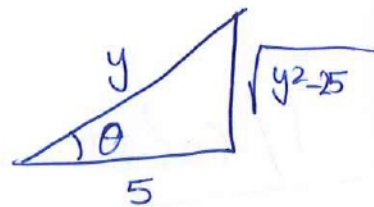


8.3 Trigonometric Substitutions

$$\boxed{12} \int \frac{\sqrt{y^2 - 25}}{y^3} dy$$

$$\text{let } y = 5 \sec \theta$$
$$dy = 5 \sec \theta \tan \theta$$



$$\int \frac{\sqrt{25 \sec^2 \theta - 25}}{125 \sec^3 \theta} \cdot 5 \sec \theta \tan \theta d\theta$$

$$\int \frac{5 \tan \theta \cdot 5 \sec \theta \tan \theta}{125 \sec^3 \theta} d\theta$$

$$\int \frac{25 \tan^2 \theta}{125 \sec^2 \theta} d\theta = \int \frac{25 \sin^2 \theta / \cos^2 \theta}{125 / \cos^2 \theta} d\theta$$

$$= \int \frac{\sin^2 \theta}{5} d\theta$$

$$= \frac{1}{5} \int \frac{1 - \cos(2\theta)}{2} d\theta$$

$$= \frac{1}{10} \left[\theta - \frac{\sin(2\theta)}{2} + C \right]$$

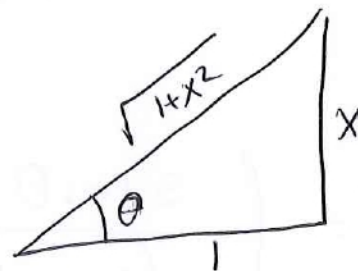
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$$\int \frac{dx}{x^2 \sqrt{x^2+1}}$$

let $x = \tan \theta$

$$dx = \sec^2 \theta d\theta$$

$$= \int \frac{\sec^2 \theta d\theta}{\tan^2 \theta \sqrt{\tan^2 \theta + 1}}$$



$$\int \frac{d\theta}{\sin^2 \theta \sqrt{\sec^2 \theta}}$$

$$= \int \frac{d\theta}{\sin^2 \theta \sec \theta} = \int \frac{\cos \theta}{\sin^2 \theta} d\theta$$

let $u = \sin \theta$

$$du = \cos \theta d\theta$$

$$= \int \frac{1}{u^2} du$$

$$= \frac{-1}{u} + C$$

$$= \frac{-1}{\frac{x}{\sqrt{1+x^2}}} + C$$

$$= \boxed{\frac{-\sqrt{1+x^2}}{x} + C}$$

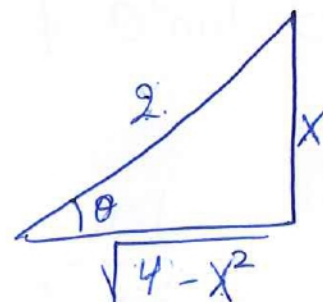
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$$\int_0^1 \frac{dx}{(4-x^2)^{3/2}}$$

let $x = 2 \sin \theta$

$$dx = 2 \cos \theta d\theta$$

$$I = \int \frac{2 \cos \theta d\theta}{(4 - 4 \sin^2 \theta)^{3/2}}$$



$$= \int \frac{2 \cos \theta d\theta}{(4 \cos^2 \theta)^{3/2}}$$

$$= \int \frac{2 \cos \theta d\theta}{8 \cos^3 \theta}$$

$$= \frac{1}{4} \int \frac{1}{\cos^2 \theta} d\theta = \frac{1}{4} \int \sec^2 \theta d\theta$$

$$= \frac{1}{4} \tan \theta$$

من المثلث

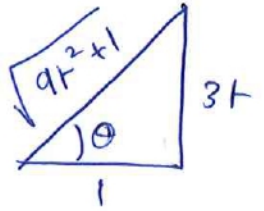
$$= \frac{1}{4} \left[\frac{x}{\sqrt{4-x^2}} \right]_0^1$$

$$= \frac{1}{4} \left[\frac{1}{\sqrt{3}} - 0 \right] = \frac{1}{4\sqrt{3}}$$

$$\boxed{30} \quad \int \frac{6}{(9t^2+1)^2} dt$$

$$\text{let } 3t = \tan \theta$$

$$3 dt = \sec^2 \theta d\theta$$



$$I = \int \frac{6}{(\tan^2 \theta + 1)^2} \frac{\sec^2 \theta}{3} d\theta$$

$$= 2 \int \frac{\sec^2 \theta}{(\sec^2 \theta)^2} d\theta$$

$$= 2 \int \frac{1}{\sec^2 \theta} d\theta$$

$$= 2 \int \cos^2 \theta d\theta$$

$$= 2 \int \frac{1 + \cos(2\theta)}{2} d\theta$$

$$= \theta + \frac{\sin(2\theta)}{2} + C$$

$$= \tan^{-1}(3t) + \frac{1}{2} \cdot 2 \sin \theta \cos \theta + C$$

$$= \tan^{-1}(3t) + \frac{3t}{\sqrt{9t^2+1}} \cdot \frac{1}{\sqrt{9t^2+1}} + C$$

$$= \tan^{-1}(3t) + \frac{3t}{9t^2+1} + C$$

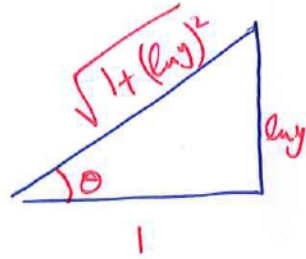
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$$\int_1^e \frac{dy}{y \sqrt{1 + (\ln y)^2}}$$

$$\text{let } \ln y = \tan \theta$$

$$\frac{1}{y} dy = \sec^2 \theta d\theta$$

$$dy = y \sec^2 \theta d\theta$$



$$\int \frac{y \sec^2 \theta d\theta}{y \sqrt{1 + (\tan \theta)^2}}$$

$$I = \int \frac{\sec^2 \theta d\theta}{\sqrt{1 + \tan^2 \theta}}$$

$$= \int \frac{\sec^2 \theta d\theta}{\sec \theta} = \int \sec \theta d\theta$$

$$= \ln |\sec \theta + \tan \theta| \Big|_{\square}^{\square}$$

$$= \ln \left| \sqrt{1 + \ln^2 y} + \ln y \right| \Big|_1^e$$

$$= \ln \left| \sqrt{2} + 1 \right| - \ln |1| = \ln (\sqrt{2} + 1)$$