Ch1_Part 2

Wednesday, September 15, 2021 12:36 PM

Exp $f(x) = |x| = \sqrt{x^2}$

Find D, R

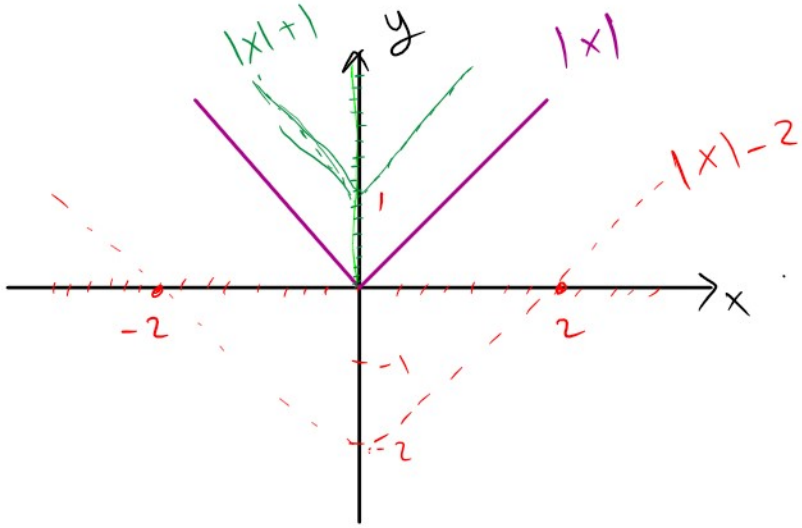
x ← x-axis

y ← y-axis

$D(f) = (-\infty, \infty) = \mathbb{R}$

$R(f) = [0, \infty)$

x	y
$\|-3\| = 3$	
$\|-2\| = 2$	
$\|0\| = 0$	
$\|7\| = 7$	
$\|10\| = 10$	



Exp $y = |x| + 1$

$= |x| - 2$

Exp $f(x) = \frac{1}{x}$ Find D, R

D(f): مجموعة الأعداد التي تجعل المقام لا يساوي صفر

$= \mathbb{R} \setminus \{0\}$

$D = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$

$y = \frac{1}{x}$

$R =$ ما هي قيم y الناتجة عن تعويض x

$f(1) = \frac{1}{1} = 1$

$f(-1) = \frac{1}{-1} = -1$

$f(2) = \frac{1}{2}$

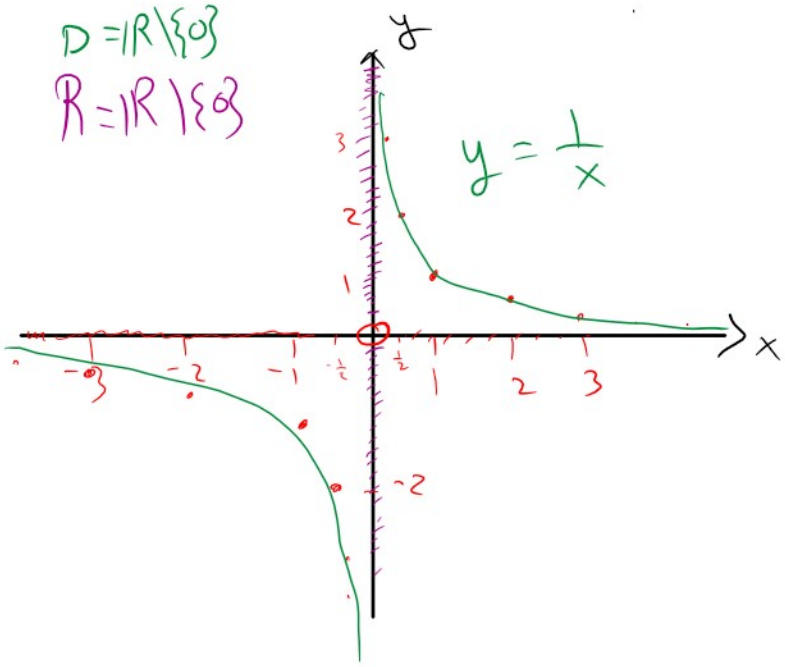
$f(-2) = -\frac{1}{2}$

$f(\frac{1}{2}) = \frac{1}{\frac{1}{2}} = 2$

$f(-\frac{1}{2}) = \frac{1}{-\frac{1}{2}} = -2$

$D = \mathbb{R} \setminus \{0\}$

$R = \mathbb{R} \setminus \{0\}$



floor function

Greatest Integer function

Exp $f(x) = \lfloor x \rfloor$

$D = \mathbb{R} = (-\infty, \infty)$

$\lfloor -4 \rfloor = -4$

$\lfloor -4.1 \rfloor = -5$

$\lfloor -4.5 \rfloor = -5$

$\lfloor -4.9 \rfloor = -5$

$\lfloor 4 \rfloor = 4$

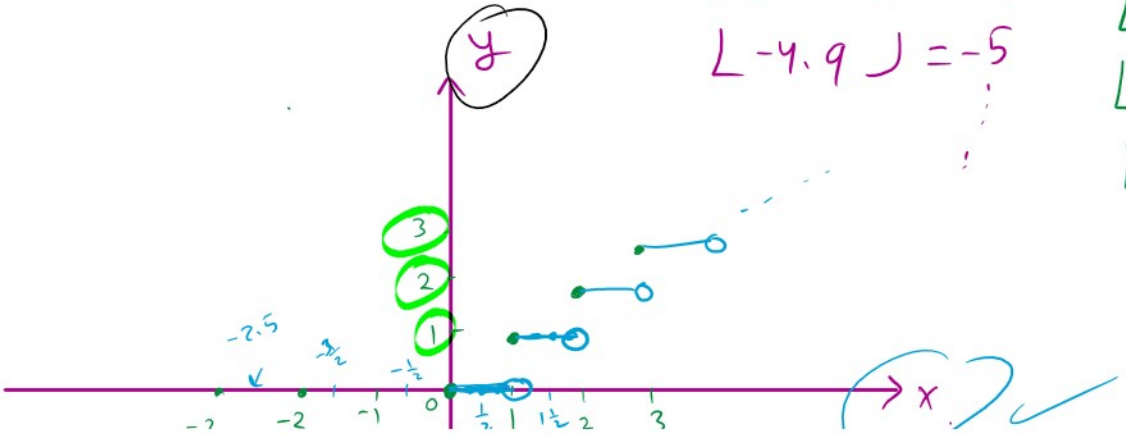
$\lfloor 4.1 \rfloor = 4$

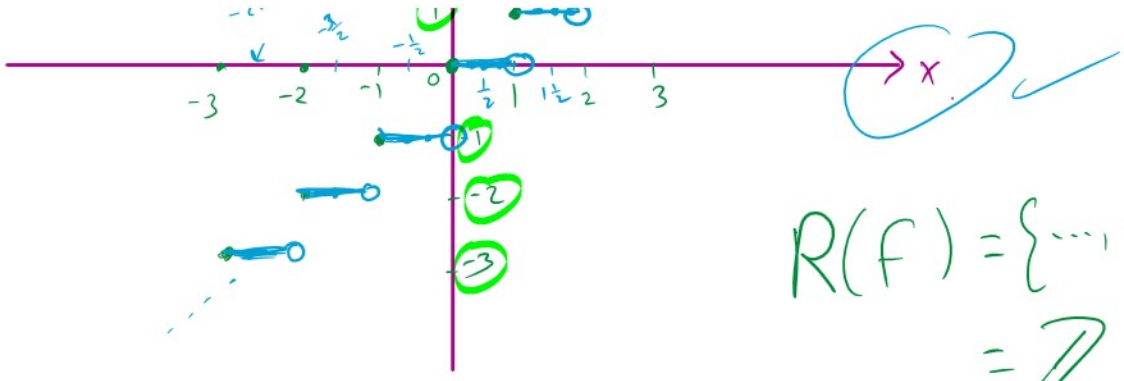
$\lfloor 4.2 \rfloor = 4$

$\lfloor 4.5 \rfloor = 4$

$\lfloor 4.8 \rfloor = 4$

$\lfloor 4.9 \rfloor = 4$

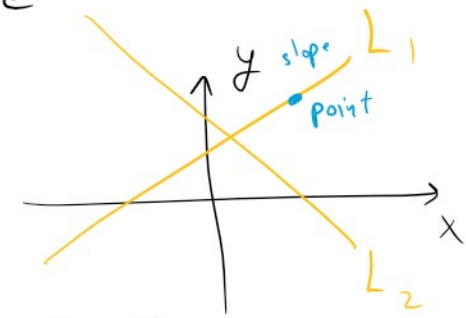




$$R(f) = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$
$$= \mathbb{Z}$$
$$= \text{Integers}$$

line equation (Tangent line) معادلة المماس

- we need two points
- or one point and slope



$$y - y_0 = m(x - x_0)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Exp Find tangent line through the points

$(\overset{x_1}{1}, \overset{y_1}{2})$, $(\overset{x_2}{3}, \overset{y_2}{7})$

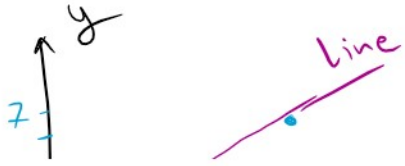
x_0 y_0

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{7 - 2}{3 - 1} = \frac{5}{2}$$

$$y - y_0 = m(x - x_0)$$

$$y - 2 = \frac{5}{2}(x - 1)$$

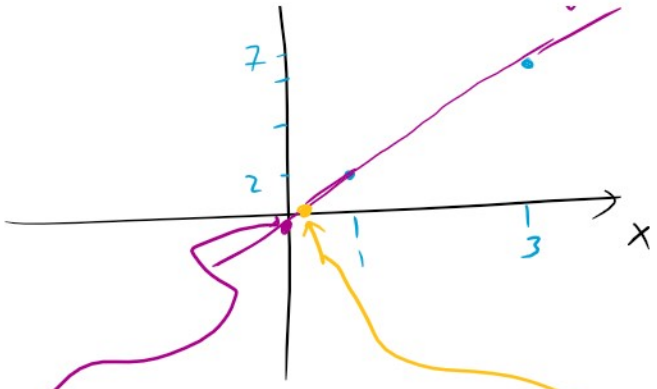


$= \frac{5}{2}x - \frac{5}{2}$

$y = \frac{5}{2}x - \frac{5}{2} + 2$

$\frac{5}{2} = 2.5$

$y = \frac{5}{2}x - \frac{1}{2}$



x-intercept $\Rightarrow y = 0$

$0 = \frac{5}{2}x - \frac{1}{2}$

$0 = 5x - 1$

$5x = 1$

$x = \frac{1}{5}$

y-intercept $\Rightarrow x = 0$

$y = \frac{5}{2}(0) - \frac{1}{2}$

$= 0 - \frac{1}{2}$

$-\frac{1}{2}$

Exp $y = \sqrt{x}$

Find D, R

$D = [0, \infty)$

$R = [0, \infty)$

$x \in D$

$y \in D$

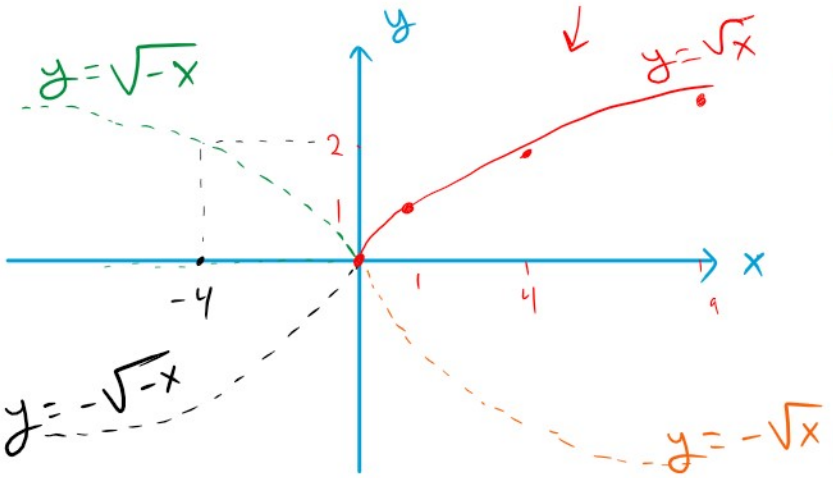
$y(0) = 0$

$y(1) = 1$

$y(4) = 2$

$y(9) = 3$

$y = \sqrt{-x}$ $\uparrow y$ $\downarrow$ $y = \sqrt{x}$ | Exp $y = -\sqrt{x}$



Exp $y=-\sqrt{x}$

Exp $y=\sqrt{-x} \Rightarrow D=(-\infty,0]$

Exp $y=-\sqrt{-x}$

$y=\sqrt{-3}$

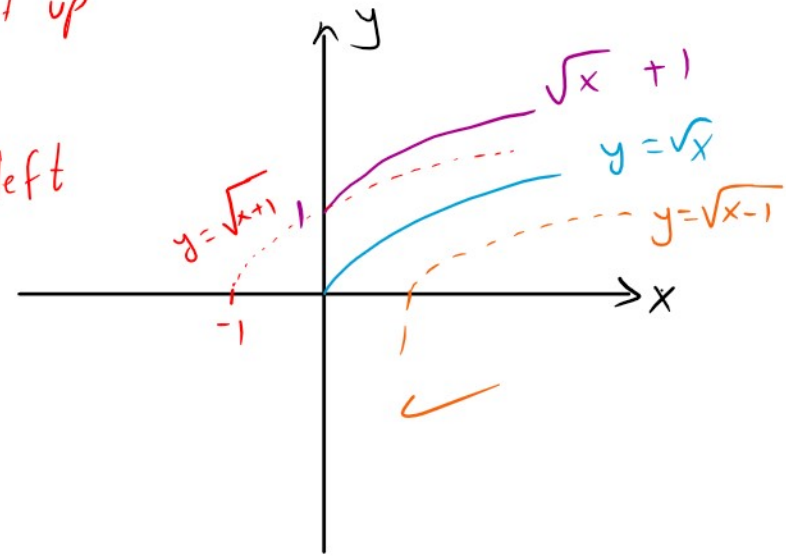
Exp ① $y=\sqrt{x}+1$ shift up

② $y=\sqrt{x+1}$ shift left

D: $x+1 \geq 0$

$x \geq -1$

$[-1,\infty)$



③ $y=\sqrt{x-1}$

D: $x-1 \geq 0$

$x \geq 1$

$[1,\infty)$

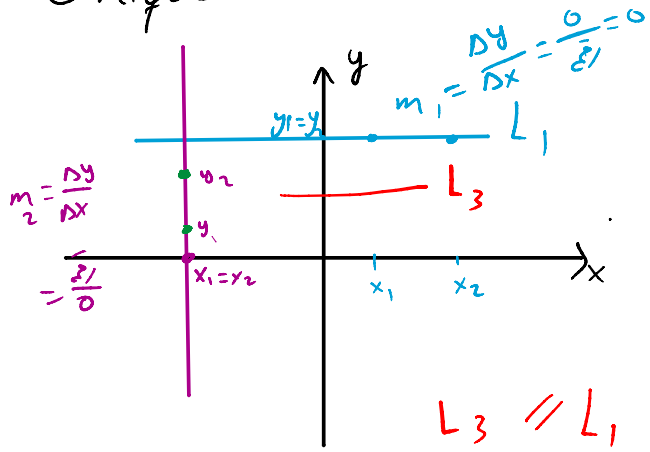
line

H. V. O.

H.
Horizontal
$m_1 = 0$

V.
Vertical
m_2 undefined

O.
Oblique



$L_3 \parallel L_1$
$m_3 = m_1 = 0$

Two lines L_1, L_2

$L_1 \parallel L_2$ iff $m_1 = m_2$ → Parallel

$L_1 \perp L_2$ iff $m_1 m_2 = -1$
perpendicular (normal), (orthogonal)

Exp L_1: $y = 2x + 1$
L_2: $y = 4 - \frac{1}{2}x$

Is $L_1 \perp L_2$

$m_1 = 2$
$m_2 = -\frac{1}{2}$
$\Rightarrow$ $m_1 m_2 = (2)\left(-\frac{1}{2}\right) = -1$

Yes

Absolute Value

(1) $|x| = a \Rightarrow x = a$ or $x = -a$

Exp $|x| = 5 \Rightarrow x = 5$ or $x = -5$

$\{-5, 5\}$

Exp $|2x-1| = 3 \Rightarrow 2x-1 = 3 \Rightarrow 2x = 4 \Rightarrow x = 2$

or $2x-1 = -3 \Rightarrow 2x = -2 \Rightarrow x = -1$

$\{-1, 2\}$

Exp $|1-4x| = -3$

$\emptyset$

(2) $|x| \leq a$ (where $a > 0$) $\Rightarrow -a \leq x \leq a$

Exp $|2x-4| \leq 8$

$-8 \leq 2x-4 \leq 8$

$-4 \leq 2x \leq 12$

$-2 \leq x \leq 6 \Rightarrow x \in [-2, 6]$

$|2(-1)-4| = |-2-4| = |-6| = 6 \leq 8$

(-1)

(3) $|x| \geq a \Rightarrow x \geq a$ or $x \leq -a$

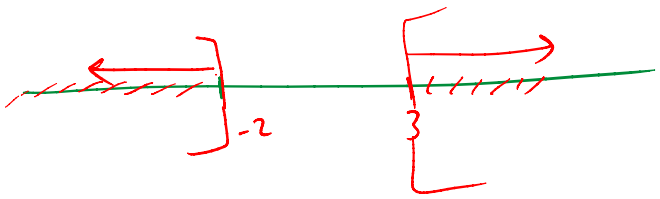
Exp $|1-2x| \geq 5$

Exp $|1-2x| \geq 5$

$1-2x \geq 5$ or $1-2x \leq -5$

$-2x \geq 4$ $\quad -2x \leq -6$

$x \leq -2$ $\quad x \geq 3$



$(-\infty, -2] \cup [3, \infty)$

Exp $|x-1| > 2$

$x-1 > 2$ or $x-1 < -2$

$x > 3$ $\quad x < -1$

$(-\infty, -1) \cup (3, \infty)$

Exp $|1-x| > 2$

$1-x > 2$ or $1-x < -2$

$-x > 1$ $\quad -x < -3$

$x < -1$ $\quad x > 3$

$x > 3$

$x < -1$

$(-\infty, -1) \cup (3, \infty)$

Ex $|x| > 5$

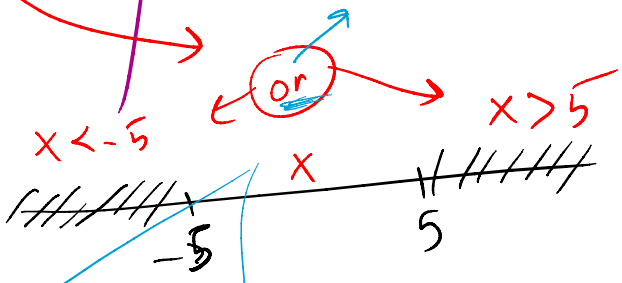
$7 > 5$

$-6 < -5$

$x = -6$ ✓

$= 7$

$=$



$(-\infty, -5) \cup (5, \infty)$

$x \geq 3$ $x \leq -2$

x

$x < -5$ and $x > 5$

Factorization

- $x^2 - a^2 = (x - a)(x + a)$
- $x^2 - 3x + 2 = 0$

$(x - 1)(x - 2) = 0$

$x_2 = 1$, $x_1 = 2$

$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$= \frac{3 \pm \sqrt{9 - 4(1)(2)}}{2(1)}$

$= 3 \pm \sqrt{9 - 8}$

$x_2 = 1$, $x_1 = 2$

$Ax^2 + Bx + C = 0$

$A=1,\ B=-3,\ C=2$

$$= \frac{3 \pm \sqrt{9-8}}{2}$$

$$= \frac{3 \pm \sqrt{1}}{2}$$

$$= \frac{3 \pm 1}{2}$$

$$x_1 = \frac{3+1}{2} = \frac{4}{2} = 2$$

$$x_2 = \frac{3-1}{2} = \frac{2}{2} = 1$$

معادلة الدائرة نصف قطرها r ومركزها (x_0, y_0)

Center (x_0, y_0), radius r — circle equation

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

Exp Find the center and radius for the circle

$$x^2 + 2x + y^2 = 4y$$

$$x^2 + 2x + \boxed{1} + y^2 - 4y + \boxed{4} = \boxed{1} + \boxed{4}$$

$$(x+1)^2 + (y-2)^2 = 5 \rightarrow (\sqrt{5})^2$$

$x^2 + 2x + 1$ $\qquad$ $y^2 - 4y + 4$

$$(x-(-1))^2 + (y-2)^2 = (\sqrt{5})^2$$

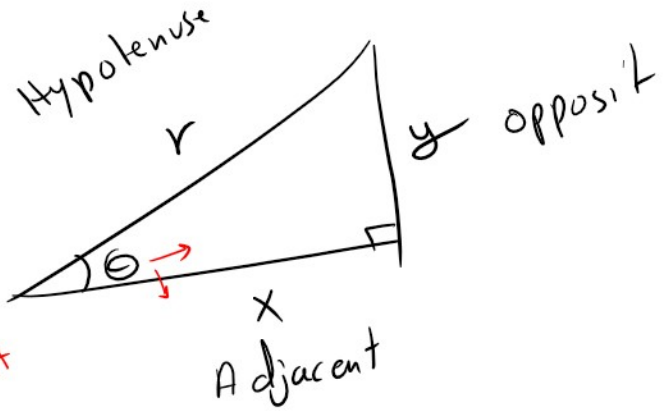
$(h, k) = (-1, 2)$

$(x-(-1))^2 + (y-2) = (\sqrt{5})^2$

center $(x_0, y_0) = (-1, 2)$

radius $= \sqrt{5}$

$(a+b)^2 = a^2 + 2ab + b^2$



$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$

co-secant

$\csc\theta = \frac{1}{\sin\theta} = \frac{r}{y}$

secant

$\sec\theta = \frac{1}{\cos\theta} = \frac{r}{x}$

co-tan

$\cot\theta = \frac{1}{\tan\theta} = \frac{x}{y} = \frac{\cos\theta}{\sin\theta}$

$x^2 + y^2 = r^2$

$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$

$\cos^2\theta + \sin^2\theta = 1$