$\Rightarrow$ 10.2

$$\sum_{n=1}^{\infty} a_n \quad \text{conv./Div} \quad ??$$

Tests

① n^{th} partial Sum Test (Conv./Div): find $S_n \Rightarrow \lim_{n\to\infty} S_n$

conv $\sum_{n=1}^{\infty} a_n = L \Leftarrow L$

DNE $\Downarrow$ $\sum a_n$ div

② n^{th} term test

if $\lim_{n\to\infty} a_n \neq 0$ then $\sum a_n$ div

if $\lim_{n\to\infty} a_n$ DNE " " "

Th Assume $\sum_{n=1}^{\infty} a_n = A$ and $\sum_{n=1}^{\infty} b_n = B$. Then

① $\sum_{n=1}^{\infty}(a_n \pm b_n) = A \pm B$

The sum (subtraction) of two convergent infinite series is convergent

② $\sum_{n=1}^{\infty} K a_n = K A$ $\Rightarrow$ Any constant multiple of convergent series is convergent.

Remarks Ⓐ If $\sum a_n$ div then $\sum K a_n$ div

K constant $\Rightarrow K \neq 0$

Remark ... k constant $\Rightarrow k \neq 0$

(B) If $\sum a_n$ conv. and $\sum b_n$ div then

$\sum (a_n + b_n)$ $\underline{\text{div}}$ and

$\sum (a_n - b_n)$ $\underline{\text{div}}$

$$\sum_{n=1}^{\infty} a_n \ ??$$

(3) Geometric Series

Geometric Series < Conv. / Div.

Geometric Series has the form:

$$\sum_{n=1}^{\infty} a r^{n-1} = a + ar + ar^2 + ar^3 + ar^4 + \cdots$$

a: 1^{st} term

$\frac{ar}{a} = r$ $\quad \frac{ar^2}{ar} = r$ $\quad \frac{ar^3}{ar^2} = r$ $\quad \frac{ar^4}{ar^3} = r$

r: ration

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Conv.

if $|r| < 1$

$-1 < r < 1$

$\Downarrow$

Div

if $|r| \geq 1$

$r \geq 1$ or $r \leq -1$

Div ← (Conv.) → Div $\quad r$

$-1 \qquad 1$

$$\sum^{\infty} \ \ {}^{n-1} \qquad a$$

$$\text{Sum} = \sum_{n=1}^{\infty} a r^{n-1} = \frac{a}{1-r}$$

Exp Find sum of

① $1 + \frac{1}{2} + \frac{1}{4} + \cdots + \left(\frac{1}{2}\right)^{n-1} + \cdots = \frac{a}{1-r} = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$

a

$r = \frac{1}{2} \in (-1,1)$

$\frac{\frac{1}{2}}{1} = \frac{1}{2}$ $\qquad \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{4} \times \frac{2}{1} = \frac{1}{2}$

Geometric Series $\Rightarrow$ conv. since $r \in (-1,1)$

$\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} = 2 \Rightarrow \sum \left(\frac{1}{2}\right)^{n-1}$ converges to 2

② $1 - \frac{1}{3} + \frac{1}{9} + \cdots + \left(\frac{-1}{3}\right)^{n-1} + \cdots = \frac{a}{1-r} = \frac{1}{1-\frac{-1}{3}} = \frac{1}{1+3}= \frac{3}{4}$

$r = -\frac{1}{3} \in (-1,1)$

$a = 1$

r: $\frac{-\frac{1}{3}}{1} = -\frac{1}{3}$ $\qquad$ r: $\frac{\frac{1}{9}}{-\frac{1}{3}} = \frac{1}{9} \times \frac{-3}{1} = -\frac{1}{3}$

Geometric Series $\Rightarrow$ conv. since $r = -\frac{1}{3} \in (-1,1)$

to $\frac{3}{4}$

③ check Con/Div $\sum_{n=1}^{\infty} \left(\frac{4}{3}\right)^n = \frac{4}{3} + \left(\frac{4}{3}\right)^2 + \left(\frac{4}{3}\right)^3 + \cdots$ ∞

a

This series is Geometric $\Downarrow$ div

$r = \frac{\left(\frac{4}{3}\right)^2}{\cdot} = \frac{\left(\frac{4}{3}\right)^3}{\cdots} = \cdots = \frac{4}{3} > 1$

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$$r = \frac{\left(\frac{4}{3}\right)}{\frac{4}{3}} = \frac{\left(\frac{4}{3}\right)}{\left(\frac{4}{3}\right)^2} = \cdots = \frac{4}{3} > 1 \Downarrow \underline{\text{div}}$$

Exp (51) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3}{2^n}$ check Conv/Div.

$$3 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2^n} = 3\left[\underbrace{\frac{1}{2}}_{a} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \cdots\right]$$

$$r = \frac{-\frac{1}{4}}{\frac{1}{2}} \overset{?}{=} \frac{\frac{1}{8}}{-\frac{1}{4}} \overset{??}{=} \frac{-\frac{1}{16}}{\frac{1}{8}}$$

$$= -\frac{1}{4} \times 2 \overset{??}{=} \frac{1}{8} \times -4 \overset{??}{=} \frac{-1}{16} \times 8$$

$$= -\frac{1}{2} \quad = -\frac{1}{2} \quad = -\frac{1}{2}$$

Geometric Series

$\Downarrow$

Conv, since $r = -\frac{1}{2} \in (-1,1)$

$$= 3\left[\frac{a}{1-r}\right]$$

$$= 3\left[\frac{\frac{1}{2}}{1 - -\frac{1}{2}}\right]$$

$$= 3\left[\frac{\frac{1}{\cancel{2}}}{\frac{3}{\cancel{2}}}\right]$$

$$= \frac{3}{3}$$

$$\sum (-1)^{n+1} \frac{3}{2^n} = 1$$

Exp Consider $\sum^{\infty} (-1)^n (x+1)^n$

① Find x s.t this series conv.

② Find its sum

Exp Consider $\sum_{n=0} (-1) \quad (x+1)$ ...

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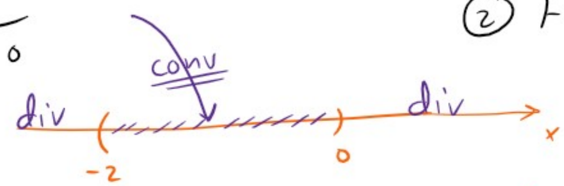
② Find its sum



$$= 1 - (x+1) + (x+1)^2 - (x+1)^3 + (x+1)^4 - \cdots$$

a

$$r = \frac{-(x+1)}{1} \overset{??}{=} \frac{(x+1)^2}{-(x+1)} \overset{?}{=} \frac{-(x+1)^3}{(x+1)^2} = -(x+1)$$

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ration

This series is $\underbrace{\text{geometric}}_{\text{Conve}}$ since r is constant

$$\Downarrow$$

$$-1 < r < 1 \Rightarrow |r| < 1$$

$$|-(x+1)| < 1$$

$$\boxed{|x+1| < 1}$$

$$-1 < x+1 < 1$$

$$\boxed{-2 < x < 0}$$

② $$\text{Sum} = \frac{a}{1-r} = \frac{1}{1 - -(x+1)} = \frac{1}{1+x+1} = \frac{1}{2+x}$$

$$\sum (-1)^n (x+1)^n = \frac{1}{2+x}$$

$-\frac{1}{2}$ $\quad$ $-\frac{1}{2}$

$$\frac{1}{2+-\frac{1}{2}} = \frac{1}{2-\frac{1}{2}} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$

Exp Express the following repeated decimals as ratio of two integers:

① $0.\overline{7} = 0.777777\cdots = \frac{a}{b}$

$= 0.7 + 0.07 + 0.007 + \cdots$

$= \frac{7}{10} + \frac{7}{100} + \frac{7}{1000} + \cdots$

a

$r = \frac{\frac{7}{100}}{\frac{7}{10}} = \frac{1}{10} = \frac{\frac{7}{1000}}{\frac{7}{100}}$

Geometric series $\Rightarrow r = \frac{1}{10} \in (-1,1) \Rightarrow$ series conv. to

$= \frac{a}{1-r} = \frac{\frac{7}{10}}{1-\frac{1}{10}} = \frac{\frac{7}{10}}{\frac{9}{10}} = \frac{7}{9}$

② $0.\overline{23} = 0.23\,23\,23\,23\cdots$

$= \frac{23}{100} + \frac{23}{(100)^2} + \frac{23}{(100)^3} + \cdots$

geom ser

$r = \frac{1}{100}$

$= \frac{\frac{23}{100}}{1-\frac{1}{100}} = \frac{\frac{23}{100}}{\frac{99}{100}} = \frac{23}{99}$

③ $0.0\overline{5} = 10^{-1} \times 0.\overline{5}$

$= 10^{-1}\ \frac{5}{9}$

$0.0\overline{5} = 0.0555\cdots$

$= \frac{5}{100} + \frac{5}{1000} + \cdots$

$=$ 5

$$= 10^{-1} \quad \frac{5}{9}$$

$$= \frac{1}{10} \quad \frac{5}{9}$$

$$= \frac{5}{90}$$

$$= \frac{5}{100} + \frac{5}{1000} \cdots$$

$$= \frac{\frac{5}{100}}{1 - \frac{1}{10}} = \frac{\frac{5}{10\cancel{0}}}{\frac{9}{1\cancel{0}}}$$

$$= \frac{5}{90}$$

Exp Find the sum $\displaystyle\sum_{n=0}^{\infty} \left(\frac{5}{2^n} + \frac{1}{3^n} \right)$

$$\sum_{n=0}^{\infty} \frac{5}{2^n} + \sum_{n=0}^{\infty} \frac{1}{3^n}$$

Geometric series

$$= \left[5 + \frac{5}{2} + \frac{5}{4} + \frac{5}{8} + \cdots \right] + \left[1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots \right]$$

$r = \frac{1}{2} \in (-1,1)$ conv.

$r = \frac{1}{3} \in (-1,1)$ conv.

$$= \frac{5}{1 - \frac{1}{2}} + \frac{1}{1 - \frac{1}{3}}$$

$$= \frac{5}{\frac{1}{2}} + \frac{1}{\frac{2}{3}}$$

$$= 10 + \frac{3}{2}$$

$$= \frac{23}{2}$$

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