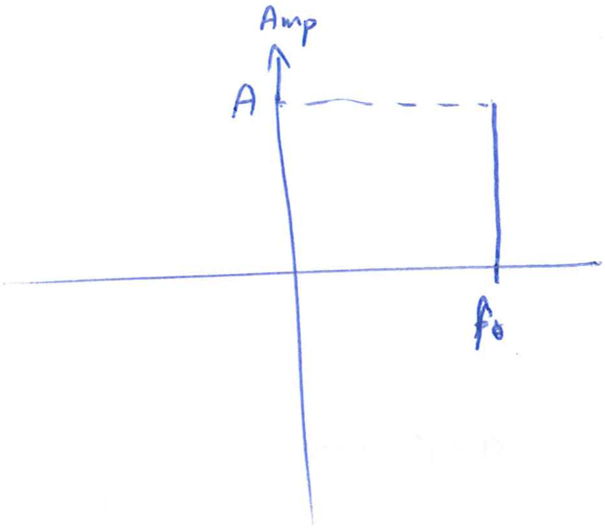
$$x^*(t) = A\cos(\omega_0 t + \theta) - jA\sin(\omega_0 t + \theta)$$

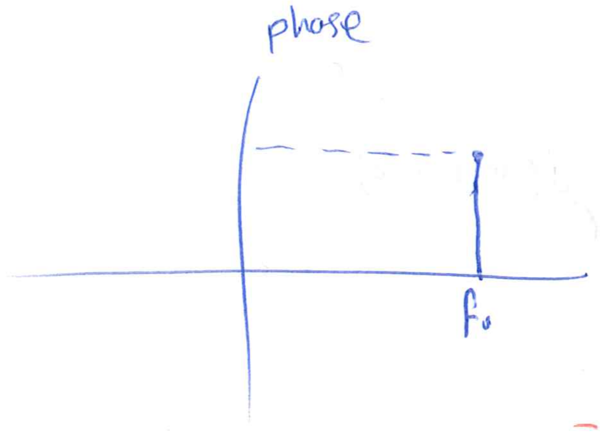
$$Re[x(t)] = A\cos(\omega_0 t + \theta) = \frac{x(t) + x^*(t)}{2}$$

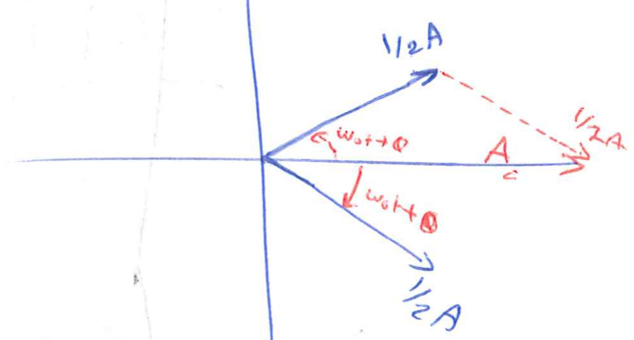
$$= \tfrac{1}{2} A\cos(\omega_0 t + \theta) + \tfrac{1}{2} A\cos(\omega_0 t + \theta)$$



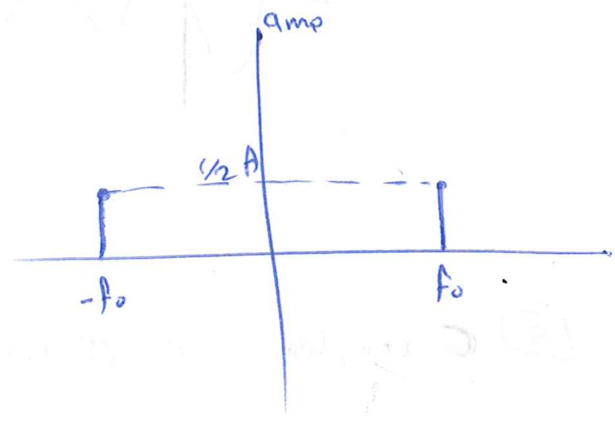
Single - sided line spectra

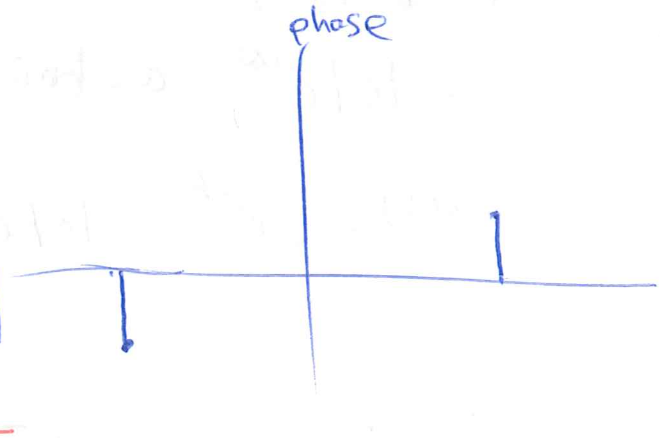






double sided line spectra





$$x(t) = e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\cos(-\theta) = \cos(\theta)$$

$$\sin(-\theta) = -\sin(\theta)$$

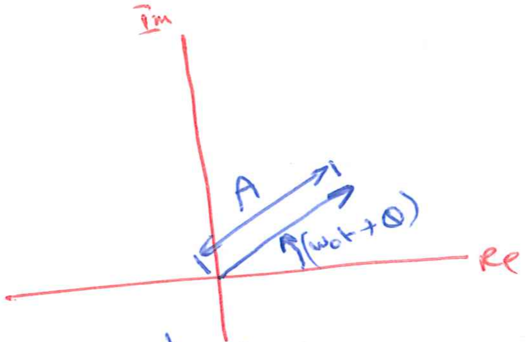
$$x(t) = e^{j(\omega t+\theta)} = A\cos(\omega t+\theta) + j\sin(\omega t+\theta)$$

$$x^*(t) = e^{-j(\omega t+\theta)} = A\cos(\omega t+\theta) - j\sin(\omega t+\theta)$$

$$\mathrm{Re}[x(t)] = A\cos(\omega t+\theta) = \frac{x(t)+x^*(t)}{2}$$

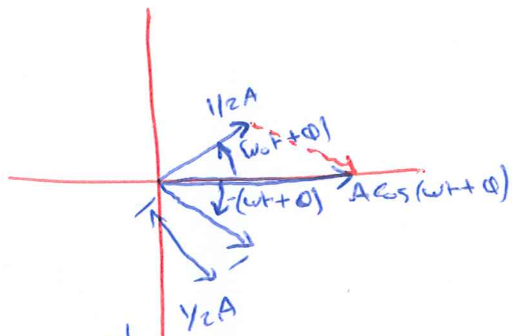
$$= \frac{1}{2}e^{j(\omega t+\theta)} + \frac{1}{2}e^{-j(\omega t+\theta)}$$

Single sided



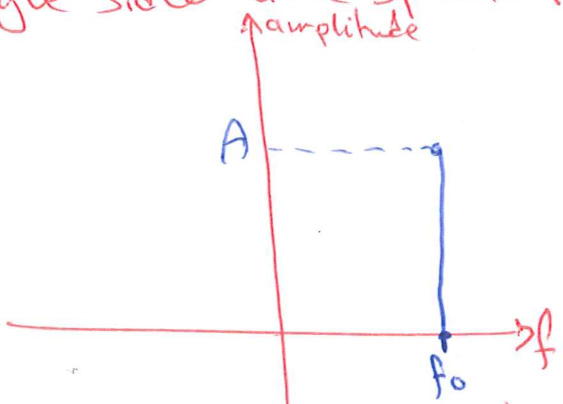
phasor

double sided



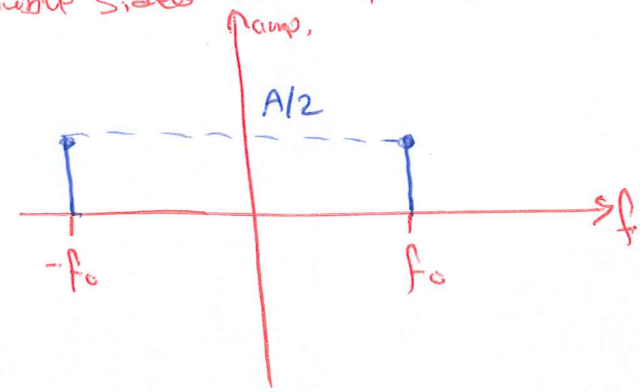
Phasor

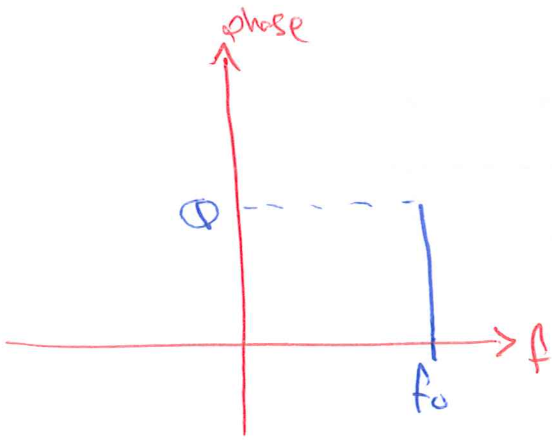
Single sided line spectra



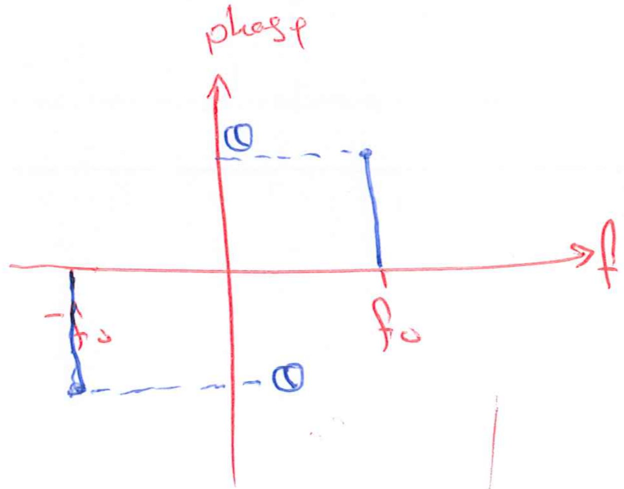
spectra - amplitude

double sided line spectra





Spectra-phase



Examples: $x(t) = 4\cos(20\pi t + \pi/4) + 3\cos(60\pi t - \pi/6)$

$+ \sin(80\pi t + \pi/6)$

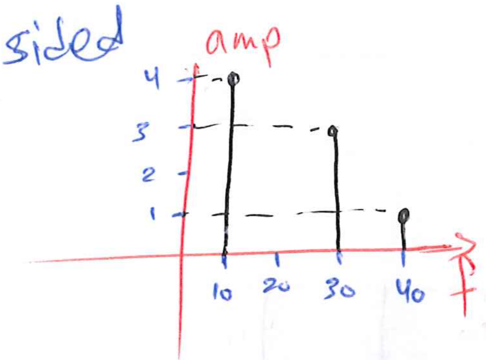
- sketch its single sided amplitude and phase spectra
- sketch its double sided amplitude and phase spectra

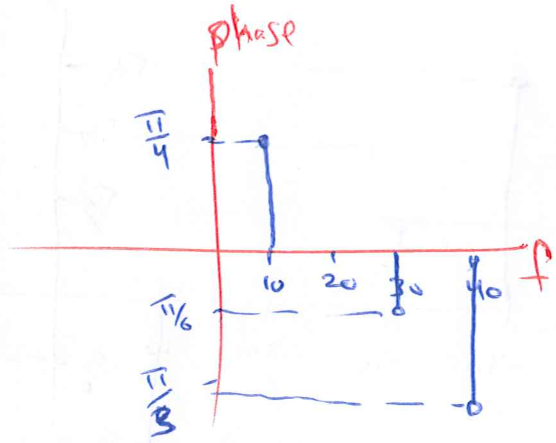
$\sin(\theta) = \cos(\theta - \pi/2)$

$$\sin(80\pi t + \pi/6) = \cos(80\pi t + \pi/6 - \pi/2)$$
$$= \cos\left(80\pi t + \frac{\pi}{6} - \frac{3\pi}{6}\right)$$
$$= \cos\left(80\pi t - \frac{2\pi}{6}\right)$$
$$= \cos(80\pi t - \pi/3)$$

$$x(t) = 4\cos(2(10)\pi t + \pi/4) + 3\cos(2(30)\pi t - \pi/6) + \cos(2(40)\pi t - \pi/3)$$

single sided



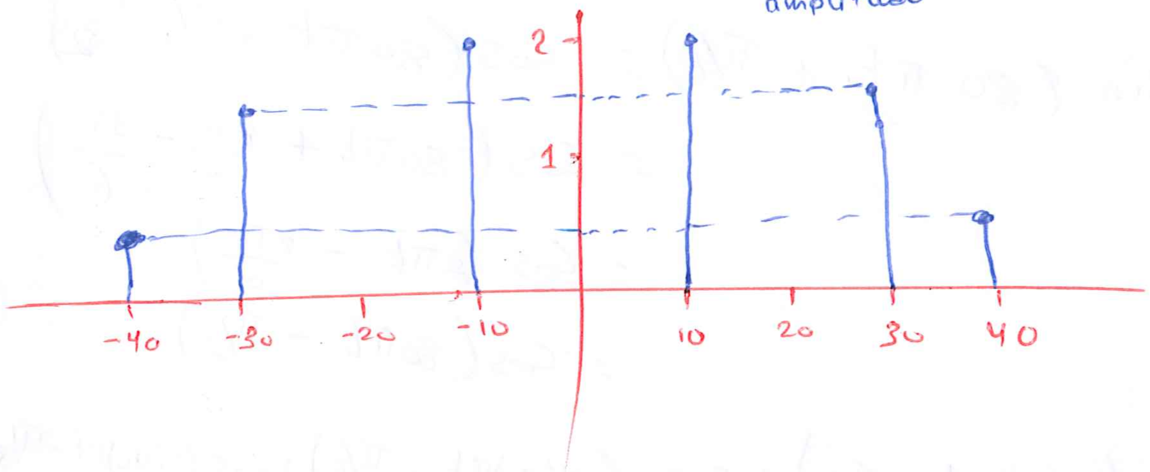


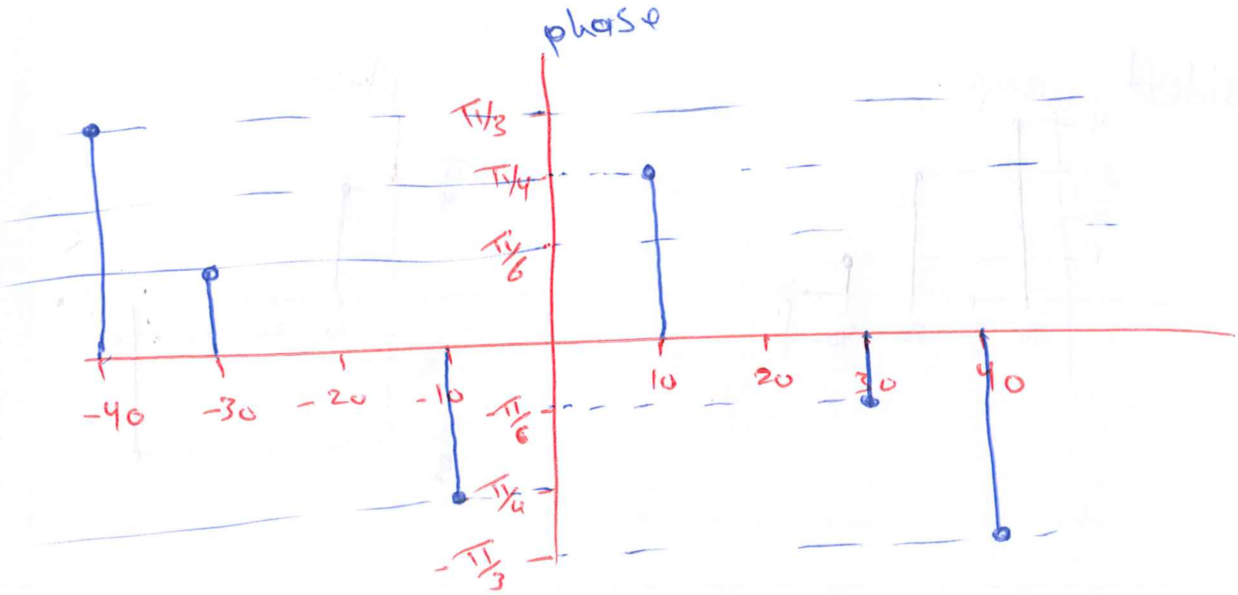
The double sided amplitude and phase spectra

$$x(t) = 4\cos\left(20\pi t + \frac{\pi}{4}\right) + 3\cos\left(60\pi t - \frac{\pi}{6}\right) + \cos\left(80\pi t - \frac{\pi}{3}\right)$$

$$= \frac{4}{2}\left[e^{j(20\pi t + \frac{\pi}{4})} + e^{-j(20\pi t + \pi/4)}\right] + \frac{3}{2}\left[e^{j(60\pi t - \pi/6)} + e^{-j(60\pi t - \pi/6)}\right]$$

$$+ \frac{1}{2}\left[e^{j(80\pi t - \pi/3)} + e^{-j(80\pi t - \pi/3)}\right]$$





Example :- given that $x(t) = 6\cos(20\pi t - \pi/3) + 4\sin^2(30\pi t - \pi/6)$

a- sketch its single-sided amplitude and phase spectra

b- sketch its double sided amplitude and phase spectra

$\sin^2(\theta) = \frac{1}{2} - \frac{1}{2}\cos(2\theta)$

$-A\cos(\theta) = A\cos(\theta \pm \pi)$

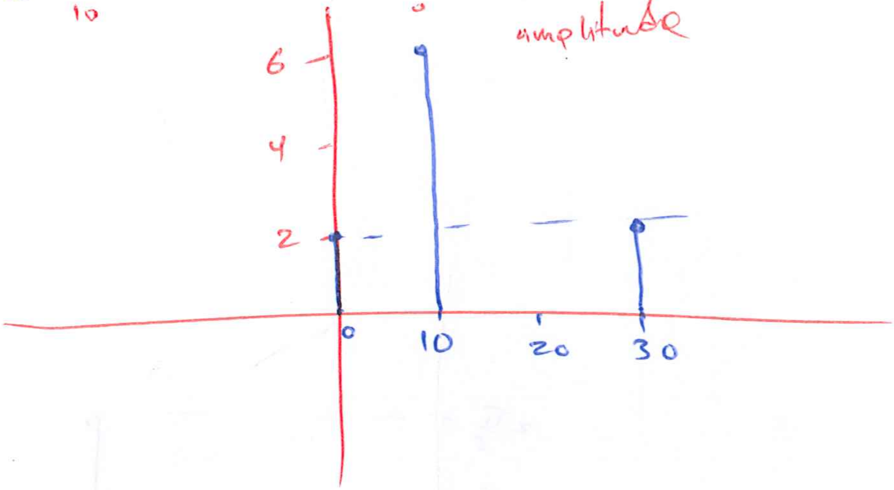
$\cos^2(\theta) = \frac{1}{2} + \frac{1}{2}\cos(2\theta)$

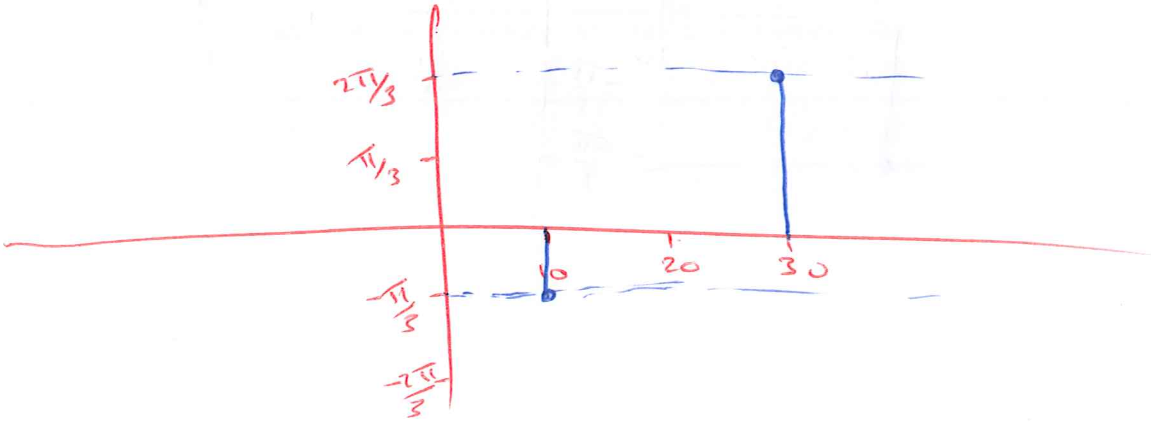
$x(t) = 6\cos(20\pi t - \pi/3) + 4\left[\frac{1}{2} - \frac{1}{2}\cos 2(30\pi t - \pi/6)\right]$

$= 6\cos(20\pi t - \pi/3) + 2 \underline{\underline{-2}}\cos(60\pi t - \pi/3)$

$= 6\cos(20\pi t - \pi/3) + 2 + 2\cos(60\pi t - \pi/3 + \pi)$

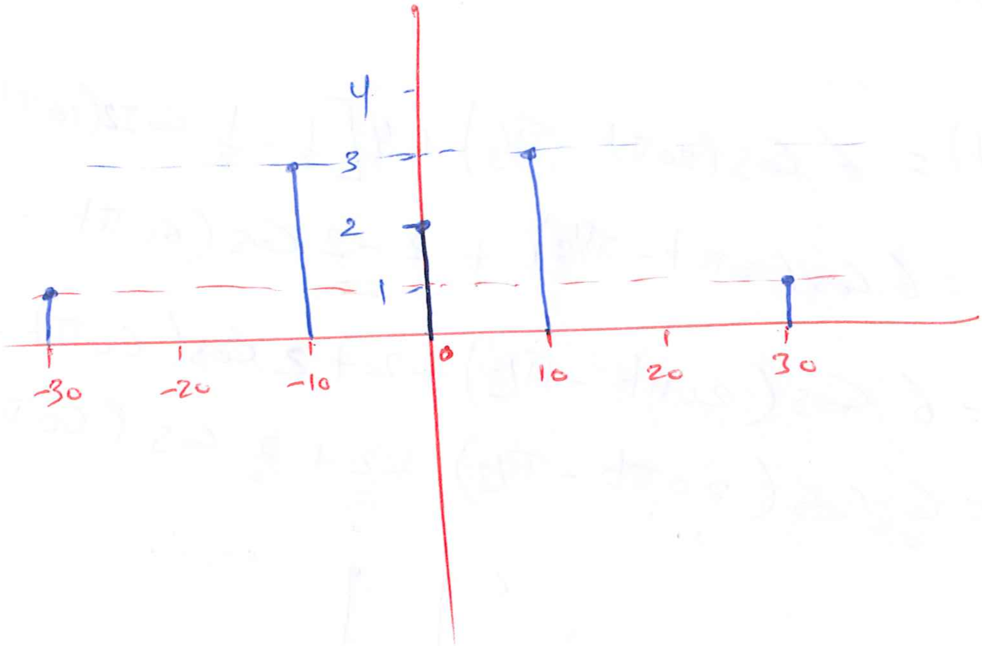
$= \underset{10}{\underline{\underline{6}}}\cos(20\pi t - \pi/3) + \underset{0}{\underline{\underline{2}}} + \underset{30}{\underline{\underline{2}}}\cos(60\pi t + 2\pi/3)$

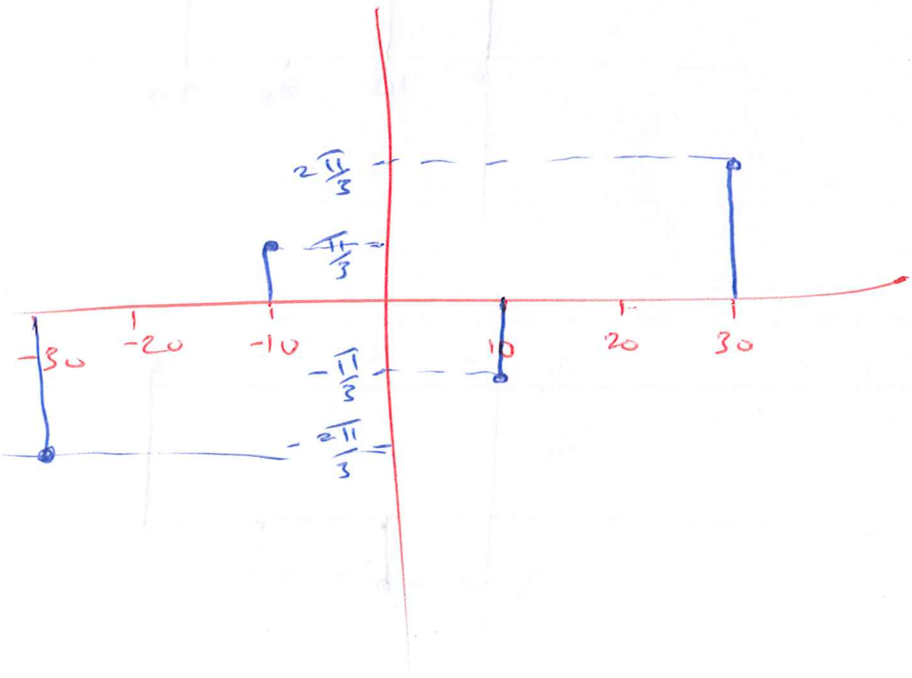




b) $x(t) = 6\cos(20\pi t - \pi/3) + 2 + 2\cos(60\pi t + \frac{2\pi}{3})$

$$= \frac{6}{2}\left[e^{j(20\pi t - \pi/3)} + e^{-j(20\pi t - \pi/3)}\right] + 2 + \frac{2}{2}\left[e^{j(60\pi t + 2\pi/3)} + e^{-j(60\pi t + 2\pi/3)}\right]$$





Energy and power signals

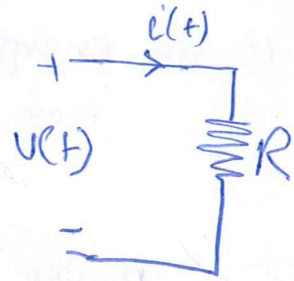
Energy signal :- The signal that have finite ore measurable Energy. The energy signal must exist over a finite duration. It is indecating the amount of energy that can be extracted from the signal when the Load is applied.

Power Signal :- The signal that have infinite or non measurable energy and finite average power. The power signal must exist over infinite duration. The signal must be periodic or it must have statistical regularity.

If both energy and average power are infinite, the the signal is neither energy nor power signal.

The Instantaneous power is defined as the power of the signal at a particular instant.

$$p(t) = v(t)\,I(t) = \frac{v^2(t)}{R} = I^2(t)(R)$$



$\therefore$ the power per unit ohm

$= v^2(t)$ or $I^2(t)$

and for any arbitrary signal $x(t)$ we may define

$p(t) = x^2(t)$

The instantaneous power integrated over its duration gives the total Energy

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt \quad \text{or} \quad E = \lim_{T\to\infty} \int_{-T}^{T} x^2(t)\, dt$$

In order that E is measurable, the signal value must tend to zero as the T approaches positive or negative infinity.

Power is the energy per unit time. To calculate power over a period (2T), we divide the energy over a period (2T) by (2T).
The average power is then calculated as

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\, dt$$

To sum.

① $x(t)$ is an energy signal if and only if $0 < E < \infty$
non periodic signals $P = 0$

② $x(t)$ is a power signal if and only if $0 < P < \infty$
periodic signals $E = \infty$
and integrable

③ $x(t)$ is neither energy nor power if:-
The magnitude of the signal is infinite at any instant of time.

$E = \infty$ $\quad$ $E = \infty$
$P = \infty$ $\quad$ $P = 0$

Examples:- Check if the following Signals are power or Energy signals

① $x(t) = A e^{-\alpha t} u(t)$

$$E = \int_0^\infty (A e^{-\alpha t})^2 \, dt = \int_0^\infty A^2 e^{-2\alpha t} \, dt = \frac{-A^2}{2\alpha} e^{-2\alpha t} \Big|_0^\infty$$

$$= \frac{-A^2}{2\alpha}(0-1) = \frac{A^2}{2\alpha}$$

another way:-

$$E = \lim_{T\to\infty} \int_0^T (A e^{-\alpha t})^2 \, dt = \lim_{T\to\infty} \int_0^T A^2 e^{-2\alpha t} \, dt = \lim_{T\to\infty} \frac{-A^2}{2\alpha} e^{-2\alpha t} \Big|_0^T$$

$$= \frac{A^2}{2\alpha}$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_0^T A^2 e^{-2\alpha t} \, dt = \lim_{T\to\infty} \frac{-A^2}{4T\alpha} \left[e^{-2\alpha T} - 1 \right]$$

$$= 0 \quad \Rightarrow \text{Energy Signal}$$

2) $x(t) = \begin{cases} A & t \geq 0 \\ 0 & t < 0 \end{cases}$

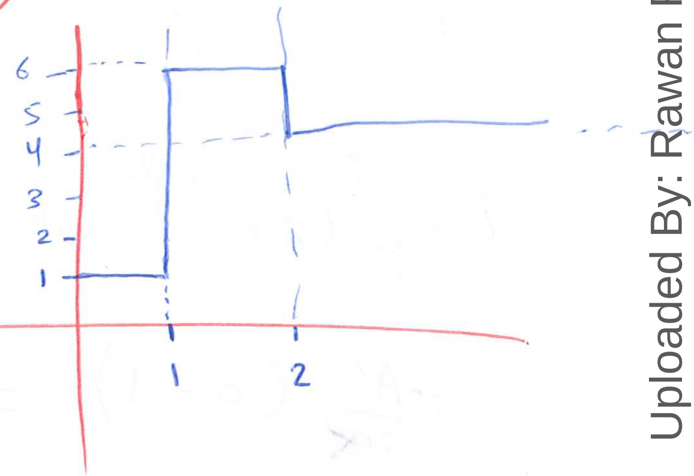
$$E = \lim_{T\to\infty} \int_0^T A^2 \, dt = A^2 \lim_{T\to\infty} t \Big|_0^T$$

$$= \infty$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_0^T A^2 \, dt = A^2 \lim_{T\to\infty} \frac{1}{2T}(T-0)$$

$$= \frac{A^2}{2} \quad \text{power Signal}$$

3) $x(t) = u(t) + 5u(t-1) - 2u(t-2)$



$$E = \lim_{T \to \infty} \int_{-T}^{T} x^2(t)\, dt$$

$$= \lim_{T \to \infty} \left[\int_{0}^{1} (1)^2 dt + \int_{1}^{2} (6)^2 dt + \int_{2}^{T} (4)^2 dt \right]$$

$$= \lim \left[1 + 36 + 16T - 32 \right] = \infty$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\, dt = \frac{16}{2} < \infty$$

$\therefore$ power signal

4) $x(t) = t\,u(t)$ neither energy nor power signal

$x(t) = \tan(t)$ neither energy nor power signal

↳ periodic but not integrable at $(\pi/2, 3\pi/2)$ ---

$x(t) = e^{at}$ neither energy nor power signal

$x(t) = t^2$ neither energy nor power signal

$x(t) = \frac{1}{t}$ neither energy nor power signal

$x(t) = t^{-1/4}$ neither energy nor power signal