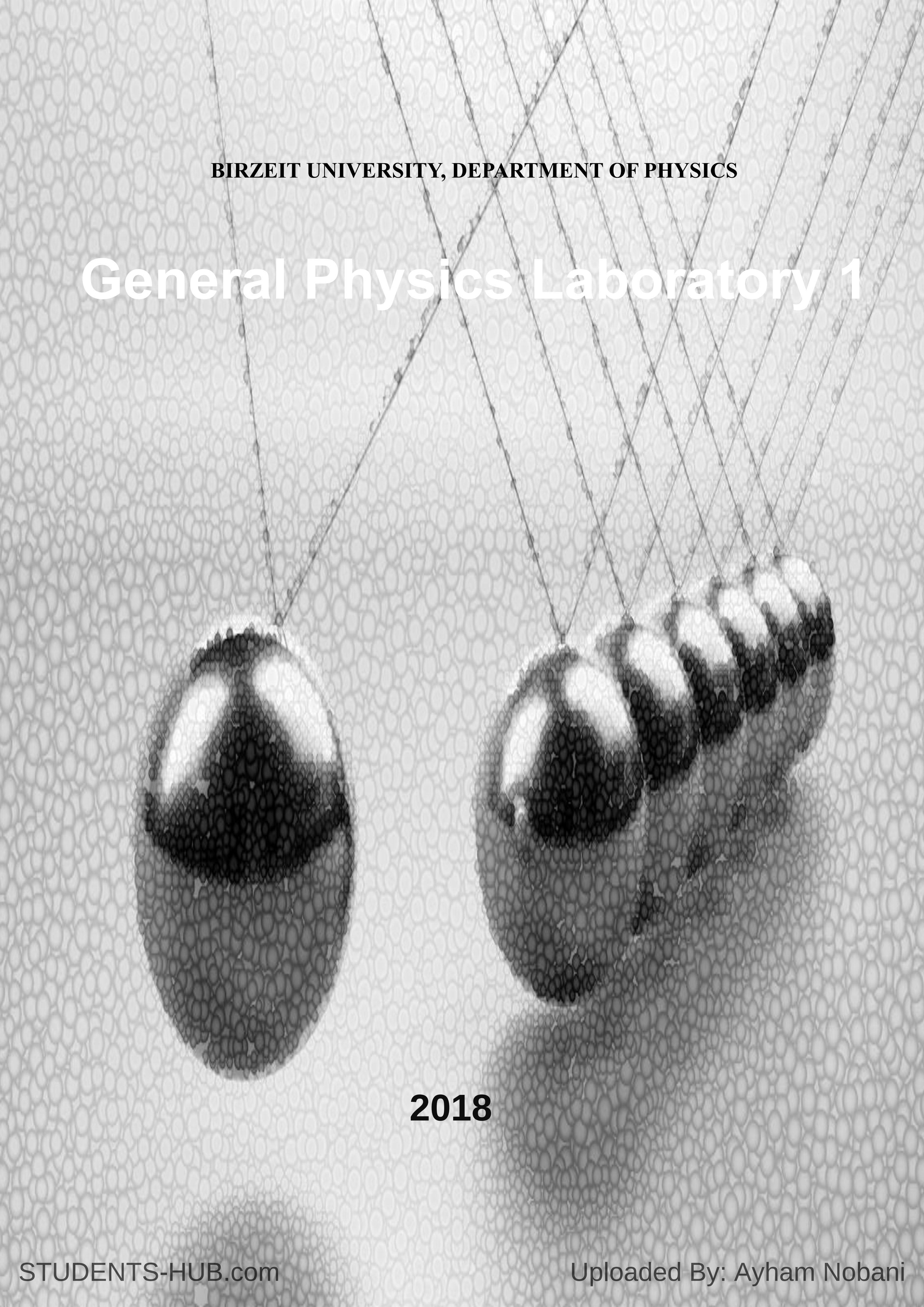

BIRZEIT UNIVERSITY, DEPARTMENT OF PHYSICS

General Physics Laboratory 1

2018

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1.INTRODUCTION

"I hear and I forget, I see and I remember, I do and I understand"

Confucius (Chinese Teacher and Philosopher 551 BC– 479 BC)

Physics is an experimental science in which theories are tested against observations. The introductory physics laboratory represents one of the first opportunities to learn the scientific method in interpreting observations and encourages you to think critically to explain different physical phenomena. Many of the experiments in this laboratory are designed to illustrate concepts encountered in the general physics course. However, other experiments such as ones covering geometrical optics, no prior knowledge is assumed and detailed instructions will be given in the laboratory.

1.1 Main Objectives of the Introductory Physics Laboratory

The introductory physics laboratory experiments and material are mainly chosen to achieve the following goals:

- Developing basic skills in experimental physics including:
  - How to do an experiment and follow and record experimental procedures.
  - How to use some measuring instruments such as vernier calipers, micrometers, voltmeters, ammeters, stop-watches, digital multimeters, signal generators and oscilloscopes.
  - How to analyze data and obtain results numerically and graphically.
  - How to estimate the uncertainty in a measurement and calculated results.
- Developing collaborative learning skills that are very important in solving real world problems.
- Encouraging the independent intellectual discoveries and knowledge.
- Encouraging the curiosity to ask questions and determine methods and apparatus to answering them.
- Developing the ability to relate abstract concepts to observable quantities.
- Comprehending the role of observation in physics and to identify the difference between predictions based on theory and the outcomes of experiments.
- Reporting on experiments and develop the necessary skills to communicate experiment findings in a scientific and concise way.

2.LABORATORY INSTRUCTIONS

- Each experiment should be read before the lab session starts and reflect on what the experiment is intended to establish.
- Many of the instruments used for the laboratory experiments are expensive and in some cases fragile. If these instruments are used incorrectly, it can become faulty or even damaged. Proper handling is therefore important. If a piece of equipment does not function, do not try to fix it. It should be reported to the laboratory instructor.
- The laboratory work is a group activity where both you and your partner contribute to the thinking process and performing the experiments.
- Each laboratory bench should be returned to its original orderly state before leaving the lab. The bench should be clean from any papers before you leave the lab.
- It is required to have an experimental setup checked and approved by the instructor before starting data collection.
- Lab report template, graph paper, sharp pencil, ruler and calculator should be brought to each laboratory session.
- Data sheet should be signed by the instructor before you leave the laboratory. This will be the only valid proof that you actually did the experiment.
- Smoking, eating, drinking or using of cellphone are not allowed inside the laboratory.
- It is permitted to share information and ideas with your classmates, but it is not accepted at all to copy any part of your report from others.

3.LABORATORY REPORTS

An experiment report is an important way to communicate it the experiment result with others. In general, the report is intended to provide information to someone who did not perform the experiment. The report should explain what are the objectives of the experiment? , How is it done? and What are the results obtained?. It is very helpful to pretend that the report is written such that it can be understood by a person who was absent from the experiment. A successful report should allow others to repeat the basic experiment. This is the only way experimental results can be checked and verified.

A laboratory report template is provided for each experiment in which experimental data should be recorded neatly. Calculations of experimental results should be included. Neatness, organization, and explanations of measurements and calculations in the laboratory report reflect the quality and confidence in the experimental work. Below are the standard sections of a typical Physics 111 lab report

- **Cover Page:** It should contain the name, number and date of the experiment as well as the experimenter names.
- **Abstract:** This section should provide a brief summary explaining *the aim of the experiment*, *the method used* and *the main result. Note that the abstract is not a conclusion and should be written after the experiment is completed.*
- **Theory:** This section should include any relevant theory along with mathematical formulas and uncertainty analysis. All uncertainties should be justified through uncertainty statistical calculations and combinations. The following should be considered when writing the Theory section
    - *Avoiding copying from the lab manual.*
    - *Writing a brief summary in the experimenter own words to explain the theoretical background of the experiment.*
    - *Explaining all symbols used in the mathematical formulas.*
    - *Using diagrams and illustrative figures as much as possible.*
- **Procedure:** This section describes in detail the way the experiment was carried out. This is very important so that anyone else could re-create the experiment exactly as it was performed. In this section usually narrative on What was measured? and How was it measured? is provided. The following should be considered when writing the Procedure section
    - *Avoiding copying from the manual.*

- *Drawing illustrative diagrams to show how the experiment is set up and how the instruments are used.*

- **Data:** measurements (data) should be recorded in clear and readable fashion. The data is recommended to be presented in table form for easier reading. The following point should be taken into consideration when writing data:
    - *Data should be written in ink.*
    - *Data should be written directly to the table. Never write the data on a scrap paper to be copied later to the table.*
    - *Data should be have units and contain the correct number of significant figures. Pure numbers with not units or clear number of significant figures are meaningless in Physics.*
    - *If a mistake in recording an entry in the data table is made, it should not be obliterated or written on the top of the wrong entry. Simply the incorrect entry should be crossed out lightly and written near it the correct entry as shown below:*

    Not this 16.9 or this 16.9, but this ~~16.9~~ 16.4

- **Calculations:** Mathematical calculations connecting fundamental physics relationships to the quantities measured should be given.

- **Conclusion:** This section presents the final result with the uncertainty associated with it. The conclusion should be based on evidence and does not reflect how the experimenter feels about the experiment. The conclusion should implicitly answer at least the following questions:
    - *Is the result acceptable? How do the result compare with the theory presented?*
    - *What is the behavior of the graph/line?*
    - *What were the possible sources of systematic uncertainties?*
    - *Were there any major experimental complications?*
    - *How the result can be improved in the future if the experiment is repeated? What should be done differently when the experiment is repeated?*

Note:

Random errors are never discussed here since these errors are unknown, unpredictable and always present.

3.1 The Laboratory Report Grade

A report should always be proof read before it is given to others. The laboratory report grade will depend on the answer of the following questions:

- Is the report written well and in good English?
- Does the theory contain full explanation of all symbols used?
- Does the theory contain the necessary illustrative figures? Do these figures look nice and informative?
- Do results and data have the correct units and number of significant figures?
- Are calculations made correctly?
- Are the experimental uncertainties correctly estimated and correctly combined them?
- Does the report convey all the essential information to reproduce the result?
- Is the result logical and consistent with what is expected?
- Are the graphs complete and properly annotated (e.g. Title, Axis Labels, Proper Scales and Sizes)?
- Does the graph indicate how the slope was calculated?
- Is the estimation of the slope reasonable and correct?
- Does the conclusion show insight and indicate ways for future improvement in the experiment?
- Are all elements (sections) of the report complete and present?
- Is the report submitted late?

Note:

If you are not sure about how to do any part of the report, remember that you can always ask your instructor for help. Do not be shy or afraid.

4. MEASUREMENTS and UNCERTAINTIES

Experimental conditions in the lab are not ideal, so it is not expect that a measurement of a physical quantity would represent its true value with infinite precision. This is because in any experiment, there are many sources of errors including:

- Choice of Instruments (its precision, calibration, etc. …).
- Environment (temperature fluctuations, gravity, background radiation, etc….).
- The way the experiment is done.
- Experimenter.
- The way the physical quantity is measured.

Each time an experiment is done, a great effort should be made to make sure that these errors as small as possible. It is worth mentioning here that measurements can never be taken without any errors. This means that the true value of a physical quantity can never obtained, however it can be practically estimated when all errors are very small.

4.1 Random Errors

4.1.1 Uncertainty in a measurement

If the position of the voltmeter is read as shown below in Figure 1a, the reading might be considered slightly greater than 5.4 $volts$. The pointer is almost halfway between 5.4 and 5.5, but is it closer to 5.44, 5.45 or 5.43? Probably it is 5.44. There is uncertainty about the reading of the voltmeter by about ± 0.1 volt in this example.

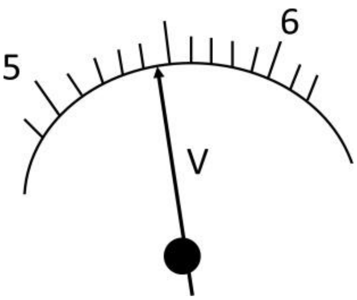


Figure 1a

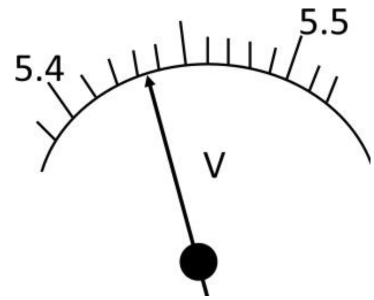


Figure 1b

To inform others about the voltmeter measurement, it should be reported as $V = 5.4 + 0.1\ volt$. This means that if the measurement is repeated, the new measurement probably will not differ from the first one by more than $\pm 0.1\ volt$.

Now suppose that the voltmeter shown in Figure 1b is used instead. This voltmeter has a more finely divided scale. The reading is slightly greater than 5.43. A gain an estimate of the position of the pointer in the space between 5.43 and 5.44 should be made. Is it 5.432, 5.433, or 5.434? Again the last figure is uncertain. You would therefore report your measurement, as $V = 5.43 \pm 0.01\, volt$. The estimated uncertainty depends on how finally the scale is divided. It also depends on how carefully the instrument is read. But no matter what always there will be some uncertainty in the last figure. Hence, all measured quantities are uncertain and the true values of all measured quantities are always unknown to infinite precision. The best that can be done is to use more precise instruments and be more careful to reduce the magnitude of these uncertainties.

In the previous example, the uncertainty was estimated by taking into consideration the instrument used and carefully using it. If only one measurement of a quantity is made, then the only way to find the uncertainty is to estimate it. However, if several measurements of the same quantity are made; say $x_1, x_2, \cdots x_N$, then the best estimate of the true value is the average value defined as

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

While the best estimate of the uncertainty is the sample standard deviation which is defined as

$$\sigma_s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

The advantage of using σ_s as a measure of the uncertainty in the measurement is that it allows the definition of a more precise meaning to "*uncertainty*". Previously it was indicated that one measurement probably does not differ from another one by more than the uncertainty. But what is meant by "*probably*"?. When σ_s is used as the uncertainty, the word "*probably*" has a clear meaning. It means that the probability is about 2/3 that any one measurement does not differ from another by more than σ_s. σ_s is called the sample standard deviation because it is the uncertainty in only one measurement, namely a sample measurement.

Example: Suppose that five students measured the voltage drop (V) across a resistance (R). The measurements in volts were as shown below:

Student No.	**1**	**2**	**3**	**4**	**5**
Voltage (Volts)	5.45	5.44	5.43	5.46	5.43

Here we have $N = 5$, $\bar{V} = \frac{1}{5}\sum_{i=1}^{5} V_i = 5.442\ volts$ and $\sigma_s = \sqrt{\frac{1}{4}\sum_{i=1}^{5}(x_i - 5.442)^2} = 0.013\ volts$.

4.1.2 Uncertainty in the mean

The best estimate of "*the true value*" is the average or the mean of our measurements. But what is the degree of confidence that the average value is close to the true value. Clearly the more measurements made, the closer the mean to the true value.

The uncertainty in the mean or the "*standard deviation of the mean*" is suggested by statistical theories to be

$$\sigma_m = \frac{\sigma_s}{\sqrt{N}}$$

We note here that σ_m falls as $\frac{1}{\sqrt{N}}$ which means for example if you want to reduce the uncertainty in the mean by half, you have to take 4 times as many measurements. The probability that the mean value does not differ from the true value by more than σ_m is about 2/3. Appendix C: contains complete instructions on how to use calculator to compute the mean value and the sample standard deviation of a set of data. You can verify that $\sigma_m = 0.0058\ volts$ for the voltage drop measurements of the previous example.

4.1.3 Importance of knowing the uncertainty

Consider the measurements of the length of a rod at two different temperatures are

Length (cm)	98.025	98.034
Temperature (℃)	10.0	20.0

Now, we would like to answer the question "Does the length of the rod depend on the temperature?". We cannot answer this question unless we know the uncertainty in the length of the rod. If the uncertainty is $\pm 0.01\ cm$, then the question cannot be answered, since the difference between the two measurements is smaller than their uncertainties. Therefore, the result of an experiment must include both the value of the measured quantity and its uncertainty.

It is worth noting that the uncertainties shown previously have plus or minus sign. This is because measured quantities are spread on both sides of the true value. If we take any other measurement we cannot predict whether it will be above or below the true value. Consequently, these certainties are called *random*. Another note is that in statistical theory the word "*error*" is usually used to mean "*uncertainty*". In this sense, "error" does not mean a mistake.

4.2 Systematic Errors

Random errors are always present in an experiment. They are equally likely to be positive or negative and cause several measurements to spread out around the true value. Conversely, systematic errors shift the measured values to be consistently bigger or smaller of the true value. If there is a systematic error (in addition to random errors), measurements spread out around a mean which is different from the true value. The following are a summary of three examples where systematic errors arise:

1) Consider a scale voltmeter that reads a value of $-0.2\ volts$ when it is not connected to anything, then any measurement that is made using this voltmeter will be smaller by $-0.2\ volts$. Now, suppose that you want to measure the voltage drop across a resistance R in your circuit using the uncalibrated voltmeter. The measured voltage across the resistance will be smaller by $-0.2\ volts$ from its actual value. This systematic error can be eliminated by checking the zero setting before doing the experiment. In other words, you can adjust the voltmeter to read zero volts when it is not connected; this process of adjusting is called *calibration* of the instrument. Note also that if instrument calibration cannot be done, the systematic error should be added (or subtracted) from all measurements.

Note:

Always check for systematic errors in any used instrument and try to eliminate them in advance.

2) Suppose you measured the length of a rod using a roughly used or old meter stick which may have ½ mm of its starting edge worn off as shown below

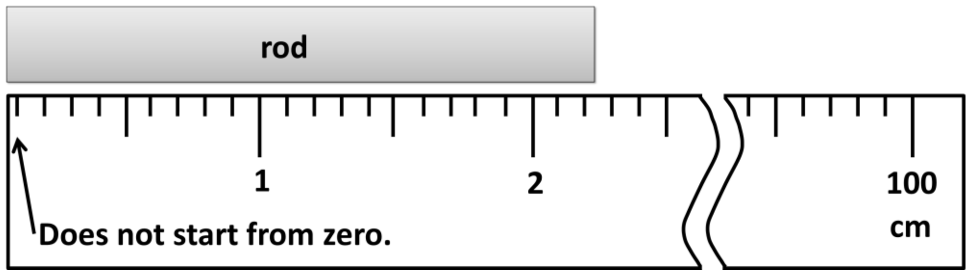


Figure 2: **Worn off ruler used to measure the length of metallic rod.**

Clearly, all measurements of the length of the rod will be ½ mm larger than it should be and there will be a systematic error of $+$ ½ mm in this case. In many other cases, there is no easy way to know the size of the systematic error in a measurement, therefore the best way to eliminate these systematic errors is by starting a measurement from a mark far away from the starting edge of the meter stick.

3) Systematic errors are not caused only by lack of calibration of instruments which will be obvious from this third example. Consider a student who tries to measure the length of a rod as shown in Figure 3. The correct way of looking is straight down perpendicular to the scale. If the student is not looking perpendicularly to the scale, then he will make an incorrect reading of the position of the edge of the rod, and thereby overestimating the length of the rod.

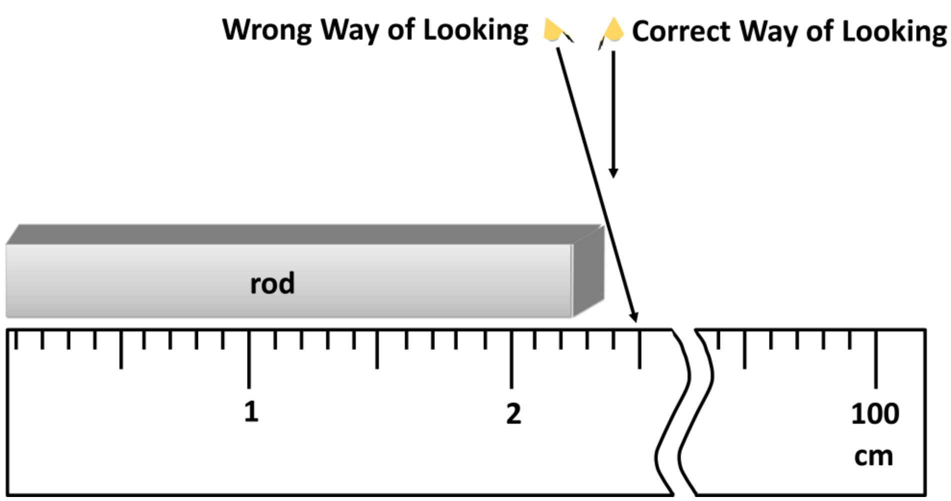


Figure 3: **A comparison between the correct way and wrong way to make a measurement using a ruler**

In principle, systematic errors can be eliminated. But there is no simple way to be sure that you have eliminated all of them in the experiment. One of the characteristics of a good experimenter is that he is able to imagine all possible sources of systematic errors and to do whatever necessary to eliminate them.

4.2.1 Precision and Accuracy

A measurement that is affected by very small random error is said to be *precise*. For example; suppose that two students made two measurements of the voltage across a resistance R using a scale voltmeter. The measurements were $V_1 = 5.443 \pm 0.002\ volts\ and\ V_2 = 5.44 \pm 0.03\ volts$. Since the measurement of student 1 has less random error compared to the random error in the measurement of student 2, the measurement of student 1 is more precise than the measurement of student 2.

Note:

Small Random Errors means High Precision.

If the systematic errors are neglected and the measured value is close to the true value, then the measurement is said to be *accurate*.

Note:

Negligible Systematic Errors means High Accuracy.

Note that a measurement can be **precise but not accurate**. This is due to the presence of systematic errors which shift the measured value from the true value. For example, suppose that a student measured the diameter of a wire using a micrometer to be $D = 2.13 \pm 0.01\ mm$. Suppose that also the student forgot to calibrate the micrometer when making the above measurement, (i.e. for example when the micrometer is closed, it reads 0.02 mm), then our measurement will be larger than its true value by 0.02 mm.

We also note that a measurement can be **accurate but not precise**. For example, A physics student measured the acceleration due to gravity and obtained the result $g = 9.9 \pm 0.2\ m/s^2$. This measurement of g is not very accurate, but it agrees with the accepted value of 9.82 m/sec^2 within the reported uncertainty.

In most experiments in this lab, the true value is known, so that it is possible to know the accuracy of a measurement. Note that the true value is the value accepted by the community of physicists, because it is the value obtained by experienced, skillful, and trust worthy experimenters.

4.2.2 A comparison between measured and accepted values

There is very little point in performing an experiment if one does not draw useful conclusions. Almost all experiments in physics have quantitative conclusions involving a statement of a numerical result. An interesting conclusion usually includes a comparison between a measurement and the accepted value. This comparison cannot be completed without carrying out rigorous error analysis.

Suppose that a student made a measurement of a physical quantity x and reported his results as

$$Result = \bar{x} \pm \Delta x$$

The meaning of the uncertainty Δx is that the true value of x "probably" lies between $\bar{x} - \Delta x$ and $\bar{x} + \Delta x$. It is certainly possible that the correct value lies slightly outside this range. Therefore a measurement can be regarded as satisfactory even if the accepted value lies slightly outside the estimated range of the measured value. However, if the accepted value is well outside the measured range (i.e. discrepancy is more than twice the uncertainty), then there is good reason to think that something went wrong with the experiment.

Example: Two students **A** and **B** measured the speed of sound in air at standard temperature and pressure as follows: $V_A = 338 \pm 2\ m/s\ and\ V_B = 325 \pm 5\ m/s.$ Since the known accepted value at standard temperature and pressure of the speed of sound is $331\ m/s$, then

For student A the discrepancy $D_A = 338 - 331 = 7\ m/s$, Note that $2\Delta V_A = 2 \times 2 = 4\ m/s$, hence $D_A > 2\Delta V_A$.

For student B the discrepancy between measured and accepted values $D_B = 331 - 325 = 6\ m/s$, and $2\Delta V_B = 2 \times 5 = 10\ m/s$, hence $D_B < 2\Delta V_B$.

The result of student A is not accepted, while the result of student B is accepted. Student A has to check his measurements and calculations to find out what went wrong. Unfortunately tracing what went wrong can be a tedious job since there are so many possibilities that might cause the results to be unacceptable. For example the student may have a mistake in taking measurements, carrying out the calculations, estimating the uncertainty or choosing the right accepted value. A discrepancy may indicate undetected source of systematic error. The detection of such systematic error will require careful checking of the calibration of all instruments and detailed review of all procedures. Therefore always check the following when you get a strange result (strange result mean that the range of the measured value does not agree with the range of the accepted value)

- Check the calibration of the instruments you use.
- Find out if the data you wrote down is the same as the data you obtained from the instruments you used.

- Find out if you made any mistakes during the calculation.
- Find out if you are comparing your result with the wrong accepted value.

4.3 Significant Figures

Suppose that you measure the length (L) of a rod using a meter stick with an uncertainty of $\pm 0.1\ cm$, then it would be silly to report the length L as $L = 2.435\ cm$, the "3" and "5" are meaningless because the "4" is uncertain by ± 1, only the "2" and "4" have a meaning. They are said to be *significant figures*. Therefore, the length of the rod should be reported as

$$L = 2.4 \pm 0.1\ cm$$

which has two significant figures.

Now suppose that you made the measurement with a micrometer with an uncertainty of $\pm 0.01\ mm\ or \pm 0.001\ cm$, then the "2", the "4", the "3" and the "5" are all four significant figures and you should report the length of the rod as

$$L = 2.435 \pm 0.001\ cm$$

Note:

The significant figures are all the figures up to and including the figure which is uncertain.

Experimental results must always be rounded such that only significant figures are included. Here are some examples of the correct use of significant figures

No.	Measurement	# of significant figures	Measurement	# of significant figures
1	4.7 ± 0.3	2	0.47 ± 0.03	2
2	473 ± 2	3	4728 ± 3	4
3	472.8 ± 0.3	4	$(4.72 \pm 0.03) \times 10^2$	3
4	472.84 ± 0.03	5	4.728 ± 0.003	4

Here are some examples of the *incorrect* use of significant figures:

$4.7 \pm 0.03, 472.8 \pm 0.03, 0.47 \pm 0.3, 472.82 \pm 0.3, 473 \pm 30$

When significant figures are correctly used, then even if we are not told the uncertainty, we can have a rough idea of how much it is. If we are told that $L = 5.43\ cm$, we know that the uncertainty is a few hundredth of a centimeter, whereas $L = 5.434\ cm$ tells that the uncertainty is a few thousands of a centimeter.

Special attention must be given to zeroes. For example, let $R = 4.7 \pm 0.03$ is incorrect but $R = 4.70 \pm 0.03$ is correct since the zero is significant and must be written. Writing $R = 4.70 \pm 0.3$ is also incorrect since the zero is not significant and should be removed.

4.3.1 Significant Figures in Calculated Values

4.3.1.1 Rounding

- If the first non-significant figure is larger than 5, then round the last significant figure up; that is increase it, as an example

$$4.37\ is\ rounded\ to\ 4.4$$
$$4.368\ is\ rounded\ to\ 4.37$$

- If the first non-significant figure is less than 5, then fix the last significant figure, as an example

$$4.43\ is\ rounded\ to\ 4.4$$
$$4.363\ is\ rounded\ to\ 4.36$$

- If the first non-significant figure equals 5, then it is not clear whether to round up or round down. The usual custom is to round up if the last significant figure is odd and round down if it is even. Thus, as an example

$$4.35\ is\ rounded\ to\ 4.4$$
$$4.45\ is\ rounded\ to\ 4.4$$

In this course, the uncertainty in a result should be rounded to one significant figure. An exception to this rule is that if the leading digit in the uncertainty is a "1" then it may be better to keep two significant figures in the uncertainty. For example, if the uncertainty is ± 0.014, then to round this to 0.01 would be a 40% reduction in the uncertainty, thus it would be better if the uncertainty is left with two significant figures as 0.014.

Note:

Experimental uncertainty should be rounded to one significant figure unless the leading digit in the uncertainty is 1, then it is left with two significant figures.

4.3.1.2 Addition and subtraction

The number with the fewest decimal places limits the number of decimal places in the result. As an example

$$R = 10.3\ cm + 108.76\ cm + 0.0349\ cm = 119.0949\ cm$$

now this result must be rounded, because according to the rule above, the result must contain one decimal place, so$R = 119.1\ cm$.

Note:

When performing long calculations, you should keep at least one extra decimal place in the result. This is to avoid round-off error, which might occur if the numbers are rounded off at each step of calculation. *Round-off error* is defined as the difference between the calculated approximation of a number and its exact mathematical value

4.3.1.3 Multiplication and division

The voltage (V) across unknown resistance and the current (I) through it were measured to be 1.7 $volts$ and 11 mA, respectively. The resistance R can be estimated using Ohms law as follows

$$R = \frac{V}{I} = \frac{1.7\ volts}{0.011\ A} = 154.545\ \Omega$$

If we only know the voltage and the current to two significant figures, we cannot know their ratio to six significant figures as shown above, the ratio should be rounded to two significant figures, that is $R = 1.5 \times 10^2\ \Omega$.

Note:

Results and measurements should always be reported with the correct number of significant figures.

Example: If $x_1 = 1.8, x_2 = 2.346\ and\ x_3 = 7.86$, then $R = x_1 x_2 / x_3$ can be calculated as

$$R = \frac{1.8 \times 2.346}{7.86} = 0.537 \approx 0.54$$

Here, the result is rounded only to two significant figures.

Note also that if you have a result as some function of a measured quantity such as

$$R = \sin(\theta)$$

The number of significant figures that should be kept in the result R must equal the number of significant figures in θ. The same applies for the Cosine function.

For the square root function of a multiplication and division, the number of significant figures in the result will be given by the fewest number of significant figures, see the examples below

- $\sin(12°) = 0.21$
- $\cos(35°) = 0.82$
- $\sqrt{\frac{2.3 \times 4.57}{1.2}} = 3.0$

4.3.2 Significant figures in the uncertainty

Suppose that the mean value of a set of measurements is $L = 6.3234\ cm$ and the standard deviation of the mean is $0.0237\ cm$, from what has been said above you should report your result as

$$L = 6.32 \pm 0.02\ cm$$

As a second example, a student measured the value of g (acceleration due to gravity). He found that

$$g = 9.81\ m/sec^2 \text{ and } \Delta g = 0.0412\ m/\sec^2$$

Then he can write his result as

$$g \pm \Delta g = 9.81 \pm 0.04\ m/\sec^2$$

4.4 Combining Uncertainties

In most experiments several quantities are measured and subsequently combined based on some theory to find the desired result. Since each measurement has inherent uncertainty, the final result will have uncertainty as well. This section discusses the methods adopted in this lab to find the uncertainty in a result using the uncertainty in measured quantities.

4.4.1 Addition and subtraction

Sometimes the desired result is obtained by adding or subtracting measured quantities. Let us consider an example that the mass of an empty bottle is $M_{empty} = 25.7 \pm 0.2\ g$ and the mass of the bottle full of liquid is $M_{full} = 72.3 \pm 0.3\ g$, this means that the first reading is somewhere between $25.5\ g$ and $25.9\ g$ and the second reading is somewhere between $72.0\ g$ and $72.6\ g$. The mass of the liquid is then

$$M_{liquid} = M_{full} - M_{empty} = 72.3 - 25.7 = 46.6\ g$$

but M_{liquid} might be as big as $72.6 - 25.5 = 47.1\ g$, or it might be as small as $72.0 - 25.9 = 46.1\ g$, thus should be reported as $M_{liquid} = 46 \pm 0.5\ g$. We see that the error in measuring the weight of the liquid is $0.5\ g = 0.3 + 0.2\ g$, so that the uncertainty in M_{liquid} is the sum of the uncertainties in the measured quantities M_{bottle} and M_{empty}.

In general if x and y are two measured quantities with uncertainties Δx and Δy, then the uncertainty in $\boldsymbol{R = x \pm y}$ is $\boldsymbol{\Delta R = \Delta x + \Delta y}$

Note that when $R = x - y$, $\Delta R = \Delta x + \Delta y$, since errors always add.

4.4.2 Constant multipliers

Suppose that $\boldsymbol{R = ax + by}$, where a and b are constants, x and y are measured quantities, then we can find ΔR by differentiation with respect to x and y as follows

$$\boldsymbol{\Delta R = a\Delta x + b\Delta y}$$

Example: A physics student measured two lengths A and B in cm: $\mathrm{A} = 57 \pm 2$ cm and $\mathrm{B} = 23 \pm 2$ cm. The student then correctly calculated $\mathrm{R} = A - 2B$ as follows

$R = 57 - 2 \times 23 = 11\ cm$ and $\Delta R = \Delta A + 2\Delta B = 2 + 2 \times 2 = 6\ cm$. So the student reports $R = 11 \pm 6\ cm$.

4.4.3 Multiplication and division

Consider a rectangular plate of width x and y, you would like to measure its area (A) and the error in estimating it, you can do that by measuring x and y, then $\boldsymbol{A = xy}$. We can find the error in A by direct differentiation with respect to x and y. If the uncertainty in measuring x is Δx and the error in measuring y is Δy, then

$$\boldsymbol{\Delta A = y\Delta x + x\Delta y}$$

dividing the latter equation by A yields

$$\frac{\Delta A}{A} = \frac{y\Delta x}{A} + \frac{x\Delta y}{A} = \frac{y\Delta x}{xy} + \frac{x\Delta y}{xy} = \frac{\Delta x}{x} + \frac{\Delta y}{y}$$

Example: The area of a rectangular plate was found by measuring its length and its width. The length was found to be $L = 8.27 \pm 0.05$ m, while the width was found to be $W = 5.12 \pm 0.02$ m. The area of the plate (A) is then equals to

$$A = LW = 8.27 \times 5.12 = 42.3424\, m^2\ and$$

$$\Delta A = L\Delta W + \Delta LW = 8.27 \times 0.02 + 0.05 \times 5.12 = 0.4214\, m^2$$

Therefore the area that should be reported is $A = 42.3 \pm 0.4\, m^2$.

4.4.4 Raising to a power

In general if $\boldsymbol{R = x^n y^m z^l}$, where x, y and z are quantities which you can measure, n, m and l, then

$$\boldsymbol{\Delta R = \left|n x^{n-1} y^m z^l\right| \Delta x + \left|x^n m y^{m-1} z^l\right| \Delta y + \left|x^n y^m l z^{l-1}\right| \Delta z}$$

Dividing the latter equation by R yields

$$\boldsymbol{\frac{\Delta R}{R} = \left|\frac{n}{x}\right| \Delta x + \left|\frac{m}{y}\right| \Delta y + \left|\frac{l}{z}\right| \Delta z}$$

If $n/x, m/y\ and\ l/z$ are positive, then

$$\frac{\Delta R}{R} = n\frac{\Delta x}{x} + m\frac{\Delta y}{y} + l\frac{\Delta z}{z}$$

Example: if $R = x^2$, then $\Delta R = 2x\Delta x\ or\ \frac{\Delta R}{R} = 2\frac{\Delta x}{x}$

Example: A student measured the following physical quantities: $\mathrm{X} \pm \Delta\mathrm{X}, \mathrm{Y} \pm \Delta\mathrm{Y}$ and $\mathrm{Z} \pm \Delta\mathrm{Z}$. If he wants to calculate $\mathrm{R} = \mathrm{Z}^2\mathrm{Y}^3/\mathrm{X}^4$, then $\Delta\mathrm{R}$ should be calculated using the following formula

$$\frac{\Delta R}{R} = 2\frac{\Delta \mathrm{Z}}{\mathrm{Z}} + 3\frac{\Delta \mathrm{Y}}{\mathrm{Y}} + 4\frac{\Delta \mathrm{X}}{\mathrm{X}}$$

4.4.5 Other functions

If the results depend on other functions, then the uncertainty can be estimated as follows:

a) Trigonometric functions such as Sine, Cosine and tan.

$$If\ R = \sin(x), then\ \Delta R = |\cos(x)|\,\Delta x$$

$$If\ R = \cos(x), then\ \Delta R = |\sin(x)|\,\Delta x$$

$$If\ R = \tan(x), then\ \Delta R = |\sec^2(x)|\,\Delta x$$

$$If\ R = \cot(x), then\ \Delta R = |\csc^2(x)|\,\Delta x$$

$$If\ R = \sec(x), then\ \Delta R = |\sec(x)\tan(x)|\,\Delta x$$

Note:

If the angle is measured in degrees, then it should be converted to radians by multiplying it by $\frac{\pi}{180}$ before using the above formulas.

Example: A student uses a protractor to measure an angle to be $\theta = 80° \pm 1°$. What should the student report for $\sin\theta$?

If $R = \sin\theta$ and $\theta = 80° \pm 1$, then $R = \sin(80°) = 0.984810775$ and $\Delta R = \cos(\theta)\left(\frac{\pi}{180}\Delta\theta\right) = \cos(80°)\left(\frac{\pi}{180}\times 1\right) = 0.0030307324$.

Therefore the student should report $\sin\theta = 0.985 \pm 0.003$

b) Natural logarithm

$$If\ R = \ln(x)\,, then\ \Delta R = \frac{\Delta x}{x}$$

c) Exponential function

$$If\ R = e^x, then\ \Delta R = e^x\Delta x$$

4.4.6 General rule

If a result R is written as a function of measured quantities x,y and z that is $R = R(x,y,z)$, then one can find the uncertainty in the result as

$$\Delta R = \left|\frac{\partial R}{\partial x}\right|\Delta x + \left|\frac{\partial R}{\partial y}\right|\Delta y + \left|\frac{\partial R}{\partial z}\right|\Delta z$$

Example: If $R(x,y,z) = x^2y^3\sin(x+z)\,, then$

$$\Delta R = |2xy^3\sin(x+z) + x^2y^3\cos(x+z)|\Delta x + |3x^2y^2\sin(x+z)|\Delta y + |x^2y^3\cos(x+z)|\Delta z$$

4.5 Exercises

Exercise 4.1: Determine the number of significant figures in the following quantities:

$$17.87\ m, 0.4730\ km, 17.9\ sec, 0.473\ kg, 18\ ns, 0.47\Omega, 1.34 \times 10^2 \frac{m}{s},$$

$2.567 \times 10^5 cm, 2.0 \times 10^{10}\ J, 1.001\ sec, 1.000\ sec, 1\ oz, 1000\ litres, 1001\ cm^2.$

Exercise 4.2: (a) Add $231.3\ to\ 5.8 \times\ 10^2$

(b) Multiply $24.7\ and\ 2.7 \times\ 10^4$

(c) Subtract 141.65 from 217.0

Exercise 4.3: A student obtained the following measurements for the height of a room ceiling, $3.13, 3.22, 3.32, 3.21\ and\ 3.17\ m$. What is the best estimate for the height of the room ceiling?

Exercise 4.4: Seven students measured the length of a rod to be as shown below:

Student No.	1	2	3	4	5	6	7
Length (cm)	87.5	87.3	87.4	87.6	87.8	87.2	87.1

(a) What is the best estimate for the length of the rod?

(b) What is the sample standard deviation in the length of the rod?

(c) What will be the uncertainty in the length of the rod?

Exercise 4.5: A group of students measured the acceleration due to gravity, g, with a pendulum and obtained the following values in $\mathrm{m/s^2}$:

$9.87, 9.75, 9.63, 9.80, 9.72, 9.73, 9.78, 9.67, 9.81$ and 9.63.

(a) What is the sample standard deviation of the measurements?

(b) What is the best estimate of g?

(c) Is the result students obtained is accepted?

Exercise 4.6: Rewrite the following measurements in their clearest forms with a suitable number of significant figures:

a) Measured mass of $3.565 \pm 0.035\ g$.

b) Measured height of $4.02 \pm 0.02315\ meter$.

c) Measured time of $1.75143 \pm 1\ sec$.

d) Measured charge of $-9.51 \times 10^{-19} \pm 1.6 \times 10^{-19}$ *coulombs*.

e) Measured wavelength of $0.000000432 \pm 0.00000007\ meters$.

f) Measured momentum of $2.362 \times 10^3 \pm 21\ g \cdot cm/sec$.

Exercise 4.7: Determine if the following results of different experiments agree with the accepted result listed to the right

(a) g = 10.4 ± 1.1 m/s2 g = 9.8 m/s2

(b) τ = 2.5 ± 0.2 sec τ = 3.1 sec

(c) k = 1368 ± 45 N/m k = 1300 ± 50 N/m

Exercise 4.8: The area of a rectangular plate was found by measuring its length and its width. The length was found to be $8.27 \pm 0.05\ cm$. The width was found to be $5.12 \pm 0.02\ cm$. What is the area of the plate?

Exercise 4.9: A student obtained the following measurements

$a = 4 \pm 1\ cm, b = 15 \pm 2\ cm, c = 14 \pm 1\ cm, t = 4.1 \pm 0.5\ sec\ and\ m = 37 \pm 1\ g$

Compute the following quantities and their uncertainties:

(a) $a + \mathrm{b} - \mathrm{c}$

(b) ct^2

(c) $\frac{\mathrm{mb}}{\mathrm{t}}$

(d) $\ln(\frac{\mathrm{c}}{\mathrm{a}})$

(e) $\exp(\frac{\mathrm{b}}{\mathrm{ac}})$

Exercise 4.10: If y can be evaluated using the following formula $y = v_i t + \frac{1}{2} g t^2$ and the quantities $v_i, g\ and\ t$ are measured to be $v_i = 2.4 \pm 0.2\, m/s,\ g = 9.80 \pm 0.02\ m/s^2\ and\ t = 3.45 \pm 0.05\ s$, then

(a) Find $y \pm \Delta y$

(b) Of the three given quantities v_i, g and t which has the most important effect on y's uncertainty? Which has the least effect on y's uncertainty?

Exercise 4.11: In order to know how one can find the random errors, do the following experiment: **Random errors in the coefficient of restitution experiment**

In this experiment you will learn how to find random errors in the coefficient of restitution experiment. You will allow a tennis ball to fall from a height $h_1$on a table; the ball will bounce back to a height h_2 which will be recorded 40 times. You will find the average of these measurements, then find the coefficient of restitution defined· by

$$r = \frac{v_2}{v_1}$$

Where $v_1$is the speed of the ball just before collision with the table surface and v_2 is the speed of the ball just after collision with the table surface.

Since $v_1^2 = 2gh_1$ and $v_2^2 = 2g\bar{h}_2$, we obtain

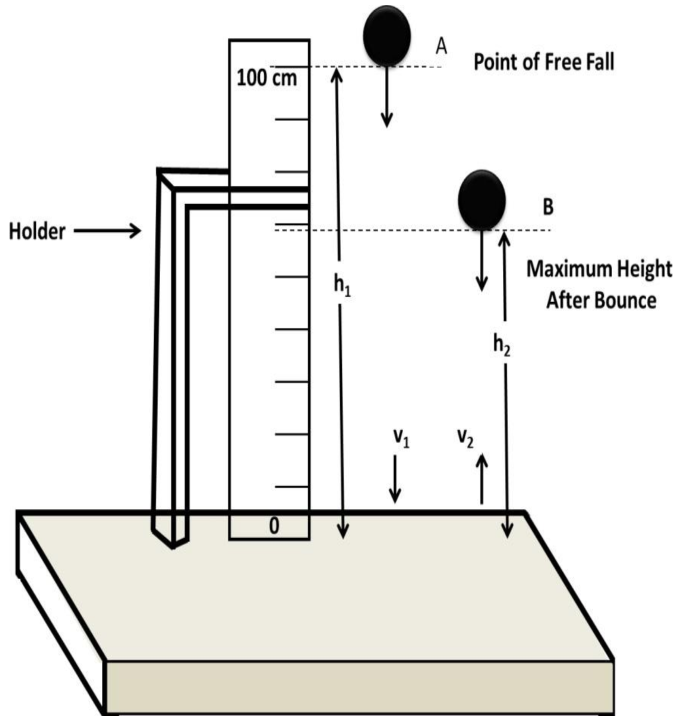
$$r = \frac{\sqrt{2g\bar{h}_2}}{\sqrt{2gh_1}} = \sqrt{\frac{\bar{h}_2}{h_1}}$$

Using the above equation, we can estimate the error in r (Δr) by direct differentiation

$$\Delta r = \frac{\Delta \bar{h}_2}{2\sqrt{\bar{h}_2 h_1}} + \frac{\Delta h_1}{2}\sqrt{\frac{\bar{h}_2}{h_1^3}}\ or\ \frac{\Delta r}{r} = \frac{1}{2}\left\{\frac{\Delta h_1}{h_1} + \frac{\Delta \bar{h}_2}{\bar{h}_2}\right\}$$

Procedure

1. Obtain at tennis ball, a meter stick with a holder.
2. Construct the setup shown below



3. Drop the ball from a height $h_1 = 100cm$.
4. Measure the maximum height the ball will reach after collision with the table surface.
5. Repeat step 4 forty times and write down your measurements in the table

$h_2\ (cm)$									

CALCULATIONS

1. Estimate the value of Δh_1.
2. Find the average value of h_2 and its sample standard deviation. (see appendix C to know how you can use the calculator to do this).
3. Find the uncertainty in the mean value of h_2
4. Find the coefficient of restitution (r).
5. Find the uncertainty in r.

5.GRAPHS

5.1 Uses of Graphs

In experimental physics, data are often displayed in graphs. There are several ways in which graphs are usually useful:

1. A graph may show more clearly than a table of data the relationship between variables, for example the relation may be linear, or non-linear which may not be obvious from the data table.

2. A graph may be used to test how well a theory corresponds to experimental facts and provide a comparison between experiment and theory.

3. Measurements which are represented on a graph can provide the value of a desired quantity. For example, in the sample lab report the value of the acceleration of gravity (g) was found by measuring the slope of the graph.

5.2 Plotting Graphs

The results of experiments are strongly dependent on how well graphs are plotted. The following instructions are recommended to produce good graphs

1. Draw the x-axis line and the y-axis line one at the bottom edge and the other at the left edge of the graph paper.

2. Label the x-axis and the y-axis with units, for example if you plot the displacement (D) of a moving object versus time ; that is the displacement (D) on the y-axis and the time (t) on the x-axis, you have to label the x-axis as "Time (sec)" and the y-axis as "Displacement (meter)".

3. Choose and use an easy scale for the axis. Choose a minimum and a maximum number of the scale such that it is easily divided.

4. Write numbers only on the main divisions of the scale. If the numbers on the scale are very large or very small, the graph will look neater if a power-of-10 is used, for example if the x-axis measurements are : 121, 133,152, 164, 171 and the y-axis measurements are : 0.012, 0.021, 0.033, 0.041, 0.052 then you can draw the points

(1.21,1.2), (1.33,2.1), (1.52,3.3), (1.64,4.1), (1.71,5.2) on the graph paper and multiply the x-axis by 10^2 and the y-axis by 10^{-2}.

5. Make graphs as large as possible. Fill the whole page.
6. Plot your points on the graph with using circles that are distinct from any other points on your graphs for examples ●.
7. Show the centroid {the point $(\bar{x}, \bar{y})$} on the graph paper by the symbol ●.
8. Give a title to the graph, since the reader should know what does the graph describe?

5.3 Linear graphs

The best straight line is the line which passes as close as possible to as many as of the measured points. Drawing the best line is usually a matter of judgment unless it is found using the least square fit method. The best line should have about the same number of points above it as below it. To determine the best line, it is necessary to look at all the points simultaneously. A plastic ruler can be used to draw the line in order to see all points at one time.

In addition to the points which represent the data, there is an additional point to help drawing the best line. It is the *centroid*. The centroid coordinates are the average values of the measured points, that is:

$$centroid = (\bar{x}, \bar{y})$$

Now the best straight line should pass through the centroid which is represented in the Figure 4 by a solid circle ● to differentiate it from experimental points.

Example: The speed of a moving object in one dimension as function of time was measured and tabulated below.

Time (sec)	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0
V (cm/s)	7.0	9.5	10.5	13	15.5	16.5	19	21

The points along with best straight line as shown below

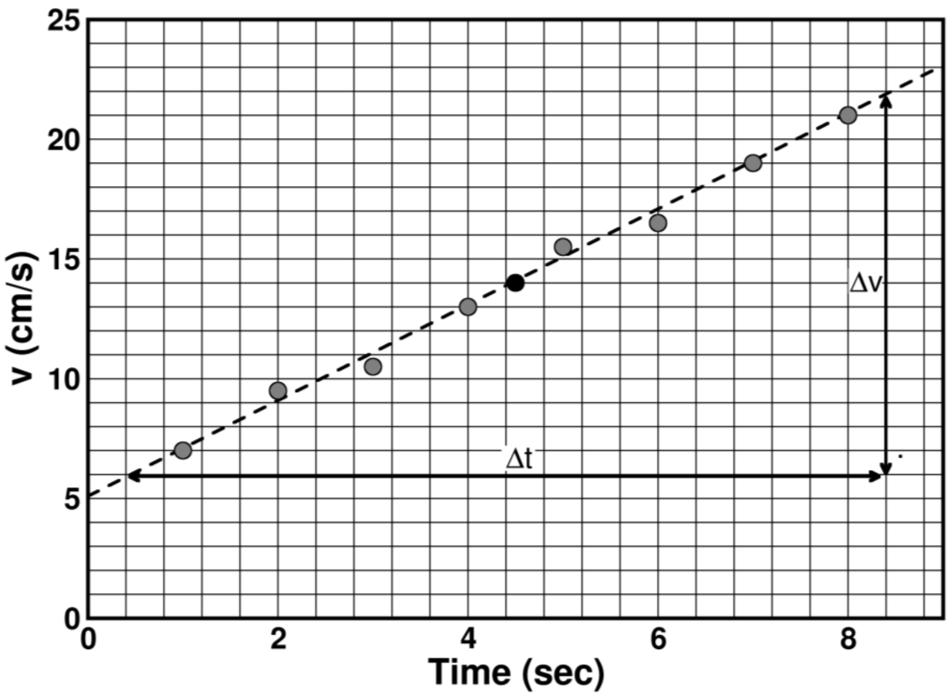


Figure 4: **Velocity of a particle measured as a function of time.**

All the information about the movement of the object are summarized in Figure 4. You know that if an object is moving under the influence of constant acceleration is represented by the equation:

$$v = v_0 + at$$

where a is the acceleration of the object $v_0$is its initial speed, t is the time. The latter equation is similar to the straight line equation $y = mx + b$, where $y = v$ and $x = t$, the $slope = m = acceleration$ (a), the y-intercept is $v_0 = b$, (the initial speed), so from the graph we can find the initial speed of the object and its acceleration.

$$slope = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{22 - 6}{8.4 - 0.4} = \frac{16}{8} = 2.0\ cm/\sec^2$$

From the graph also, the y-intercept =$v0 = b = 5.1\ cm/\sec^2$.

5.4 Working with Exponential Functions

There are many phenomena in nature which are described by exponential functions, for example the number of bacteria in a biologist's culture dish increases exponentially with time, the pressure of the earth atmosphere decreases exponentially with height above the earth's surface, the intensity of the x-rays transmitted through a conductor decreases exponentially with its thickness and the decay of a given radio-active element decreases exponentially with time.

5.4.1 Exponential growth functions

Figure 5 is a plot of an exponential growth function $y = y_0 e^{+kx}$, where y_0 is the value of y at $x = 0$, and k is a constant of units cm^{-1}.

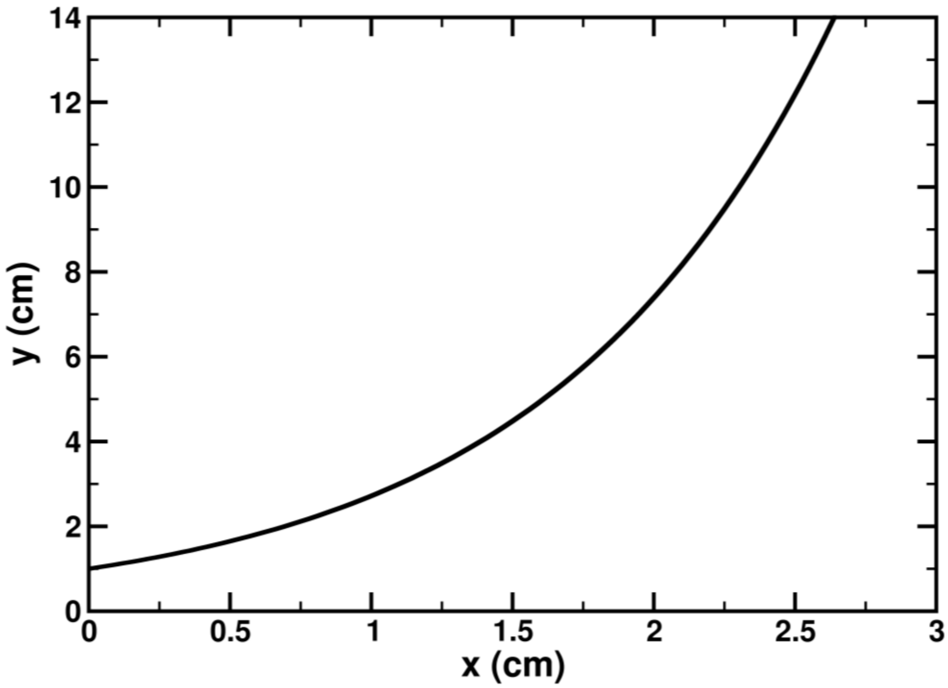


Figure 5: **Exponential growth,** $y = y_0 \exp(kx)$. $y_0 = 1$ *and* $k = 1$.

5.4.2 Exponential decay functions

Figure 6 is a plot of an exponential decay function, $y = y_0 e^{-kx}$. Where y_0 is the value of y at $x = 0$, and k is called *the decay constant*. In this case the units of k is cm^{-1}. Note also that the exponent kx is a pure number without units. You can see from the graph of the decay function that when $x = \infty$, y goes to zero.

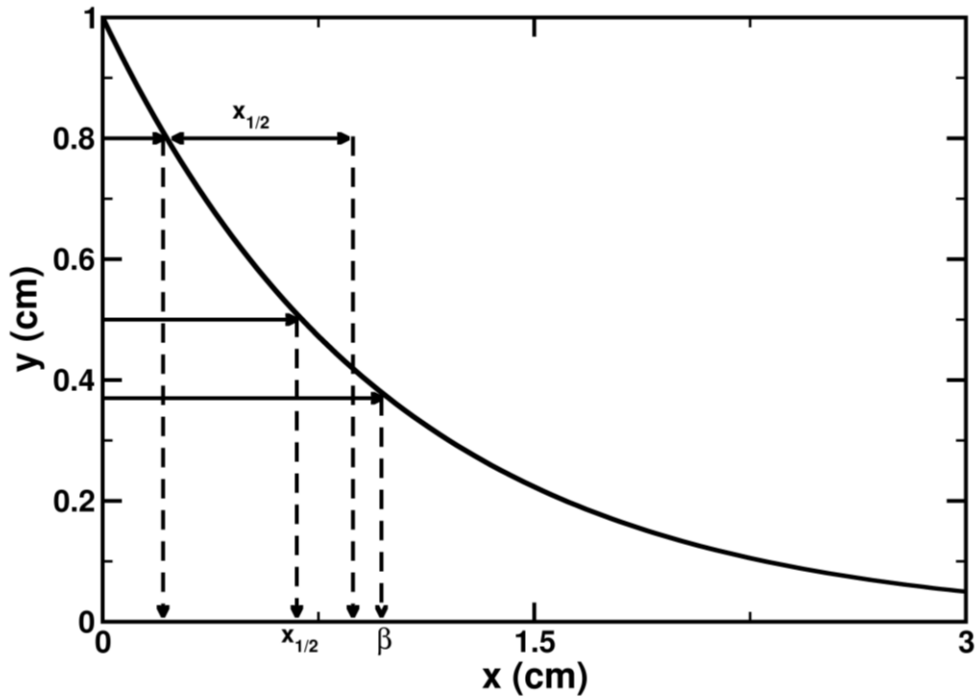


Figure 6: **Exponential decay.** $\boldsymbol{y = y_0 \exp(-kx)}$. $\boldsymbol{y_0 = 1, k = -1}$.

To describe how rapidly the exponential decay function approaches zero, it is the custom to specify one or the other of the two parameters $\beta$and $x_{1/2}$ described below

- The value of x at which the exponential decay function has decreased to 0.37 of its initial value is called β, this occurs when $kx = 1$, means that $k\beta = 1$ or $\beta = 1/k$, in our example the initial value of the exponential decay function is 1 cm. Notice that $y = 0.37\ cm$ when $kx = 1$, that is $y = 1cm\ e^{-1} = 0.37cm$.
- The value of x at which y has decreased to ½ of its initial value is called $x_{1/2}$ (*x-half*). Notice that one can find $x_{1/2}$ in terms of the decay constant k.

$$y = y_0 e^{-kx}$$

When $y = y_0/2$, $x = x_{1/2}$, so

$$\frac{y_0}{2} = y_0 e^{-kx_{1/2}}\ or\ \frac{1}{2} = e^{-kx_{1/2}} \rightarrow \ln\left(\frac{1}{2}\right) = \ln(e^{-kx_{1/2}})$$

which can be solved for $x_{1/2}$ as follows

$$-\ln(2) = -kx_{1/2} \text{ or } x_{1/2} = \frac{\ln(2)}{k} = \frac{0.693}{k}$$

Since $\beta = \frac{1}{k}$ implies that $x_{1/2} = \beta \ln(2)$. Notice in Figure 6 how $x_{1/2}$. and β are found.

5.5 Drawing Exponential graphs

The following data satisfies the relation $y = y_0 e^{-kx}$ and we are interested in finding the decay constant k, and $x_{1/2}$

$x\ (cm)$	$y(cm)$	$x(cm)$	$y(cm)$
0	100	1.0	37
0.1	90	1.5	22
0.2	82	3.0	5
0.3	74	4.0	1.8
0.4	67	5.0	0.71
0.5	61	6.0	0.25
0.6	55	7.0	0.11

There are more than one way to find the constants k, β and $x_{1/2}$ which are listed below:

(a) By plotting a graph of y vs. x using a linear graph paper as shown in Figure 5, the value of β can be found directly from the graph. Recall β is the value of x at which $y = 0.37\, y_0$ which occurs at $x = 1\ cm$. Therefore $\beta = 1\ cm$, and $k = 1/\beta = 1\ cm^{-1}$. $x_{1/2}$ can be found either from the formula derived earlier ($x_{1/2} = \beta \ln(2) = 0.693$) or directly from the graph, which is about $0.68\ cm$.

(b) By plotting a graph of y vs. x on a semi-log graph paper. In a semi-log graph paper, one axis has an ordinary scale while the other axis has a log scale in which the spacing between the divisions is proportional to log y. This is of great convenience because it makes it unnecessary to look up in a table of $\log y$. The graph paper automatically assigns the points at positions corresponding to $\log y$. Figure 6 is a plot of the same data as in Figure 5 but using a semi-log graph paper. In this case the y-axis will be a log scale while the x-axis is a linear scale.

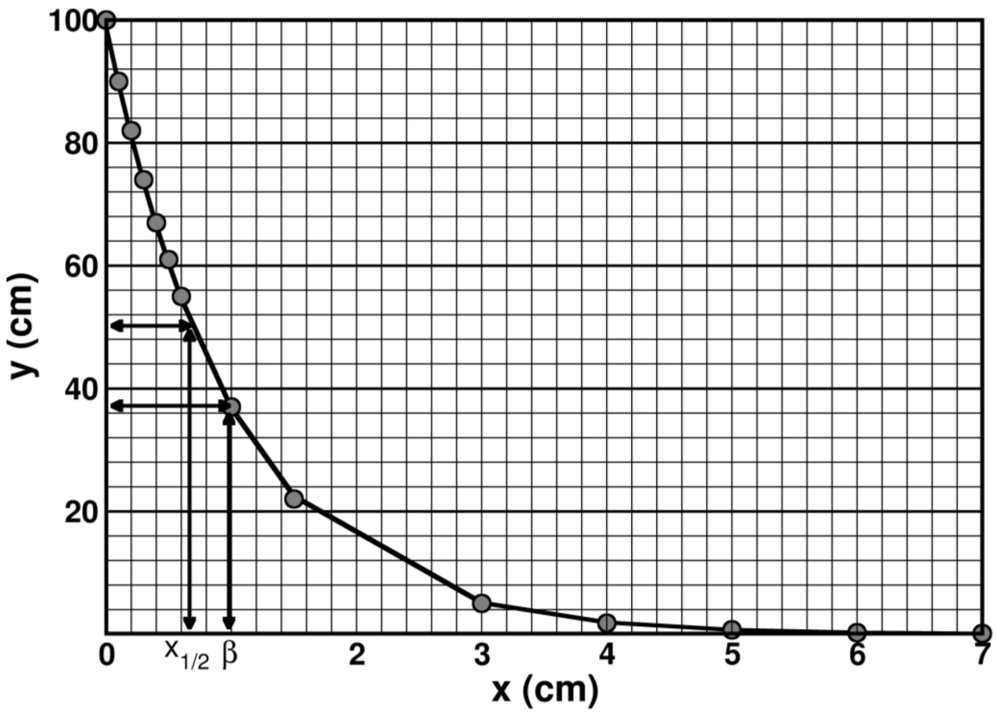


Figure 7: Data from a process that exhibits an exponential decay plotted on a linear graph paper.

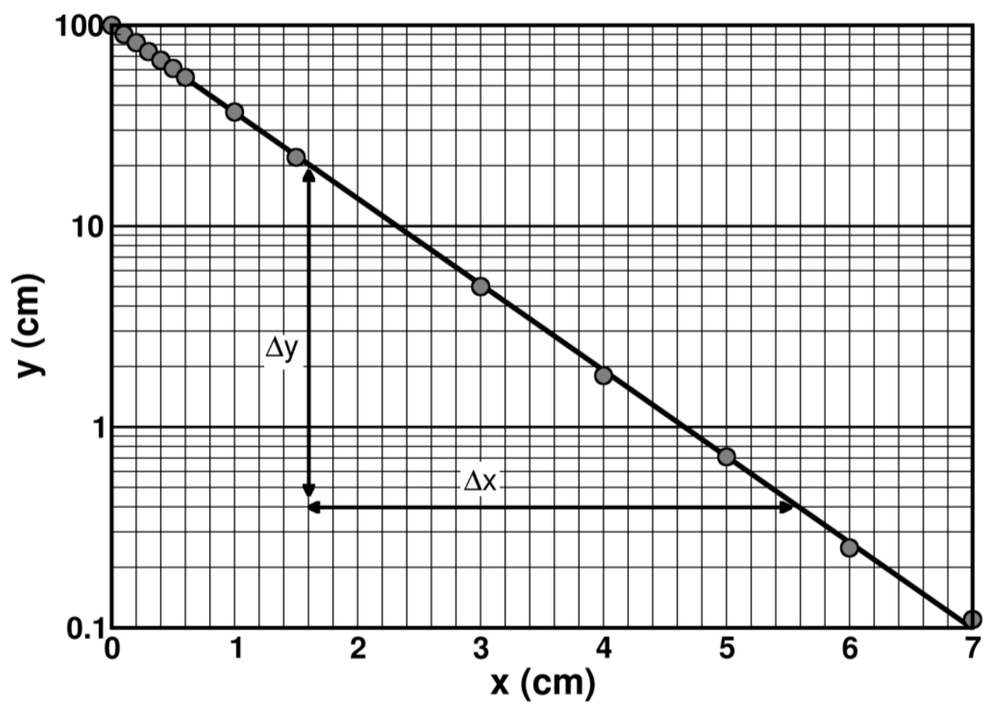


Figure 8:Data from a process that exhibits an exponential decay plotted on a semi-log graph paper.

The value of k, β and $x_{1/2}$ can be obtained from the semi-log graph paper shown above. Taking the natural logarithm of both sides of the equation $y = y_0 e^{-kx}$ yields

$\ln(y) = \ln(y_0) - kx$. The latter equation will be a linear equation if plotted in $\ln(y)$ $vs. x$ graph. The y-intercept will be $\ln(y0)$ and the slope will equal to $-k$. This means that the data obtained from an exponential decay process will be linear on a semi-log graph paper because as we said earlier, the y-scale in the semi-log graph automatically plots the points corresponding to $\log y$. The slope of the line in Figure 6 is

$$slope = \frac{\ln(y_2) - \ln(y_1)}{x_2 - x_1} = -k$$

Subsequently, $\beta = \frac{1}{k}$, and $x_{1/2} = \beta \ln(2)$.

Exercise 5.1: Put the following relations into a straight line formula to find the constants a, b and n.(a) $t = aL^n$ where t and L are variables. (b) $b = \frac{uv}{u+v}$ where u, v are variables. (c) $e = a/t$ where e, t are variables.

Experiment 1: Density of a Metal and Distance between its Atoms

BACKGROUND AND THEORY

Mass density of a material at room temperature provides a possible way to identify the composition of the material. For a piece of material, the density, ρ, can be determined using

$$\rho = \frac{M}{V} \tag{1.1}$$

where M is the mass of the material, and V is its volume.

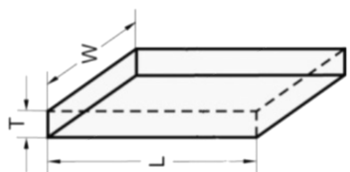


Figure 1. 1: Rectangular Piece of Material.

If the piece of material has rectangular shape of thickness T, width W and length L, then its volume is given by

$$V = L \times W \times T \tag{1.2}$$

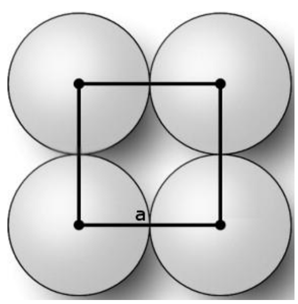


Figure 1. 2: Simple Cubic Lattice.

In a metal, the atoms are almost spherical and identical. They are usually organized in a specific lattice structure. The conceptually simplest lattice structure is the simple cubic crystal in which atoms are located on a grid (see Figure 1.2).

Let us now find an approximate expression for the distance between the neighboring atoms of the material. If the mass of the piece of material is M and its volume is V, then one can estimate the total number of atoms (N) in that piece of material as follows

$$N = nN_A = \frac{M}{A_W} N_A \tag{1.3}$$

where n is the number of moles, A_W is the atomic mass of the material and N_A. is Avogadro's number $6.023 \times 10^{23}\ atom/mole$. If we also assume that each atom is contained inside a box of edge length a and volume a^3, then another estimate of the total number of atoms from

$$N = \frac{M}{\rho a^3} \tag{1.4}$$

From equations (1.3) and (1.4) an expression for the spacing between neighboring atoms in a metal can be obtained

$$a = \sqrt[3]{\frac{A_W}{\rho N_A}} \tag{1.5}$$

Uncertainty in Density

The uncertainty in ρ can be computed as follows

$$\Delta\rho = \frac{\Delta M}{V} + \frac{M}{V^2}\Delta V \tag{1.6}$$

which can be rewritten as

$$\frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} + \frac{\Delta V}{V} \tag{1.7}$$

The uncertainty in the mass can be estimated from the balance scale used to measure it. As for the uncertainty in the volume it can be obtained from

$$\Delta V = WT\Delta L + LT\Delta W + LW\Delta T \tag{1.8}$$

which can be expressed as

$$\frac{\Delta V}{V} = \frac{\Delta L}{L} + \frac{\Delta W}{W} + \frac{\Delta \mathrm{T}}{\mathrm{T}} \tag{1.9}$$

APPARATUS

Metal block, Vernier Caliper, Micrometer and balance scale.

PROCEDURE

1) Obtain one of the small metal blocks which are provided, a caliper and a micrometer.
2) Measure the length L and the width W of the block using the vernier caliper. Repeat the measurements six times from different places.
3) Measure the thickness T of block with the micrometer. Repeat the measurements from different places six times.
   - ✓ *Note that your measurements may be in mm units, divide by 10, so that the units of the measurements are in cm.*
4) Use a balance scale to measure the mass of the block, write down your measurements in table below

Block #:

$M = \quad \pm \quad g$							Average (cm)
Length (cm)							
Width (cm)							
Thickness (cm)							

CALCULATIONS

1) Calculate the mean values $\bar{L}, \bar{W}$ *and* $\bar{T}$ and their standard deviation of the mean $\sigma_m(L),\ \sigma_m(W)$ *and* $\sigma_m(T)$.

   - ✓ *Note that* $\Delta L = \sigma_m(L), \Delta W = \sigma_m(W)$ *and* $\Delta T = \sigma_m(T)$.

2) Calculate the best volume $V = \bar{L} \times \bar{W} \times \bar{T}$ in units of cm^3.
3) Calculate the uncertainty in V from equation (1.8) and express your results as $V + \Delta V$
4) Estimate the uncertainty in your measurement of the mass of the block (ΔM).
5) Calculate the density of the block from $\rho = M/V$.
6) Calculate the uncertainty in the density ρ, and write your result as $\rho \pm \Delta\rho$.
7) See the table in appendix A to identify the material of the block.

8) From your calculated value of the density ρ, and using appendix B to find the atomic weight, estimate the atomic spacing (distance a). Express you results in Angstroms, where $1\ \text{Å} = 10^{-10} m$.

Note:

It is not meaningful to calculate the uncertainty in the atomic spacing a, because we are not sure that the atoms are arranged in a cubic lattice as we assumed. The value of the atomic spacing a is probably not precise to more than two significant figures.

QUESTIONS

1) Why did you repeat the measurements in L, W and T in different locations?
2) Do you expect systematic errors to affect standard deviation?
3) What measurements should be improved to reduce the uncertainty in ρ most effectively?

Note:

You can always use the answers to the questions above to enrich the results and conclusion part of your lab report.

Experiment 2: Conservation of Linear Momentum

BACKGROUND AND THEORY

The linear momentum of an object of mass m and velocity $\vec{v}$ is defined by

$$\vec{p} = m\vec{v} \qquad (2.1)$$

Consider an isolated system consisting of N objects, if the *i-th* object is moving with a velocity $\vec{v}_i$, then the total momentum of the system is given by

$$\vec{P} = \sum_{i=1}^{N} m_i \, \vec{v}_{\mathrm{i}} \qquad (2.2)$$

This system of N objects is said to be *isolated* if no external resultant force acts on the system. The law of conservation of linear momentum states that *"the linear momentum of an isolated system is conserved"*, that is, it does not change.

In this experiment you will test the law of conservation of linear momentum in a collision between two balls which are constrained to move in one dimension, so there is no need to use vector notation.

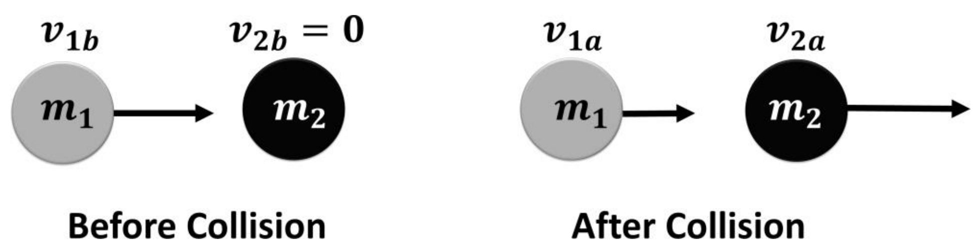


Figure 2. 1: Schematic representation of collision between two balls.

The collision between the two balls is depicted in the above Figure, the mass of ball No. 1 is m_1 while the mass of ball 2 is m_2, if before collision the speed of ball No. 1 is v_{1b},while the speed of ball No. 2 is zero, that is $v_{2b} = 0$, and if after collision the speed of ball No. 1 is v_{1a} and the speed of ball No. 2 is v_{2a}. Let us first define the ratio R as

$$R = P_a/P_b \qquad (2.3)$$

where

$$P_b = m_1 v_{1b} \text{ and } P_a = m_1 v_{1a} + m_2 v_{2a} \qquad (2.4)$$

Substituting the latter equations in equation 2.3 yields

$$R = \frac{m_1 v_{1a} + m_2 v_{2a}}{m_1 v_{1b}} \tag{2.5}$$

The ratio R should be theoretically equal to 1. Our objective is to verify this experimentally.

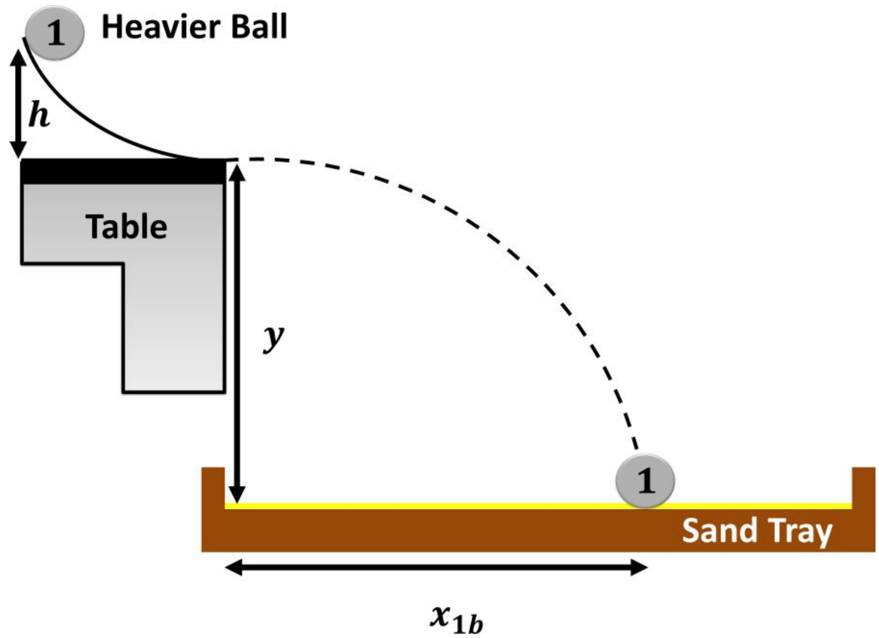


Figure 2. 2: Ball 1 range measurements before collision.

In the first part of this experiment as depicted in Figure 2.2, the heavier ball (ball No. 1) rolls down along a curved track from a height h above a table, then it follows a parabolic trajectory and falls inside a tray full of sand. The ball hits the sand at a horizontal distance X_{1b} from the place where it leaves the track. The vertical distance from the point of collision to the tray is $y = \frac{1}{2}gt^2$, where g is the acceleration due to gravity and t is the time of flight of ball No. 1.

From the last equation one can find the time of flight:

$$t = \sqrt{\frac{2y}{g}} \tag{2.6}$$

The horizontal speed of ball No.1 is given by

$$v_{1b} = \frac{x_{1b}}{t} = \frac{x_{1b}}{\sqrt{\frac{2y}{g}}} \rightarrow P_b = \frac{m_1 x_{1b}}{\sqrt{\frac{2y}{g}}} \tag{2.7}$$

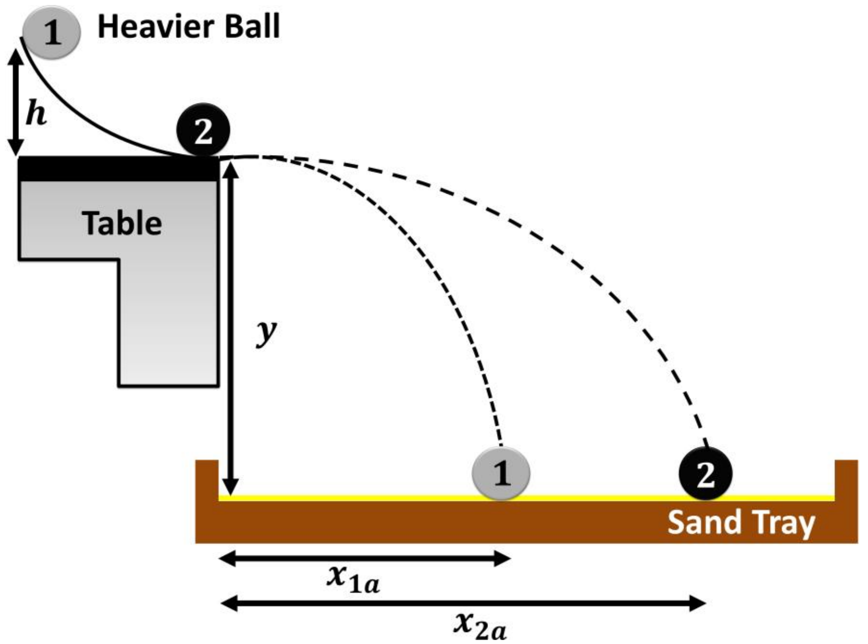


Figure 2. 3:Both balls range measurements after collision.

Let us consider the collision between the two balls. Figure 2.3 illustrates that ball No. 1 rolls down the curved track from the same height h, then it hits ball No.2 which is initially at rest at the end of the track. The two balls subsequently fall down inside the tray at the end of their parabolic trajectories hitting it at horizontal distances X_{1a} and X_{2a} from the point of collision.

Since the two balls fall down the same distance y and their initial y-component of the velocity is zero for both balls at the point of collision, then the time of flights of both balls are the same and are given by equation 2.6. The horizontal speeds and momenta of the two balls are constant after collision and are given by

$$v_{1a} = \frac{x_{1a}}{t} = \frac{x_{1a}}{\sqrt{\frac{2y}{g}}} \rightarrow P_{1a} = \frac{m_1 x_{1a}}{\sqrt{\frac{2y}{g}}} \quad (2.8)$$

and

$$v_{2a} = \frac{x_{2a}}{t} = \frac{x_{2a}}{\sqrt{\frac{2y}{g}}} \rightarrow P_{2a} = \frac{m_2 x_{2a}}{\sqrt{\frac{2y}{g}}} \quad (2.9)$$

Substituting equations 2.7, 2.8 and 2.9 into equation 2.5 yields

$$R = \frac{m_1 x_{1a} + m_2 x_{2a}}{m_1 x_{1b}} \quad (2.10)$$

In this experiment you will measure x_{1b}, x_{1a} and x_{2a} many times, from which we determine the ratio R. The uncertainty in R can be determined as follows:

We start with

$$R = \frac{m_1\bar{x}_{1a} + m_2\bar{x}_{2a}}{m_1\bar{x}_{1b}} = \frac{\mathrm{A}}{\mathrm{B}} \tag{2.11}$$

Clearly,

$$\frac{\Delta R}{R} = \frac{\Delta A}{A} + \frac{\Delta \mathrm{B}}{\mathrm{B}} \tag{2.12}$$

Where

$$\Delta A = m_1\Delta x_{1a} + x_{1a}\Delta m_1 + m_2\Delta x_{2a} + x_{2a}\Delta m_2 \tag{2.13}$$

and

$$\Delta B = m_1\Delta x_{1b} + x_{1b}\Delta m_1 \tag{2.14}$$

APPARATUS

Curved track with support stand, Two different balls, Meter stick, Wooden tray, sand.

PROCEDURE

1) Adjust the track such that the end is at the edge of the table; do not bend it.
2) Use a balance to determine m_1 and m_2. Estimate the uncertainty in m_1 and m_2.
3) Use a plumb-bob to locate the point on the tray directly below the point of collision.
4) Try several practices to determine a good starting point for ball No. 1 (the heavier ball) so that the ball will fall inside the tray.
5) Make 7 measurements of x_{1b}.
6) Make 7 measurements of x_{1a} and x_{2a}.
7) Write down you measurements in the table shown below

$m_1 =$ $\pm$ g					$m_2 =$ $\pm$ g			Average
$x_{1b}(cm)$								
$x_{1a}(cm)$								
$x_{2a}(cm)$								

CALCULATIONS

1) Calculate the mean value and the standard deviation in the mean of x_{1b}, x_{1a} and x_{2a}; that is ($\bar{x}_{1b}, \bar{x}_{1a}, \bar{x}_{2a}, \Delta\bar{x}_{1b}, \Delta\bar{x}_{1a}$ *and* $\Delta\bar{x}_{2a}$).
2) Calculate R and its uncertainty ΔR.

QUESTIONS

(Answer these questions implicitly in the results and conclusion section of your experiment report).

1) Does your experimental value agree with theory with experimental error?
2) If the lower end of the track is not horizontal, how would this affect your result?
3) List any sources of systematic errors (if any), and write how they affect your result.

Experiment 3: Density of Liquids

BACKGROUND AND THEORY

A fluid is a material which flows such as gases and liquids. Fluids exert forces on the walls of their containers or on any surface they touch. If the fluid is at rest, then the force that it exerts on a surface is perpendicular to the surface. The pressure of the fluid on a surface is defined as the force exerted by the fluid per unit area

$$Pressure = \frac{Force}{Area} \; or \; P = \frac{F}{A} \tag{3.1}$$

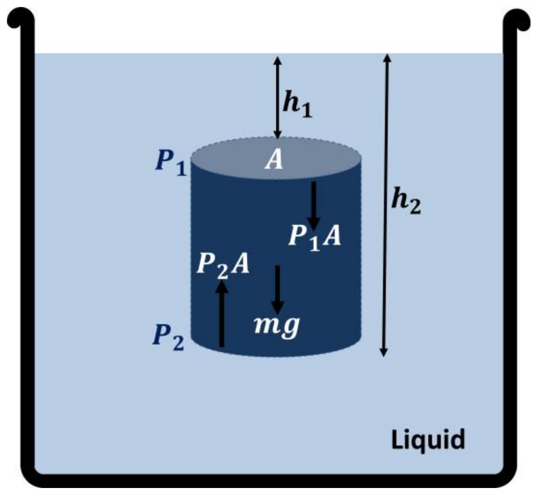


Figure 3. 1: Forces acting on a fluid imaginary volume element.

The pressure may be different at different points in a fluid; in particular, the pressure is larger at points deeper below the surface of the liquid. Figure 3.1 shows a container filled with a liquid of density ρ. Consider a portion of the liquid in the form of a cylinder with area A and height $h_2 - h_1$. The pressure at the top surface is P_1, while on the bottom surface is P_2. Note that there are three forces acting on this portion of the liquid; the liquid above pushes down with force P_1A, the liquid below pushes the portion up with a force P_2A, and the weight of the cylindrical portion acts down with force equals mg.

Since the cylindrical portion of liquid is in static equilibrium which means that the net force acting on it is zero;

$$P_2A - mg - P_1A = 0 \tag{3.2}$$

But since $mass = density \times volume \; or \; m = \rho V = \rho A(h_2 - h_1)$

equation 3.2 can be rewritten as

$$P_2 - P_1 = \rho g(h_2 - h_1) \tag{3.3}$$

This shows that the difference in pressure depends only on the difference in vertical height.

In this experiment you will use a U-tube shown in Figure 3.2. It contains two liquids with densities ρ_1 and ρ_2 ($\rho_1 > \rho_2$)

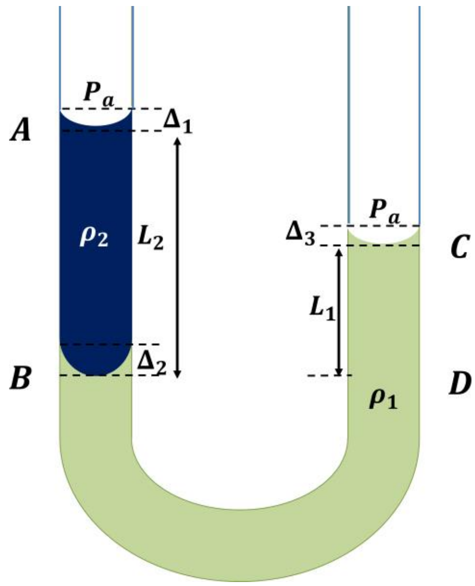


Figure 3. 2: U-Shaped tube filled with two different liquids.

From equation 3.3, we conclude that

$$P_B - P_A = \rho_2 g L_2 \tag{3.4}$$

$$P_D - P_C = \rho_1 g L_1 \tag{3.5}$$

but $P_A = P_C = P_a$, where P_a is the atmospheric pressure since both sides of the tube are open to the atmosphere. Furthermore, $P_B = P_D$ because the two points B and D are at the same vertical height in liquid 1. Therefore we add the two equations (3.4) and (3.5) can be reduced to the following

$$\rho_1 L_1 = \rho_2 L_2 \tag{3.6}$$

In this experiment, liquid 1 will be water which has a density of $1\frac{g}{cm^3}$. This means that $L_1 = \rho_2 L_2$ or $L_1 = \rho L_2$ (we renamed $\rho_2$as ρ). A graph of L_1 vs. L_2 is a straight line with a slope equal to ρ.

The uncertainty in ρ is given by

$$\rho = \frac{L_1}{L_2} \rightarrow \Delta\rho = \frac{\Delta L_1}{L_2} + \frac{\Delta L_2 L_1}{L_2^2} \; or \; \frac{\Delta\rho}{\rho} = \frac{\Delta L_1}{L_1} + \frac{\Delta L_2}{L_2} \tag{3.7}$$

APPARATUS

U tube with wooden stand, Different oils, Dropper, Colored water.

PROCEDURE

1) Clean the tube by adding few cm of acetone in the tube and shake it well. Pour acetone out from the tube.
2) Add colored water to the tube until it is about 1/3 full.
3) Use a dropper to add about 2 cm of the unknown liquid to one side of the tube (see Figure 3.2). Wait enough time so that all added liquid settled down.
4) Estimate Δ_1, Δ_2 *and* Δ_3(see Figure 3.2).
5) Measure L_1 and L_2.
6) Repeat step 6 times. Write down your measurements in the table shown in the lab. form of the experiment.

							Average
$\boldsymbol{L_1\ (cm)}$							
$\boldsymbol{L_2\ (cm)}$							

CALCULATIONS

1) Find $\Delta L_1$and ΔL_2 ($\Delta L_1 \approx \Delta_3 + \Delta_2$ *and* $\Delta L_2 \approx \Delta_1 + \Delta_2$).
2) Plot a graph of L_1 vs. L_2. (Show the centroid in your graph) Measure the slope.
3) Calculate the density of the unknown liquid from the slope of the graph.
4) Knowing the uncertainty in L_1 and L_2 (ΔL_1 and ΔL_2), calculate the uncertainty in the density of the unknown liquid ($\Delta\rho$).

QUESTIONS

1) From the result of your experiment and table of densities, Appendix A, identify the unknown liquid. Does your result identify one liquid only, or is there a possibility of a number of liquids? Explain with reference to your accuracy.
2) What is the effect of having dirt in the tube on the result of the experiment?
3) Why did you wait for a minute after adding the liquid before taking measurements?

Experiment 4: DC Circuits

BACKGROUND AND THEORY

The resistance R of a metallic conductor is defined by

$$R = \frac{Voltage}{Current} = \frac{V}{I} \qquad (4.1)$$

where V is the potential difference applied between the endpoints of the conductor and I is the current flowing through the conductor.

For a metallic conductor, such as a copper wire, the resistance R is a constant provided that the temperature of the wire stays essentially constant, that is R does not depend on I or V. Units: V is measured in volts (V), I in Amperes (A) or milli-amperes (mA), and R in Ohms (Ω).

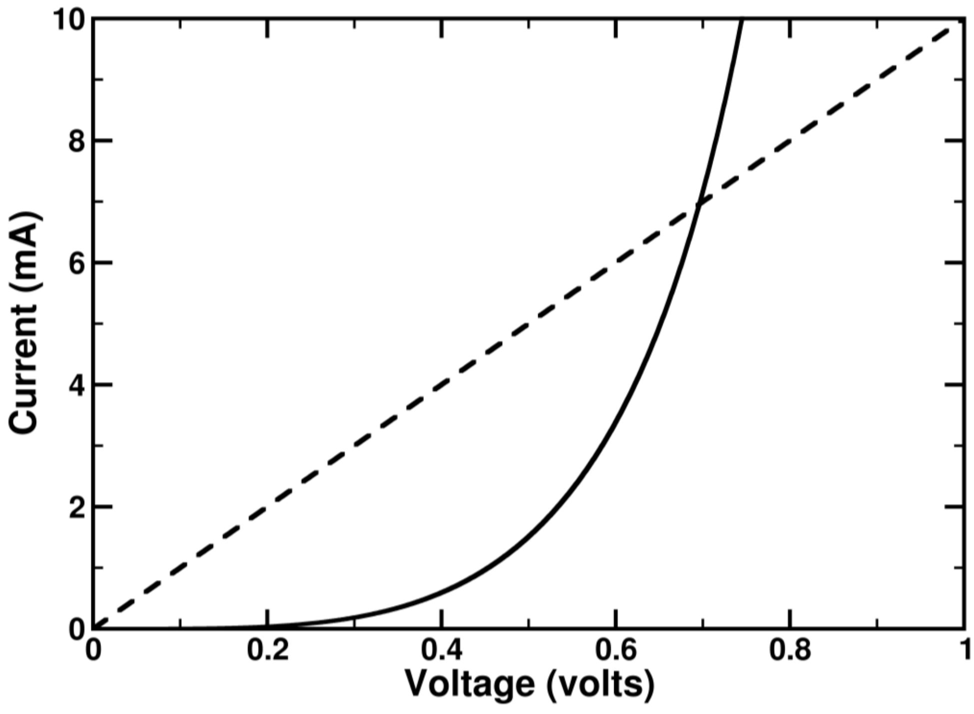


Figure 4. 1: Example of Ohmic and Non-Ohmic Materials

Conductors obey Ohm's law and are called ohmic materials. On the other hand materials that do not obey Ohm's law are called non-ohmic materials. We can test if a material is ohmic or not by measuring the potential difference V across the material against a current I that pass through it while keeping the temperature of the material constant. If V does not depend linearly on I, then the material is not ohmic.

Equivalent Resistance of Two Resistors

Consider two resistors R_1 and R_2 connected in series, then these resistors can be replaced by a single equivalent resistor R_s where

$$R_s = \mathrm{R}_1 + \mathrm{R}_2 \tag{4.2}$$

If the two resistors are connected in parallel, then they are equivalent to a single resistor of magnitude R_P where

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} \; or \; R_p = \frac{R_1 R_2}{R_1 + R_2} \tag{4.3}$$

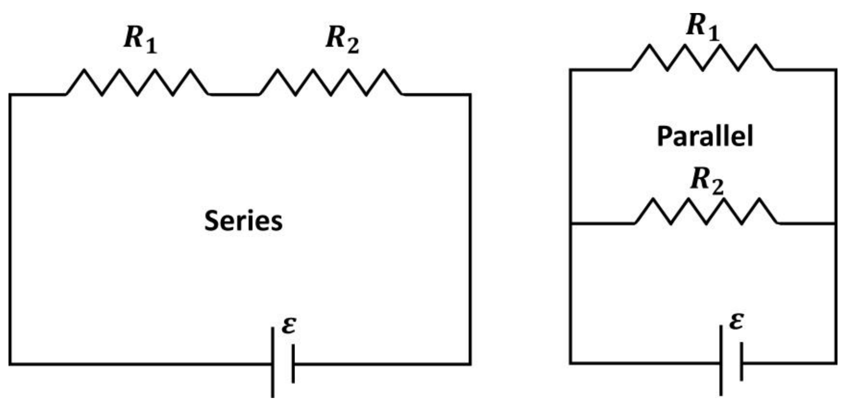


Figure 4. 2: Two resistors connect in series (left) and in parallel (right).

Figure 4.2 shows how two resistors are connected in series and in parallel. When the two resistors are connected is series, the current passing through them is the same, while when they are connected in parallel, the potential difference (voltage) across their end points is the same.

The uncertainty in R (ΔR) is given by

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I} \tag{4.4}$$

APPARATUS

DC power supply, two resistors, Variable resistor, Ammeter, Voltmeter.

PROCEDURE

Part A: DC Circuit with One Resistor

1. Connect the following circuit using only one resistance either R_1 or R_2. (You are provided with a 4 or 5 volt power supply).

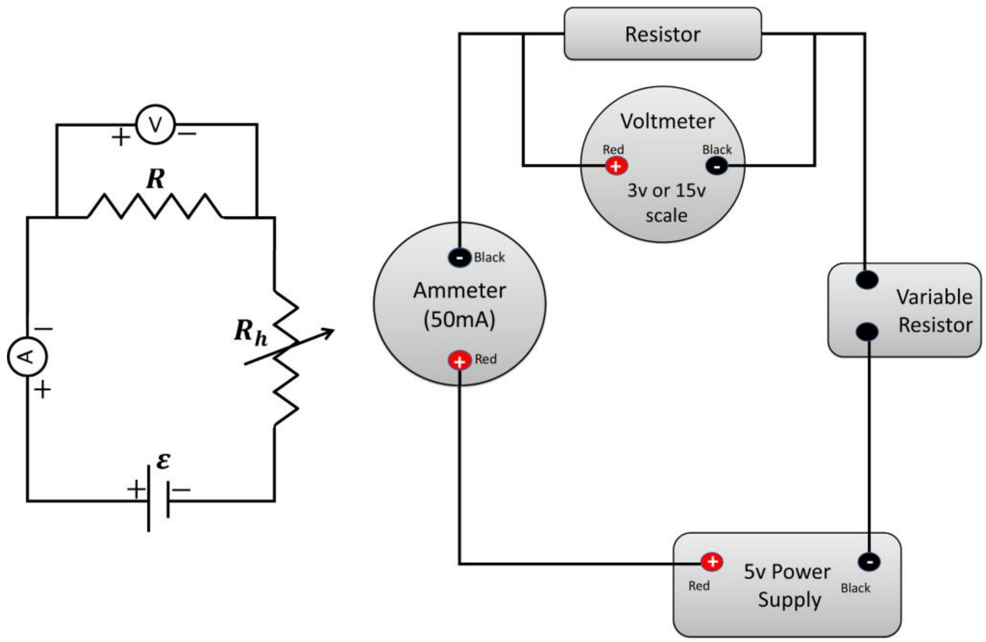


Figure 4.3: DC circuit with one resistor.

Note:

You should not connect the circuit to the power supply until it is approved by your instructor.

2. From the ammeter and the voltmeter scale, estimate the uncertainty in the current I (ΔI) and the uncertainty in the voltage (ΔV).
3. Measure the current I in the resistor and the potential difference V across it.
4. Change the current I by adjusting the variable resistor R_h, then again measure I and V.
5. Repeat step 4 for at least for 6 measurements of I and V, over as large range as possible.
6. Write down your measurements in the table shown below:

No.	1	2	3	4	5	6	7	Average
I (mA)								
V(volts)								

Part B: DC Circuit with Two Resistors Connected in Series

1. Connect the following circuit with the resistors R_1 and R_2 in series as shown in the following circuit:

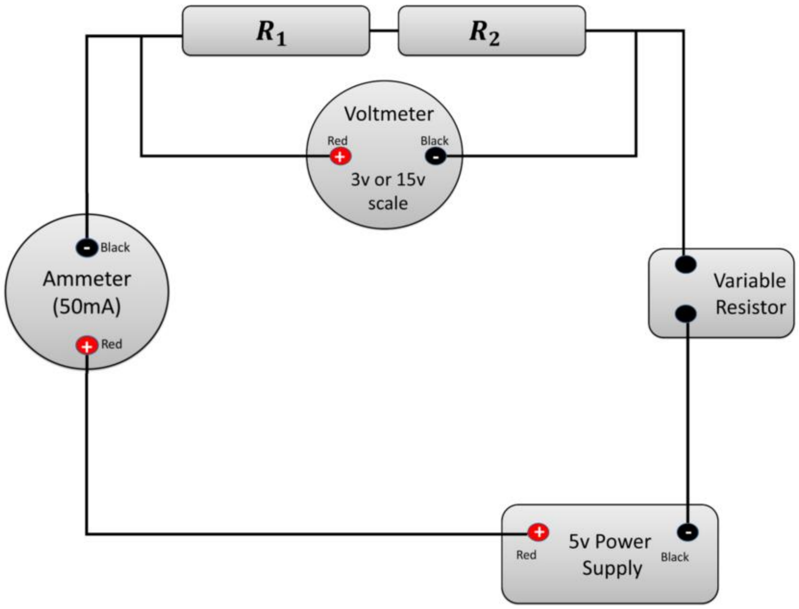


Figure 4.4: DC circuit with two resistors connected in series.

2. Estimate ΔI and ΔV from the scales of the ammeter and the voltmeter.
3. Write down the readings of the ammeter (I_s) and the voltmeter (V_s) only once.
4.

Part C: DC Circuit with Two Resistors Connected in Parallel

1. Connect the resistors R_1 and R_2 as shown in Figure 4.5 in parallel.
2. Estimate ΔI and ΔV from the scales of the ammeter and the voltmeter.
3. Write down the readings of the ammeter (I_P) and the voltmeter (V_P) only once.
4. Write down the values of the two resistors using the color code (see appendix D).

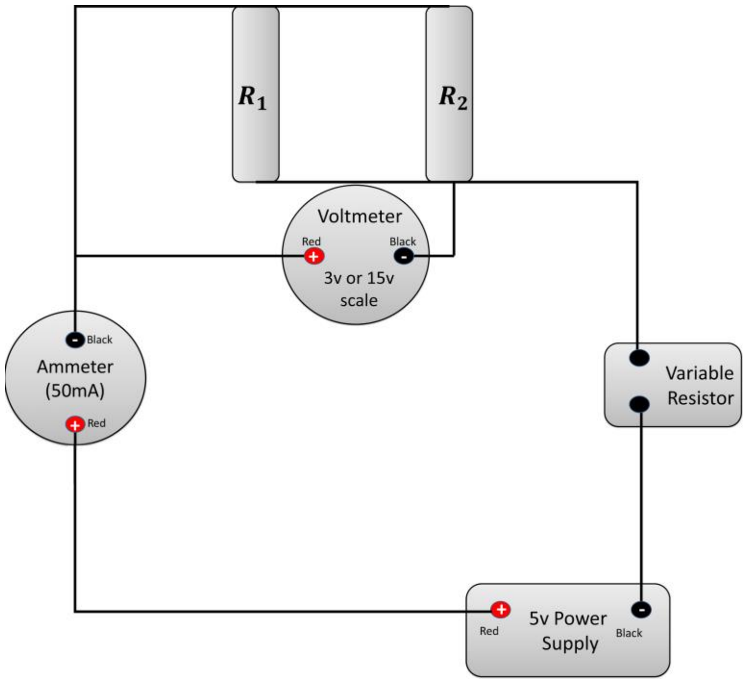


Figure 4.5: DC circuit with two resistors connected in parallel

CALCULATIONS

<u>Part A:</u>

1. Plot a graph of V vs. I.
2. Find R from the slope.
3. Calculate the uncertainty in R (ΔR), compare your result $(R + \Delta R)$ with the values obtained from the color codes on the resistor.

<u>Part B:</u>

1. Find the experimental value of the equivalent resistance for the two resistors connected in series $((R_s)_{exp} = \frac{V_s}{I_s})$.
2. Find the uncertainty in $(R_s)_{exp}$. (note :$\frac{\Delta(R_s)_{exp}}{(R_s)_{exp}} = \frac{\Delta I_s}{I_s} + \frac{\Delta V_s}{V_s}$)
3. Compare your result with the values you get from the color codes $(R_s = R_1 + R_2)$.

Part C:

1. Find the experimental value of the equivalent resistance for the two resistors connected in parallel ($(R_p)_{exp} = \frac{V_p}{I_p}$.
2. Find the uncertainty in $(R_p)_{exp}$. (note : $\frac{\Delta(R_p)_{exp}}{(R_p)_{exp}} = \frac{\Delta I_p}{I_p} + \frac{\Delta V_p}{V_p}$)
3. Compare your result with the values you get from the color codes ($R_p = \frac{R_1 R_2}{R_1 + R_2}$).

QUESTIONS

Discuss if your measured values of R (R_1 or R_2), R_s and R_p are consistent with the values you obtained from color codes, if not why not?

Experiment 5: Focal Length of a Convex Lens

BACKGROUND AND THEORY

When an object is placed at a distance u from a thin lens, an inverted image is formed on the other side of lens as shown in Figure 5.1.

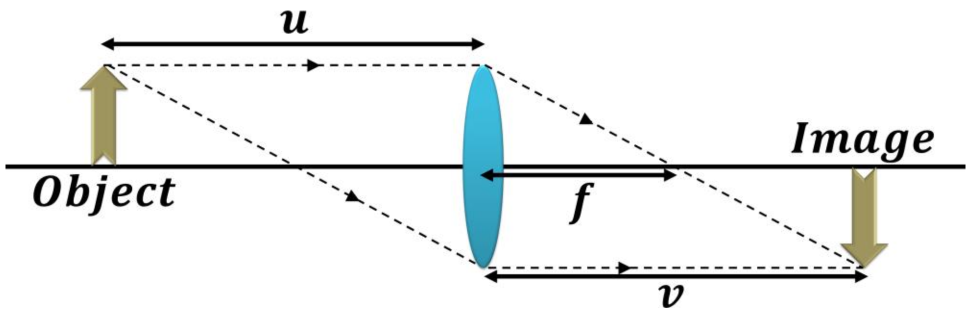


Figure 5. 1: Schematic diagram illustrating the image of an object formed by a convex lens.

If the image of an object at distance u from a lens is formed at a distance v from it, then v and u are related by:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \tag{5.1}$$

where f is the focal length of the lens, defined as the distance from the lens to the point of convergence of the light rays coming from infinity. Notice that when the object is placed at infinity (i.e. $u = \infty$), then the image will be formed at f(i.e. $v = f$).

Note that from equation 5.1

$$\frac{1}{v} = -\frac{1}{u} + \frac{1}{f} \tag{5.2}$$

It is clear that a graph of $1/v$ vs. $1/u$ is a straight line with a slope of -1, furthermore;

$$\frac{1}{u} = \frac{1}{f} \text{ when } \frac{1}{v} = 0 \tag{5.3}$$

and

$$\frac{1}{v} = \frac{1}{f} \text{ when } \frac{1}{u} = 0 \tag{5.4}$$

Thus you notice that there are two intercepts of the line; the y-axis intercept when $(1/u) = 0$ is $b_y = 1/f_y$, the other is when $(1/v) = 0$ is $b_x = (1/f_x)$. Theoretically, $f_y = f_x$, but experimentally f_y may differ from f_x because of errors.

In this experiment you are going to take several measurements of u and v, plot a graph of $1/v$ vs. $1/u$ and find the x-axis and y-axis intercepts from which you can find the focal length of the lens as the average of f_x and f_y, $f = \frac{f_x+f_y}{2}$.

The uncertainty in f (Δf) directly from equation 5.1 as follows

$$|-u^{-2}\Delta u| + |-v^{-2}\Delta v| = |-f^{-2}\Delta f| \text{ or } \frac{\Delta u}{u^2} + \frac{\Delta v}{v^2} = \frac{\Delta f}{f^2} \quad (5.5)$$

APPARATUS

Light source with exposed object, Convex lens with holder, Meter Stick, Screen.

PROCEDURE

1) Setup a lens, an illuminated object, and a screen, all on the same straight line axis as shown in figure E5.2. The lens and the screen are to be perpendicular to the axis.

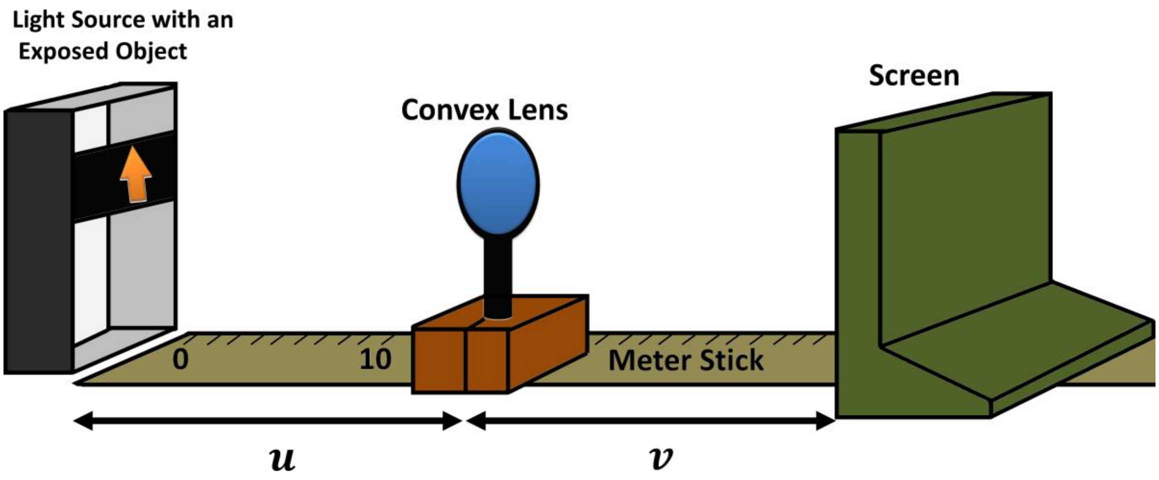


Figure 5. 2: Schematic diagram illustrating the experiment setup.

2) Start with an object distance $u \approx 80.0\ cm$, move the screen until you see a sharply focused image of the illuminated object.
3) Measure u and v using a meter stick and write down your measurements in the following table

	Average

$u\ (cm)$							
$v\ (cm)$							
$\frac{1}{u}\ cm^{-1}$							
$\frac{1}{v}\ cm^{-1}$							

4) Move the lens toward the illuminated object about 15 cm, then measure u and v, write them down in the table shown above.
5) Calculate the values of $1/u$, $1/v$, $\overline{\left(\frac{1}{u}\right)}, \overline{\left(\frac{1}{v}\right)}, \bar{u}, \bar{v}$ and write them down in the table shown above.

CALCULATIONS

1) Plot a graph of $1/v$ vs. $1/u$. Show the centroid and the origin (0,0) on the graph.
2) Measure the two intercepts with the x-axis (b_x) and with the y-axis (b_y).
3) Find the reciprocal of each intercept, then find the focal length $f = \frac{f_x+f_y}{2}$. Write down the letter shown on the lens you used.
4) To calculate the uncertainty in f (Δf), your instructor will setup an apparatus like yours, with a fixed value u; he will ask several students to measure v independently. After the measurements, each student have to write the values down in a table, calculate σ_S of these measurements then $\Delta v = \sigma_m(v) \approx \Delta u.$
5) Using the results of equation 5.5 the uncertainty in f is given by

$$\Delta f = f^2 \left(\frac{\Delta u}{\bar{u}^2} + \frac{\Delta v}{\bar{v}^2}\right) \qquad (5.6)$$

QUESTIONS

1. Write down any systematic errors and estimate them.

(Note: do not forget to write down the letter appearing on your lens; either A, B or C)

2. Can you justify using the average of f_x and f_y for f?

Experiment 6: Index of Refraction

BACKGROUND AND THEORY

When light passes from one medium to another its path bends. Examples of different media are glass, plastic, water and air. Different colors of light bend by different amounts at the boundary between the two media. The bending of light is called "refraction". We note that not all media bend a given light by the same amount. This means that each medium has its own refractive index n which is defined as:

$$Index\ of\ refraction = \frac{speed\ of\ light\ in\ vacuum}{speed\ of\ light\ in\ medium} = \frac{c}{v} \quad (6.1)$$

This refractive index is a measure of how much bending will occur for the light when it falls on a medium.

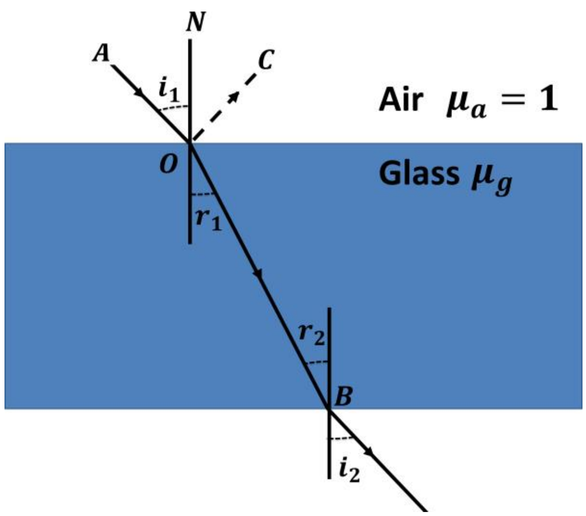


Figure 6. 1: Schematic diagram that demonstrate light refraction

In Figure 6.1,a light ray falls on a block of glass from air; AO represents a ray of light traveling in air and is incident on the surface of the block. OC represents the reflected ray while OB represents the refracted ray, the line ON represents the normal to the block surface, the angle of incidence is "i" while the angle of refraction is "r".

Snell's law relates the angle of incidence "i" and the angle of refraction "r" by

$$\mu_a \sin(i) = \mu_g \sin(r) \quad (6.2)$$

where μ_a is the index of refraction of air which is very near that of vacuum ($\mu_a \approx 1$) and μ_g is the index of refraction of glass. Thus equation 6.2 can rewritten as

$$\sin(i) = \mu_g \sin(r) = \mu \sin(r) \tag{6.3}$$

From the latter equation one can find the index of refraction of glass by plotting a graph of $\sin(i)$ vs. $\sin(r)$ which gives a straight line with slope equal to μ.

The uncertainty in the index of refraction ($\Delta\mu$) can be calculated as follows

$$\Delta\mu = \left|\frac{\cos(i)\Delta i}{\sin(r)}\right| + \left|\frac{-\sin(i)\cos(r)\Delta r}{\sin^2(r)}\right| \text{ or } \frac{\Delta\mu}{\mu} = \left|\frac{\cos(i)}{\sin(i)}\right|\Delta i + \left|\frac{\cos(r)}{\sin(r)}\right|\Delta r \tag{6.4}$$

APPARATUS

Light source with a slit, glass or plastic block and protractor.

PROCEDURE

1) Place the block on a piece of white paper, draw the borders of block on the paper.
2) Mark the angles of incidence as shown in Figure 6.2; you can choose the first angle of incidence near 10°, the second angle of incidence near 20° the third near 30° and so on.

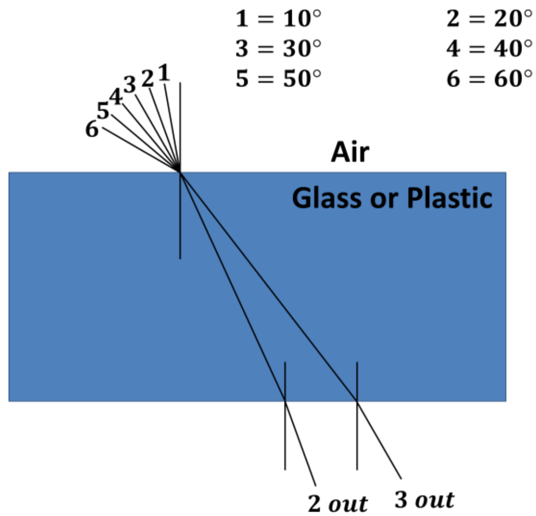


Figure 6. 2: Assigning 6 angles of incidence and finding the corresponding angles of refraction.

3) Shine a narrow beam of light such that the beam is exactly on path 1 in figure 6.2, then mark the path of the outgoing beam and label it (write down the beam number which is 1). See for example the lines labeled by 2 out and 3 out.
4) Repeat the procedure in part 3 five more times for the other angles from 20 degrees to 60 degrees.
5) Remove the block, for each outgoing beam, draw a perpendicular line to the block boundary at each exit point.
6) For each incident and outgoing beam connect the incident point with the exit point.

7) From the divergence of the beam, estimate the uncertainty in the measurement of i and r, Δi and Δr, (Refer to figure 6.3), in radians.

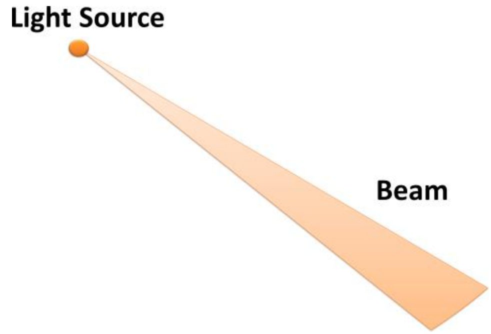


Figure 6. 3: Beam Divergence.

8) For each incident and outing beam, measure the angles (i_1, i_2) and (r_1, r_2) and write down your measurements in the table below.

No.	Angle i (°)		$\bar{\imath}$	$\sin(\bar{\imath})$	Angle r (°)		$\bar{r}$	$\sin(\bar{r})$
	i_1	i_2			r_1	r_2		
1								
2								
3								
4								
5								
6								

CALCULATIONS

1) Plot a graph of $\sin(i)\ vs.\ \sin(r)$. Find the value of the index of refraction μ of the material. Show the centroid on the graph.
2) Calculate the values of $\overline{cos(\bar{\imath})}$ and $\overline{cos(\bar{r})}$.
3) Find the uncertainty in the index of refraction $\Delta\mu$ using equation

$$\frac{\Delta\mu}{\mu} = \left|\frac{\cos(\bar{\imath})}{\sin(\bar{\imath})}\right|\Delta i + \left|\frac{\cos(\bar{r})}{\sin(\bar{r})}\right|\Delta r$$

Remember that Δi and Δr must be in radians, $1° = \frac{\pi}{180} radians$.

QUESTIONS

1. The index of refraction of glass is approximately 1.52, and for plastic it is about 1.46. Is the result of your experiment consistent with these values?
2. What are the main sources of error in your result? How can you improve your result?

Experiment 7: Measuring of g at BZU

BACKGROUND AND THEORY

Consider an ideal simple pendulum which consists of a point mass particle of mass m connected to a massless string of length L swinging back and forth as shown in Figure 7.1.

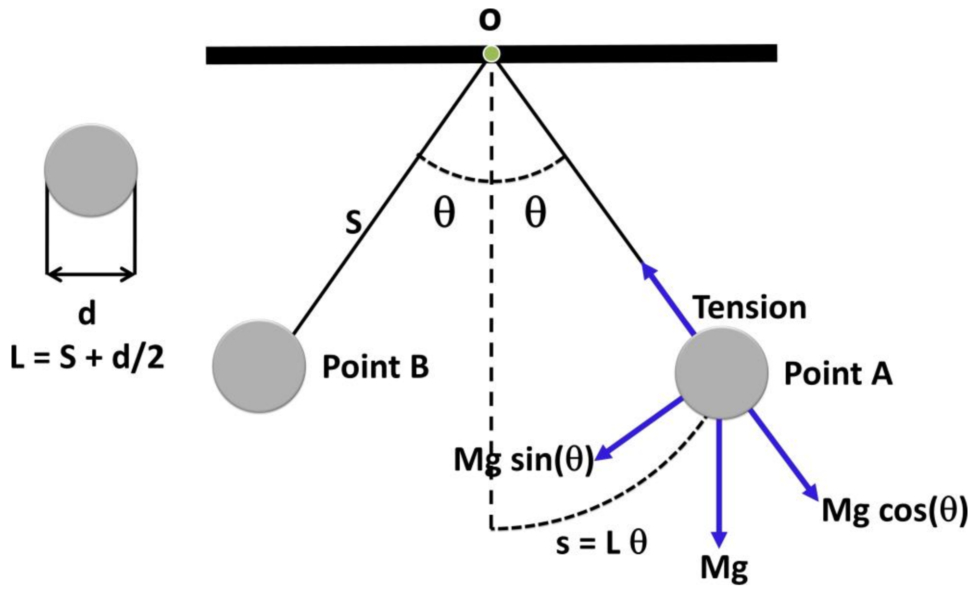


Figure 7. 1: Simple Pendulum.

We would like to find the period of the pendulum (T) which is the time required by the pendulum to complete one oscillation from point A to point B and then back to point A.

The total force acting on the mass m can be decomposed into radial and tangential components. Applying Newton's second law on the *radial* direction yields

$$F_r = Tension - Mg\ \cos(\theta) = \frac{Mv^2}{L} \tag{7.1}$$

while applying Newton's second law on the *tangential* direction yields

$$-Mg\sin(\theta) = M\frac{d^2s}{dt^2} \tag{7.2}$$

For small angle θ, $\sin(\theta)$ can be approximated by θ. Hence the latter equation becomes

$$\frac{d^2s}{dt^2} + g\theta \approx 0 \tag{7.3}$$

Since the length of the arc at any time is given by $s = L\theta$, equation 7.3 can be simplified to contain only one dependent variable as follows

$$\frac{d^2s}{dt^2} + \frac{g}{L}s \approx 0 \tag{7.4}$$

Equation 7.4 is a homogenous linear second order differential equation which has a solution of the form

$$s(t) = s_0 \sin(\omega t + \delta) \tag{7.5}$$

where b_0 and δ are constants that can be found from the initial position and velocity of the pendulum. Substitution of equation 7.5 in equation 7.4 yields

$$-s_0\omega^2 \sin(\omega t + \delta) + \frac{g}{L}\sin(\omega t + \delta) = 0 \rightarrow \omega^2 = \frac{g}{L} \tag{7.6}$$

From the angular frequency ω, the period of the pendulum can be calculated as follows

$$Period = T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{g/L}} = 2\pi\sqrt{\frac{L}{g}} \rightarrow T^2 = \frac{4\pi^2}{g}L \tag{7.7}$$

A graph of T^2 vs. L is a straight line of the form $y = m\,x + b$ with a slope $m = 4\pi^2/g$ and $b = 0$, thus a measurement of the slope and its uncertainty will allow us to determine the value of g and its uncertainty $\Delta g = \frac{4\pi^2}{m^2}\Delta m$. In this experiment you will find the value the slope m, the y-intercept b and their uncertainties using the least square fit method.

APPARATUS

String, spherical mass, meter stick, stand with a clamp and stopwatch

PROCEDURE

Before doing the experiment, you should keep in mind the following:

- The maximum angle of displacement should not exceed 15 degrees.

- If the mass of the string is neglected, then the length L is the distance from the support to the center of the ball that is $L = S + d/2$, where S is the length of the string and d is the diameter of the ball which can be measured using the caliper.

1) Start with a length $L \approx 40\ cm$, measure the time needed to do 10 periods using a stop-watch.
2) Repeat step 1 for a total of 5 to 6 different values of L ranging from $L \approx 50\ cm$ to $L \approx 150\ cm$.
3) For each value of L, find the average value of the time for 10 periods, then divide by 10 to find the average value of a single period T. Find also T^2 and add it to table below

The diameter of the ball d =								
No.	**S (cm)**	**L (cm)**	**t_1 (sec) 10 periods**	**t_2 (sec) 10 periods**	**t_3 (sec) 10 periods**	**T average (sec)**	**One Period T (sec)**	**T^2 sec.^{2}**

CALCULATIONS

1) Using the method of least square fit, find the best slope (m) and the error in it Δm as well as the best y-intercept (b) and the error in it Δb.
2) From the slope found in step 1, estimate the value of g.
3) Calculate the uncertainty in g using the uncertainty in the slope found in step 1.
4) Using a linear graph paper, make a scatter plot of T^2 vs. L (show only measured points, do not draw the best line through the points).
5) Draw the straight line equation $y = m\,x + b$; on the graph of step 4, this line will be the best straight line for the data points.

QUESTIONS

1) Is the measured value of g acceptable within the experimental error?
2) Does the line plotted in step 5 of the calculation fit your data well? Does it pass through the centroid?
3) Does the period of the pendulum depend on the mass hanging from the string? How can you check for this experimentally?
4) Can you derive the relation $\sin(\theta) \approx \theta$ for small angles? You may use either algebraic or geometrical methods.
5) What do you think is the effect of air resistance on your result? How can you reduce this effect?

Experiment 8: Half-life of a Draining Water Column

BACKGROUND AND THEORY

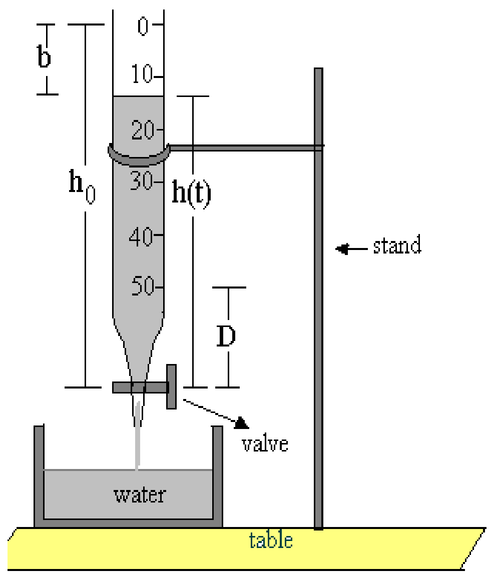


Figure 8. 1: Decaying Water Column.

Consider the calibrated burette shown in Figure 8.1; at time $t = 0$, the tube contains water to height h_0. When one opens the valve, the water will start draining out of the tube. This process is called "decaying of the water column". The rate of the decay of the water column is proportional to the weight of the water column above the valve, which is also proportional to its height h, that is;

$$-\frac{dh}{dt} \propto h(t) \text{ or } \frac{dh}{dt} = -\lambda h(t) \tag{8.1}$$

where λ is a constant. Multiplying the last equation by dt and dividing by $h(t)$ yields

$$\frac{dh}{h} = -\lambda dt \tag{8.2}$$

Integrating both sides of equation yields

$$\int_{h_0}^{h(t)} \frac{dh}{h} = -\lambda \int_0^t dt \qquad (8.3)$$

which implies

$$\ln h(t) - \ln h_0 = -\lambda t \text{ or } \ln \frac{h(t)}{h_0} = -\lambda t \rightarrow h(t) = h_0 e^{-\lambda t} \qquad (8.4)$$

Equation 8.4 gives the height of water at any given time. The constant λ is called the decay constant. As an example, a graph of h vs. t for three different values of λ is shown in Figure 8.2. Notice that when λ is large the decay is faster.

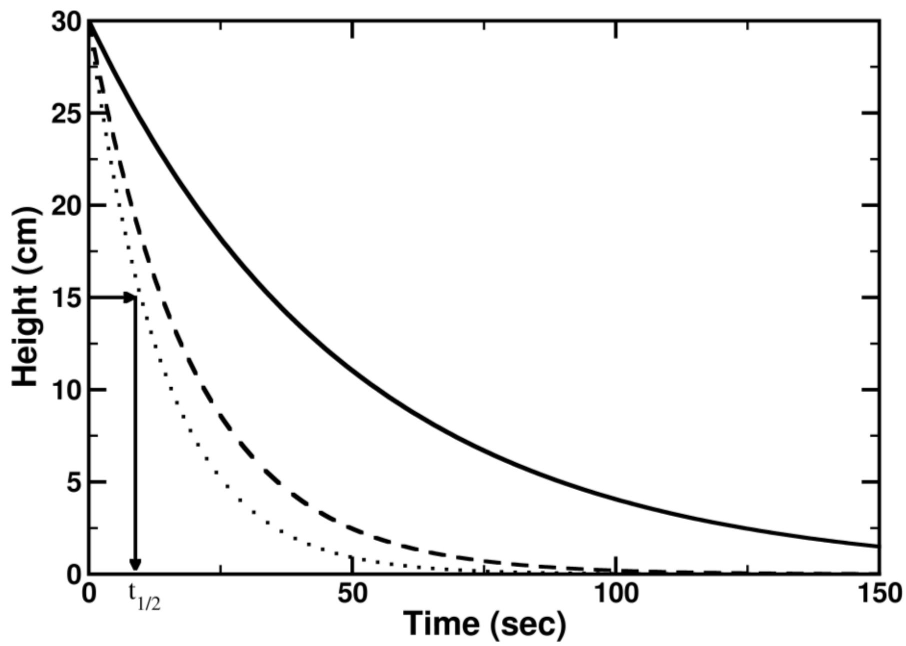


Figure 8. 2: Height of water column in a burette as a function of time for three different decay constants.

Notice that when $h(t) = h_0/2$, the time t is equal to $t_{1/2}$, substituting this in equation 8.5 we get

$$\frac{h_0}{2} = h_0 e^{-\lambda t_{1/2}} \rightarrow \ln 2 = -\lambda t_{\frac{1}{2}} \rightarrow t_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \qquad (8.5)$$

APPARATUS

Burette with stand, colored water in a beaker, measuring ruler, stopwatch, funnel.

PROCEDURE

1) Measure the total burette length h_0 in the burette calibration units, refer to Figure 8.1, $h_0 = 50\ units\ +\ D$ (in burette units), you can measure the distance D in units of cm then convert it to burette units.
2) Fill the burette with water using the funnel. Adjust the valve such that water will drain in about 2.5 minutes, do not change the setting of the valve during the experiment; the adjustment is made in just one time. Make sure that the burette is clean and vertical.
3) Measure the reading of the burette (b) (see Figure 8.1) every 10 seconds; use your finger tip to close the opening of the burette.
4) Fill the burette again with water to same initial height, do not change the setting of the valve, repeat step 2 for two more times, write down your measurements in a table like that shown below

Time (s)	Burette reading (u)				$h(t) = \left(h_0 - \bar{b}\right)$	$\ln(h)$
	b_1	b_2	b_3	$\bar{b}$		
0	0	0	0			
10						
20						
30						
40						
50						
60						
70						
80						
90						
100						
110						
120						
130						
140						
150						
160						
180						

CALCULATIONS

1) Using a linear graph paper, plot $h(t)$ vs. time and obtain 6 different values of $t_{1/2}$.
2) Find the average value of $t_{1/2}$ and the uncertainty in it $\Delta t_{1/2}$.
3) Plot a graph of $\ln(h)$ vs. t. From equation 8.5 this will give a straight line with a slope $-\lambda$ and a y-intercept $\ln(h_0)$.
4) Using equation 8.6 and using the value you obtained for the decay constant λ in step 2, find the value of $t_{1/2}$.

Experiment 9: RC Circuit

BACKGROUND AND THEORY

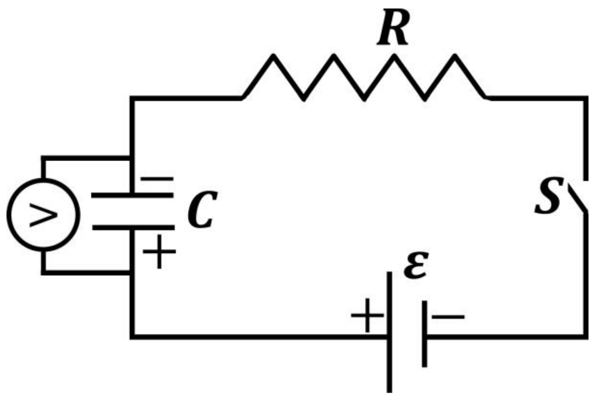


Figure 9. 1: Charging Circuit. At t=0, Q=0.

Consider the series RC circuit shown below in figure E9.1, the circuit consists of a capacitor C and a resistance R connected in series to a voltage source ε, initially the capacitor is uncharged, this means that Q_0=0 at t=0. When the switch is closed, charge will start accumulating on the capacitor.

Part A: Charging

Let us find the charge $Q(t)$ on the capacitor as a function of time. From Kirchhoff's second rule, the sum of all voltage drops over a closed loop is zero, that is

$$\sum V_j = 0 \tag{9.1}$$

The latter implies

$$\varepsilon - IR - \frac{Q}{C} = 0 \tag{9.2}$$

But

$$I = \frac{dQ}{dt} \tag{9.3}$$

hence

$$\varepsilon - R\frac{dQ}{dt} - \frac{Q}{C} = 0 \tag{9.4}$$

Rearranging equation 9.4 gives

$$\frac{dQ}{\left(\frac{\varepsilon}{R} - \frac{Q}{RC}\right)} = dt \tag{9.5}$$

Integrating both sides of equation 9.5 yields

$$\int_0^{Q(t)} \frac{dQ}{\left(\frac{\varepsilon}{R} - \frac{Q}{RC}\right)} = \int_0^t dt \tag{9.6}$$

Equation E9.4 implies that the charge Q on the capacitor is;

$$Q(t) = \varepsilon C\left(1 - e^{-\frac{t}{RC}}\right) \tag{9.7}$$

Since the voltage on the capacitor is related to the charge on it by the relation $V = Q/C$, thus;

$$V(t) = \varepsilon\left(1 - e^{-\frac{t}{RC}}\right) \tag{9.8}$$

Let us consider the case when the time $t = RC$, this is called the time constant $(\tau)$of the circuit, substituting this in equation 9.8, one gets;

$$V(\tau) = \varepsilon(1 - e^{-1}) = 0.63\varepsilon \tag{9.9}$$

Thus the time constant of the circuit is the time needed for the potential difference on the capacitor $V(t)$ to reach 0.63 of the maximum voltage (the maximum voltage is ε). In Figure 9.3 equation 9.8 is plotted as a function of time, it also shown in this Figure how one can find the time constant from the charging curve.

Part B: Discharging

When the initial charge on the capacitor is not zero, that is the capacitor has an initial potential difference ε and initial charge $Q_0 = C\varepsilon$. When the initially charged capacitor is connected in series with a resistance R, the capacitor will start discharging through the resistance, (see Figure 9.2).

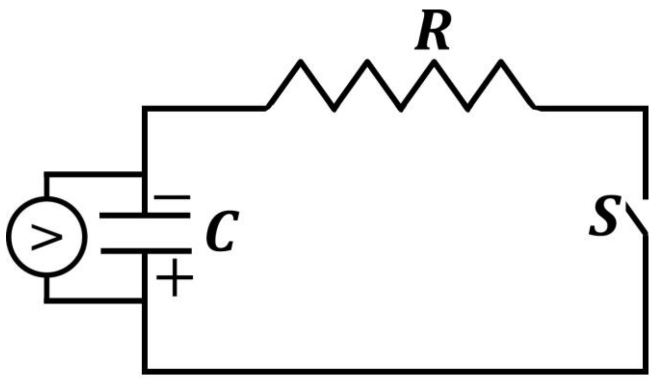


Figure 9. 2:Discharging Circuit. $Q = Q_0$ and Initial Capacitor Voltage = ε, at t=0.

Following Kirchhoff's second rule again (equation 9.1), we have

$$-IR - \frac{Q}{C} = 0 \tag{9.10}$$

But

$$I = \frac{dQ}{dt} \tag{9.11}$$

Hence

$$-\frac{dQ}{dt}R - \frac{Q}{C} = 0 \tag{9.12}$$

Dividing both sides of equation 9.12 by RQ we get;

$$\frac{dQ}{Q} = -\frac{dt}{RC} \tag{9.13}$$

Integrating both sides of equation 9.13 yields

$$\int_{Q_0}^{Q(t)} \frac{dQ}{Q} = -\frac{1}{RC}\int_0^t dt \tag{9.14}$$

The charge $Q(t)$ on the capacitor is then given by

$$Q(t) = Q_0 e^{-\frac{t}{RC}} = \varepsilon C e^{-\frac{t}{RC}} \tag{9.15}$$

The voltage on the capacitor $V(t)$ can be found from the relation between the voltage and the charge

$$V(t) = \frac{Q(t)}{C} = \varepsilon e^{-\frac{t}{RC}} \tag{9.16}$$

The voltage on the capacitor in the discharging process when $t = \tau = RC$ is

$$V(RC) = \varepsilon e^{-1} = 0.37\varepsilon \tag{9.17}$$

Equation 9.15 is plotted in Figure 9.3, from this discharging curve, one can find the value of the time constant τ by taking 0.37 of the maximum (ε).

The time constant can be found using another method; let us take the natural logarithm of both sides of equation 9.15

$$\ln V(t) = \ln\left(\varepsilon e^{-\frac{t}{RC}}\right) \rightarrow \ln V(t) = \ln(\varepsilon) - \frac{t}{RC} \tag{9.18}$$

The graph of $\ln V(t)$ vs. time gives a straight line with a y-intercept equals to $\ln(\varepsilon)$ and with a slope equals to $-1/RC = -1/\tau$ so one can find the slope, then the value of the time constant τ

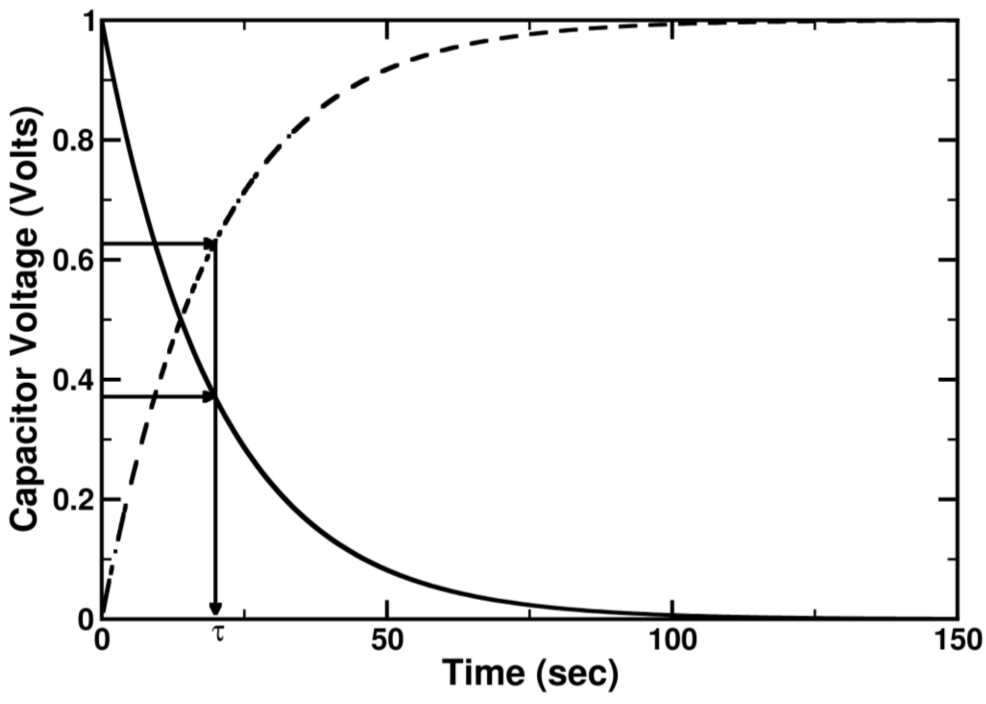


Figure 9. 3: The Capacitor Voltage in an RC circuit during charging and discharging process.

In this experiment you will be given a capacitor, its value will be unknown to you, you will obtain three values of the time constant τ two values from the charging and discharging curves (say τ_c and τ_d), and the third value (τ_s) can be found by plotting the voltage $V(t)$ for <u>discharging</u> vs. time on a semi-log graph paper, refer to the graph section in this manual so that you know how to plot graphs using semi-log graph papers.

You will find the average time constant $\bar{\tau}$, then you can find the value of the unknown capacitor using the relation

$$C = \frac{\bar{\tau}}{R} \tag{9.1}$$

Using the last equation one can find the error in the capacitor C (ΔC)

$$\frac{\Delta C}{C} = \frac{\Delta \bar{\tau}}{\bar{\tau}} + \frac{\Delta R}{R} \tag{9.2}$$

The error in $\tau, \Delta\bar{\tau} = \sigma_m(\tau)$; it is the standard deviation of the mean value of the three measurements, the resistance R can be measured using the ohmmeter, you can also find it using the color code appearing on the resistance. The uncertainty in R (ΔR) can be estimated from the ohmmeter or from the color code.

APPARATUS

DC power supply, Resistor ($R \approx 1M\Omega = 1 \times 10^6 \Omega$), Capacitor, Digital multi-meter, Stop watch.

PROCEDURE

1) Setup the circuit shown in Figure 9.1, you will be provided by 4-5 volts from the main power supply. Note that the voltmeter which you will use is a digital voltmeter; you have to adjust it to read DC volts, also select the appropriate scale, (refer to the instrument section before you do this experiment). If the capacitor is of electrolyte type, make sure you connect its positive terminal to the higher potential side.
2) Make sure that the capacitor has no charge on it when you start doing the experiment, this can be done by shortening the terminals of the capacitor by a wire.
3) Remove the wire and close the circuit, at the same instant start the stop watch, write down the reading of the digital voltmeter for the time (t) values shown in table below.
4) Disconnect the circuit from the voltage source and replace it by a short at the same instant start the stopwatch; this will be the circuit of Figure 9.2, before you close the circuit the voltage on the capacitor will be maximum, the voltage on the capacitor will start decreasing as a function of time, write down the capacitor voltage for the time values shown in table below.

Time (sec.)	$V_{capacitor\text{-}charging}$ (volts)	$V_{capaccitor\text{-}discharging}$ (volts)
0	0	$\boldsymbol{\varepsilon} =$

5		
10		
15		
20		
25		
30		
35		
40		
45		
50		
55		
60		
70		
80		
90		
100		
110		
120		
140		
160		
180		
200		

CALCULATIONS

1) On a single linear graph paper, plot $V(t)$ vs. time (t) for both, charging and discharging.
2) From the two curves, find the two values of the time constant τ_c and τ_d refer to Figure 9.3 to help you find these values.
3) On a semi-log graph paper, plot a graph V vs. time (t), (the y-axis should be the log scale while the x-axis is the linear time scale), choose two arbitrary points along the best straight line (V_1,t_1), (V_2, t_2), find the slope of the line using the relation

$$slope = m = \frac{\ln(V_2) - \ln(V_1)}{t_2 - t_1}$$

4) Refer to equation 9.18 to find the third value of the time constant τ_s from the slope (m) of the line.
5) Find the average value of the three measurements of the time constant $\bar{\tau}$.
6) Find the error in the time constant $\tau, \Delta\bar{\tau} = \sigma_m(\tau)$.
7) Measure the resistance R using an Ohmmeter or the color code, refer to the instrument section to know how you can do that, also estimate the uncertainty (ΔR) using the ohmmeter or the color code.
8) Calculate the value of the unknown capacitor C, and its uncertainty using the relations 9.19 and 9.20.

QUESTIONS

1) Does your measured value of C agree with the manufacturer's stated value written on the capacitor?
2) What is the effect of increasing the resistance R on both the charging and discharging processes.
3) Show that the unit of "RC" is the time unit "seconds".
4) Another application of exponential decay: Newton's Law of Cooling. A large room is kept at a constant temperature T_0. A small object in the room is heated to a higher temperature T. It is assumed that the rate at which the object cools down is proportional to how much it is hotter than the room, that is $dT/dt \propto -k(T-T_0)$. Solve this equation and show that you get a result similar to the discharge of the capacitor. Why should the room be large and the object small?

Experiment 10: Oscilloscope

BACKGROUND AND THEORY

The Cathode-Ray Oscilloscope (CRO) is an instrument which can display graphs of potential differences vs. time; it can trace and redraw waves and signals as being processed through different stages in electronic circuits, it can also be used to measure AC and DC voltages. Another useful feature of the CRO is that it can be used to measure the amplitude and frequency of a given AC signal as well as the phase (ϕ) between two given AC signals.

Structure of the CRO

The main part of the Oscilloscope is an evacuated glass tube called the cathode ray tube (CRT) which is shown in Figure 10.1, the filament heats the cathode to emit electrons into the vacancy inside the tube. These electrons are accelerated due the high positive potential at the accelerating anode; these toward the florescent screen which is located at the other end of the tube. When an electron hits the screen, the material covering the screen emits light in the visible range, so one is able to see a spot on the screen, another part of the tube is the grid which is a metallic mesh negatively charged. By varying the value of its negative potential one can control the intensity of the emitted light.

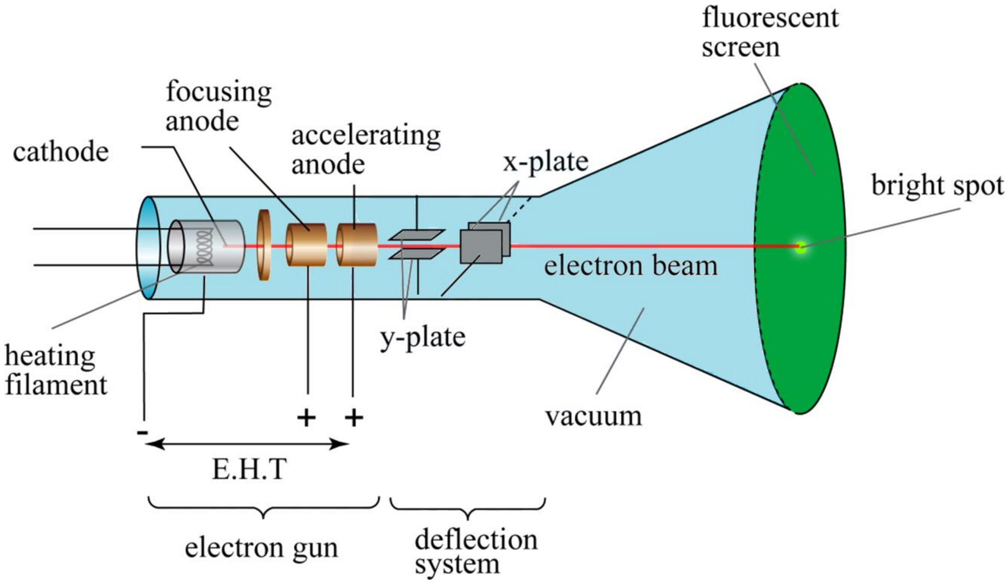


Figure 10.1: Schematic Diagram of A CRO.

The focusing anode is positively charged and it is used to allow the beam to be focused at the screen. The vertical deflection plates can deflect the beam of electrons either up or down while the horizontal deflection plates can deflect the electron beam sideways.

Operation of the CRO

The CRO can function in two modes:

- The external mode: it is selected by turning the time base button to the x-y external mode. When the external mode is selected, the CRO screen will act as an x-y plotter for external voltages applied to both x and y inputs of the CRO.
- The internal mode: Once the internal mode is selected using the time base button, the x-axis will be connected to a sweep generator that periodically scans the x-axis with a constant velocity, such a sweep generator produces a "sawtooth" potential difference V_x which is connected to the horizontal deflection plates, this is shown in figure 10.2. Thus the x- axis becomes a time axis while the y-axis can receive inputs from any external source.

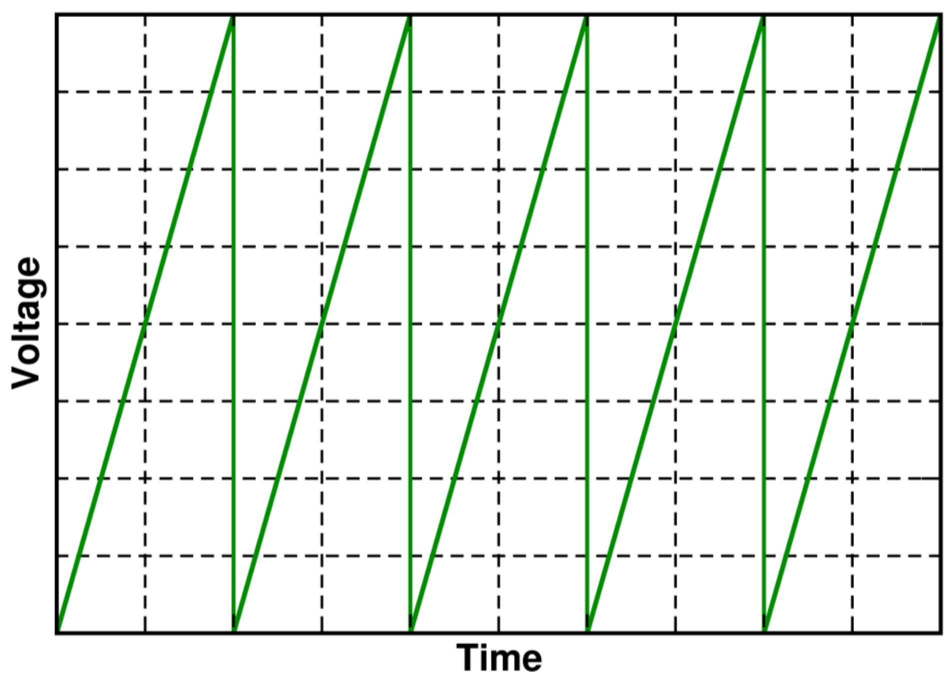


Figure 10.2: Example of a sawtooth wave.

In general the CRO can be used in either of its two modes, depending on what you want to measure. The CRO has two inputs x-input and the y-input, where you can connect two external signals to it (one to the x-input the other to the y-input).

Let us show you the front panel of the oscilloscope for one of the models used in the physics 111 lab. This is shown in Figure 10.3

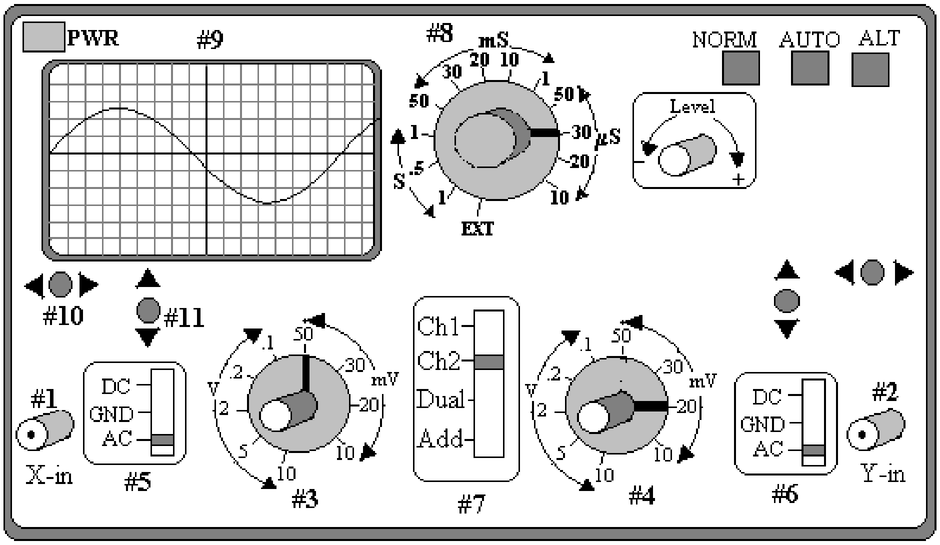


Figure 10. 3: Model Oscilloscope front panel.

#1 is the x-input which can receive external signals through a cable.

#2 is the y-input which can also receive external signals by through a cable.

#3 is the x-voltage multiplier control. Notice that in this example, the reading of the x-voltage multiplier is $50\,mV$. This means that each division on the y-axis of the screen equals $50\,mV$. From the sinusoidal wave appearing on the CRO screen, one can calculate the peak-peak voltage as shown below

$$V_{PP} \approx 5.5\ divisions \times \frac{50mV}{division}\ if\ the\ signal\ is\ connected\ to\ X-in$$

$$V_{PP} \approx 5.5\ divisions \times \frac{2}{division}\ if\ the\ signal\ is\ connected\ to\ Y-in$$

#4 is the y-channel voltage multiplier control. Its reading is $2\ volts$. This means that each division along the y-axis equals $2\ volts$ for the y-channel.

#5 has three selections **DC** (constant input voltage), **GND** which is ground and **AC** (alternating input voltage). If **GND** is selected no signal will be observed on the x-channel.

#6 is same as #5 but for y-channel.

#7 is used to select the signal wanted to be displayed on the screen. If **Ch1** is selected, this means that the signal is connected to x-input is displayed on the CRO screen. If **Ch2** is selected, this means that the signal connected to y-input is displayed on the CRO screen. If **Dual** is selected, then the two signals (x-input and y-input) will be displayed on the CRO screen simultaneously and finally if **ADD** is selected, the two signals will be added and sum will be displayed on the screen.

#8 is the time base control. The external mode is selected when this piece is rotated all the way counter clockwise to **EXT.** In Figure 10.3 the CRO is used in its internal mode. Notice the reading of the time base is 30 μs. This means that each division along the x-axis of the CRO screen equals 30 μs. The period of the sinusoidal wave screen on the CRO screen can be measured directly.

$$\boldsymbol{Period = T \approx 17\ division \times \frac{30\mu s}{division} = 17 \times 30 \times 10^{-6} sec}$$

#9 is the CRO screen.

#10 is the horizontal position control which used to move the input electrical signals sideways (left to right).

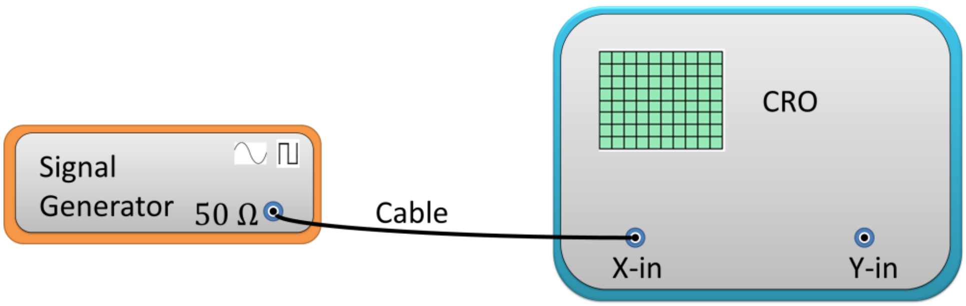
#11 is used to move the input electrical signals up or down (there are two of them, one for the x-input and the other for the y-input).

#12 is used for focusing and adjusting the thickness of the line when the CRO is used in the internal mode and to adjust the size of the light spot when the CRO is used in its external mode.

#13 is used to control the brightness of the line or the light spot.

PROCEDURE

1) Turn the CRO on, turn the time base control button to the external mode (counterclockwise), observe the light spot and bring it to the center of your screen using the x and y deflecting knobs. Use the focus and intensity knobs to convert the light spot into a focused sharp point.



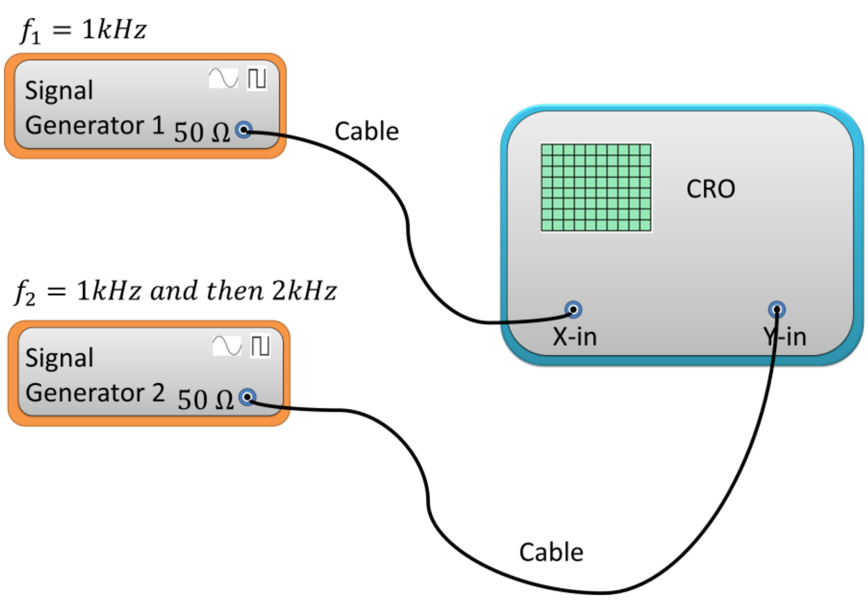
2) Apply a sinusoidal signal to y-input of the CRO, adjust the frequency of the signal to about 10 Hz, this can be done by pushing 10 Hz button in the signal generator. Increase the amplitude of the signal, notice the harmonic motion of the spot up and

down, connect the sinusoidal signal to the x-input, notice the harmonic motion of the spot sideways.

3) Turn the CRO to the internal mode (by turning the time base button clockwise), connect the sinusoidal signal to the y-input, increase the frequency of the signal to few KHz, measure the peak-to-peak voltage, this is done by counting the number of squares on the y-axis from the minimum to the maximum of the wave then multiply this number by the y-channel voltage multiplier. The peak voltage (V_P) of the wave will be V_{PP}, record your answer in table 1

4) Select a frequency from the signal generator near 10 KHz, measure the period (T) of the wave by counting the number of boxes along the x-axis for one complete wave $(0,2\pi)$ then:

$$\boldsymbol{T = No\ of\ boxes\ for\ one\ wave \times time\ base\ reading}$$



Calculate the frequency of the wave $f = 1/T$, compare the frequency which you found by the frequency from the signal generator, fill table 2

5) Leave the sinusoidal signal connected to the y-channel, connect another sinusoidal signal to x-input using another signal generator, turn the time base control knob to the x-y external mode, change the frequency of the two signals such that $f_1 = f_2$, draw the shape you saw in part C of the data, change the frequency such that $f_1 = 2f_2$, draw this shape in part C of the data.

Table 1

No. of squares on y-axis (max-min)	y-channel voltage multiplier (volts)

Table 2

No of boxes on x-axis for 1 complete wave	Time base reading (sec)	Frequency reading Signal Generator (Hz)

CALCULATIONS

1) From table 1 find the peak to peak voltage (V_{PP}).
2) Find the peak voltage V_p.
3) From table 2 find the period of the wave (T).
4) Find the frequency of the wave, compare this frequency with the frequency obtained from the signal generator.

Experiment 11: RC Circuit using Oscilloscope

BACKGROUND AND THEORY

You have learned in experiment 9 how to measure the RC circuit time constant τ using a power supply and a stopwatch which was possible because the time constant was large enough for you to be able to measure it using the stopwatch. The values of R and C given to you gave a time constant ($\tau = RC$) of the order of a minute or more. If you have used values of R and C much less than those used in experiment 9, you will not be able to measure τ using a stop watch, but will be possible using the CRO.

In this experiment, you will construct a simple series RC circuit, but with values of R and C much smaller than the values you have used previously. You will measure the time constant (τ) and the half-life time ($t_{1/2}$) using a signal generator and the CRO

Consider the circuit consisting of a resistor R, a capacitor C and a signal generator connected in series (see Figure 11.1). The signal generator provides a square wave voltage to the circuit.

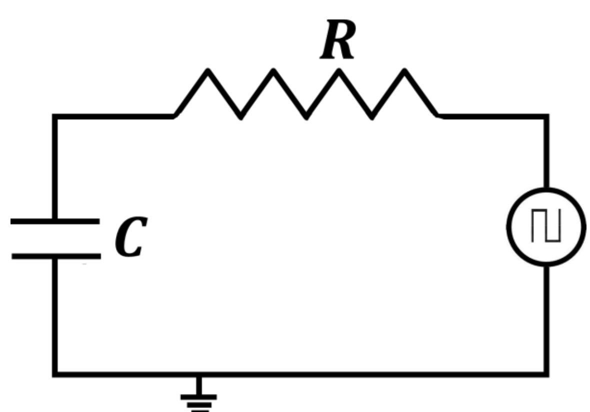


Figure 11. 1: RC circuit connect in series to square wave signal generator.

Recall from experiment 9 that the voltage on the capacitor for charging is

$$V(t) = \frac{Q_0}{C}\left(1 - e^{-\frac{t}{RC}}\right) \tag{11.1}$$

where t is time and Q_0 is the final charge on the capacitor at $t = \infty$. The charge on the capacitor for discharging is

$$V(t) = \frac{Q_0}{C} e^{-\frac{t}{RC}} \tag{11.2}$$

Therefore we expect to see the charging and discharging curves on the CRO screen as shown in Figure 11.2.

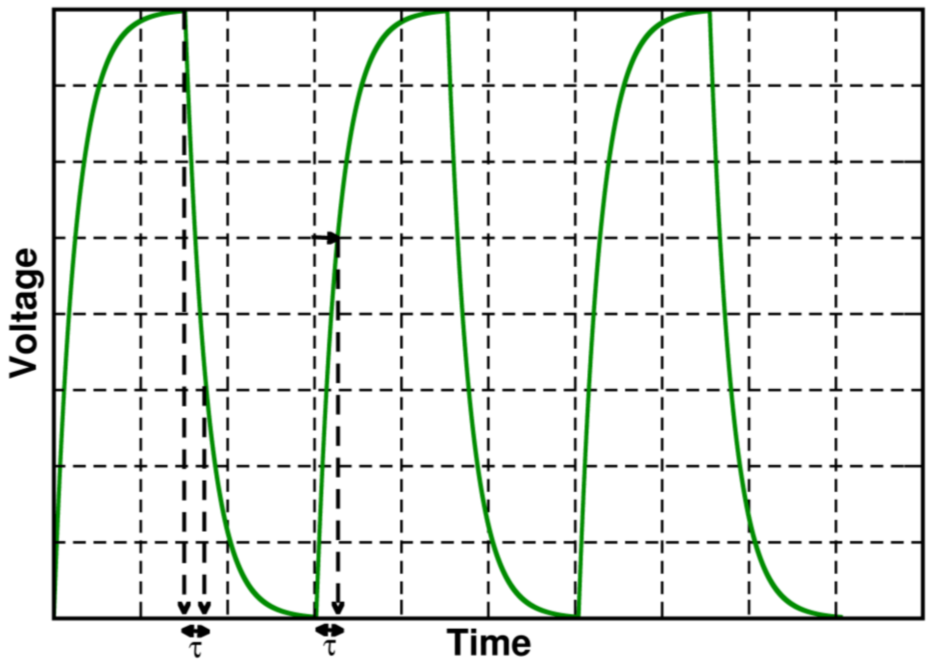


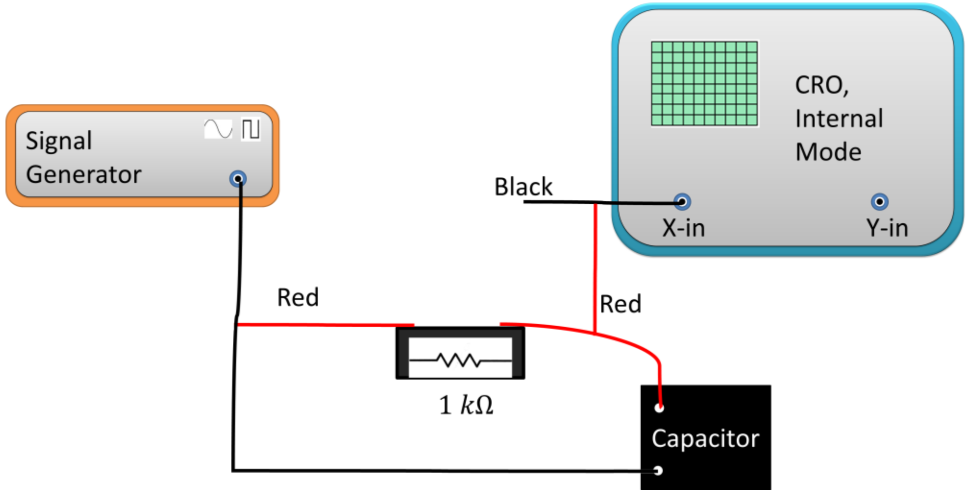
Figure 11. 2: Capacitor voltage as function of time as shown on CRO.

Let us find the theoretical relation between the time constant τ and $t_{1/2}$ for the RC circuit. We can use equation 11.1 or 11.2. Note that when $t = \tau$, the final voltage equals 0.63 of the initial voltage. When $t = t_{1/2}$, $V(t) = \frac{Q_0}{2C}$. Substituting the later in equation 11.2 yields

$$V(t_{1/2}) = \frac{Q_0}{2C} = \frac{Q_0}{C}\left(1 - e^{-\frac{t_{1/2}}{RC}}\right) \rightarrow -\frac{1}{2} = -e^{-\frac{t_{\frac{1}{2}}}{RC}} \rightarrow \; t_{1/2} = \tau \ln 2 \tag{11.1}$$

PROCEDURE

1) Connect the simple circuit shown in Figure 11.3. Use a square voltage wave from the signal generator. Refer to the instrument appendix to know how you can use the signal generator. Use a value of $R = 1k\Omega$ and a capacitor $C = 0.01\mu F$ or any other suitable value.
2) Connect point B to the x-input or the y-input of the CRO. You will be able to see the charging and discharging patterns on the CRO.



3) Change the frequency of the signal generator so that you get the best shapes of the charging and discharging curves.

4) Use the vertical and horizontal position control so that the charging curve begins at a chosen origin.

5) Draw the shapes of the charging and discharging curves on a linear graph paper.

6) Measure τ for charging by taking 0.63 of the maximum voltage, and then find the corresponding time by drawing a line from the point parallel to the y-axis. The line will intersect with the x-axis. Count the number of boxes from the origin to the point of intersection. Multiply this number with the reading of the time base control. This will be τ.

7) Measure τ for discharging by taking 0.37 of the maximum and repeating the same procedure. Write down your measurements in the table shown below.

8) From the discharging curve, measure $t_{1/2}$ and write down your result in the table shown below

$\tau_{charging}$ (sec)	$\tau_{discharging}$ (sec)	$\tau_{average}$ (sec)	$(t_{1/2})_{exp}$ (sec)

Calculation

1) Calculate the time constant τ for the circuit and compare your value with the corresponding experimental one.
2) Calculate the half-life $t_{1/2}$ of the circuit and compare your calculation with experimental value.

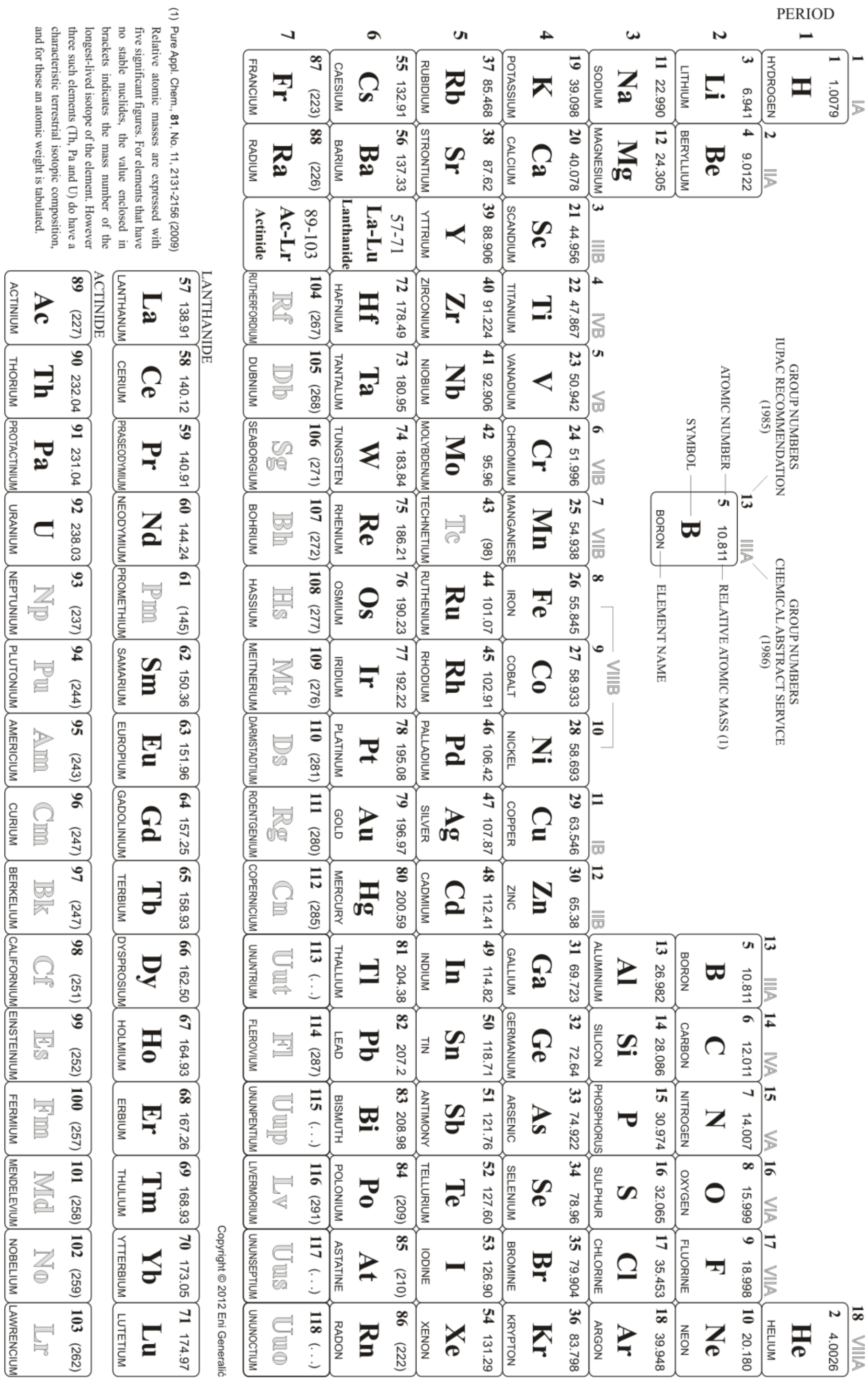
QUESTIONS

1) Does your experimental measurements agree with those predicted by theory, if not explain why?
2) Why you cannot measure the time constant of the circuit using a stopwatch and a digital voltmeter for $R = 2\ k\Omega$ and $C = 0.002\mu F$?

Appendix A: Table of Densities

[redacted]							
Aluminum	2.7	Molybdenum	10.22	Yttrium	3.8		
Antimony	6.62	Neodymium	6.95	Zinc	7.1	[redacted]	
Arsenic	5.73	Nickel	8.88	Zirconium	6.4	Acetone	0.792
Barium	3.5	Osmium	22.5	Alloys		Alcohol	
Beryllium	1.85	Palladium	12.16			Ethyl	0.791
Bismuth	9.78	Phosphorous		Bell metal	8.7	Methyl	0.810
Boron	2.53	Red	2.20	Brass	8.4-8.7	Benzene	0.899
Cadmium	8.65	Yellow	1.83	Bronze	8.8-8.9	Glycerin	1.26
Calcium	1.55	Platinum	21.45	Phosphor	8.8	Oil:	
Carbon:		Potassium	0.86	Constantan	8.88	Lubricating	0.90-0.92
Graphite	2.25	Praseodymium	6.48	Invar	8.00	Olive	0.92
Diamond	3.51	Rhodium	12.44	Magntium	2.0-2.5	Paraffin	0.82
Cerium	6.90	Rubidium	1.53	Manganin	8.5	Seawater	1.025
Cesium	1.87	Ruthenium	12.1	Steel	7.8	Normal H_2SO_4	1.0304
Chromium	7.14	Samarium	7.75	Wood's metal	9.5-10.5	Normal HCl	1.016
Cobalt	8.71	Scandium	3.02			Normal HNO_3	1.032
Copper	8.96	Selenium		[redacted]		Normal NaCl	1.0388
Gallium	5.93	(Amorph.)	1.92	Asbestos	2.0-2.8	Normal KOH	1.048
Germanium	5.46	Silicon		Celluloid	1.4	Normal KCl	1.0116
Gold	19.3	(Amorph.)	2.42	Cork	0.22-0.26		
Hafnium	13.3	Silver	10.49	Ebonite	1.15		
Indium	7.28	Sodium	0.97	Glass	2.4-2.8		
Iodine	4.94	Strontium	2.56	Ice, 0° C	0.917		
Iridium	22.42	Sulfur		Mica	2.6-3.2		
Iron:		(Amorph)	1.92	Fused Silica	2.1-2.2		
Pure iron	7.88	Tantalum	16.6	Paraffin Wax	0.9		
Wrought	7.85	Tellurium		Woods (oven			
Cast	7.6	Crystal	6.25	Ash	0.52-0.64		
Steel	7.83	Thallium	11.86	Balsa	0.12-0.20		
Lanthanum	6.15	Thorium	11.3	Beech	0.65-0.67		
Lead	11.34	Tin	7.3	Elm	0.55-0.67		
Lithium	0.534	Titanium	4.5	Mahogany	0.54-0.67		
Magnesium	1.74	Tungsten	19.3	Oak	0.67-0.98		
Manganese	7.41	Uranium	18.7	Teak	0.58		
Mercury		Vanadium	5.98				
(Solid -39° C)	14.19						

Appendix B: Periodic Table of Elements



Appendix C: Using the Scientific Calculator

The standard calculator used at the Physics 111 Lab is the Casio fx-82MS shown in the picture to the right. You are advised to learn how to use it, especially in the Statistics mode to calculate various statistical quantities such as the mean and the sample standard deviation. The table below shows some of the basic operations of this calculator and includes a link to the more detailed user manual of it.

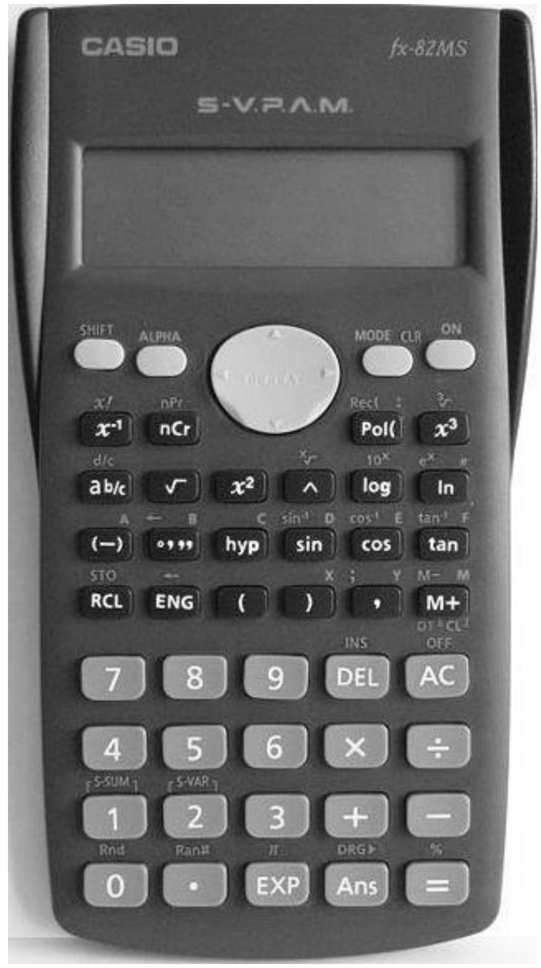


Figure C. 1: Picture of Casio fx-82MS

Casio fx-82MS Calculator	
Use Scientific Notation	[Mode] [Mode] [Mode] [2]
Set Deg/Rad	[Mode] [Mode] [1] for Degrees [Mode] [Mode] [2] for Radians
Enter Statistics Mode	[Mode] [2]
Data Entry	Enter data pressing [M+] after each entry.
To calculate statistical quantities:	[Shift][2][1][=] for the mean [Shift][2][3][=] for the sample standard deviation
Online Manual	http://support.casio.com/en/manual/004/fx-82SX_220PLUS_etc_EN.pdf

Suppose that you have N measurments $x_1, x_2, \ldots, x_N$ and suppose you want to calculate the mean, the sample standard deviation, the uncertainty in the mean. These and other statistical quantities can be calculated on the Casio fx-82MS calculator using the following steps:

- Turn on the calculator by pressing the [ON] button.
- Clear the calculator memory form any stored data by pressing [Shift][Mode][3][=][=].
- Enter the Statistics mode by pressing [Mode][2]. The letters SD will appear in small font in the upper left side of the calculator screen.
- Enter the first measurement on the calculator followed by the [M+] button, then enter the second measurement followed by the [M+] button, and so on until you finish from all measurements.
- The mean of the measurement ($\bar{x}$) now can be calculated by pressing [Shift][2][1][=].
- The sample standard deviation (σ_s) can be calculated by pressing [Shift][2][3][=].
- The uncertainty in the mean (σ_m) can be calculated by dividing σ_s by $\sqrt{N}$.

$$\sigma_m = \frac{\sigma_s}{\sqrt{N}}$$

- The quantity ($\sum_{i=1}^{N} x_i^2$) can be calculated by pressing [Shift][1][1][=].
- The number of data points entered can be found by pressing [Shift][1][3][=].

Appendix D: Electric and Electronic Elements

1) **Resistors** are commonly used components in electronics. There are many types of resistors ·used including:

   a. **Carbon Resistors:** Carbon resistors are color coded, indicating their value of resistance and tolerance. Most of the resistors you will encounter in the physics 111 lab will have four bands as shown below:

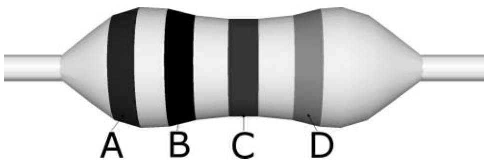


Figure D. 1: Schematic Representation of A Carbon Resistor.

- **Band A** corresponds to the first significant digit.
- **Band B** corresponds to the second significant digit.
- **Band C** represents the decimal multiplier.
- **Band D** indicates the tolerance.

The colors are coded as follows

Color	Band A&B	Band C	Band D
Black	0	10^0	Non $\pm$ 20%
Brown	1	10^1	Silver $\pm$ 10%
Red	2	10^2	Gold $\pm$ 5%
Orange	3	10^3	Red $\pm$ 2%
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	

Examples:

- A resistor with Red, Black, Brown and Gold colors has a resistance of $20 \times 10^1 \Omega$ and its tolerance is equal to $20 \times 10^1 \times 0.05 = 10\ \Omega$. Therefore the resistance of the resistor is $(2.0 \pm 0.1) \times 10^2 \Omega$
- A resistor with Orange, Brown, Red and Silver colors has a resistance of $31 \times 10^2 \Omega$ and its tolerance is equal to $31 \times 10^2 \times 0.1 = 31 \times 10^1 \Omega$. Therefore the resistance of the resistor is $(3.1 \pm 0.3) \times 10^3 \Omega$.

b. **Variable resistors (Potentiometer):** These are resistors in which the resistance can be adjusted. There are two common types of these variable resistors, carbon strip type and wire wound type. The carbon strip type of a carbon strip, usually a semicircle in shape in shape, with a movable contact (the wiper) that is attached to a shaft and slides on the carbon strip. In the wire wound type, rather than having a carbon strip, it has a long length of un insulated wire wound around a form.

Figure D. 2: A Potentiometer

2) **Capacitor:** A capacitor is an arrangement of one or more pairs of conductors separated by insulators between which an electric field can be produced. The conductors are called electrodes or plates, and the insulator is called the dielectric. The capacitor may have a fixed or variable va1ue. Its capacitance is measured in units of Farad. The practical units of capacitance are the microfarad. The simplest capacitor consists of two conducting parallel palates separated by a dielectric material like air.

Figure D. 3: An electrolytic capacitor.

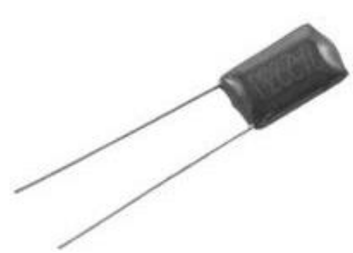
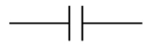
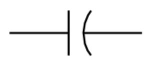
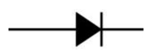
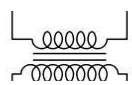
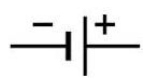
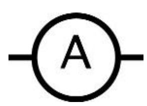
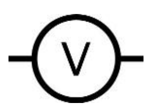
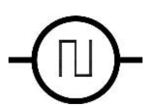
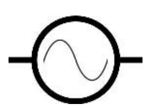


Figure D. 4: A ceramic capacitor.

Appendix E: Standard Symbols for Electrical Circuit Elements

Connecting Wires	
Fixed Resistor	
Variable Resistor, (two terminals Reohstat)	
Variable Resistor (Three terminal)	
Non-Electrolytic Capacitor	
Electrolytic Capacitor	
Diode	
Transistor	
Inductance	
Transformer	
Battery	
Fuse	
Switch	
Ammeter	
Voltmeter	
Square Wave Signal Generator	
Sine Wave Signal Generator	

Appendix F: Sample Lab Report

Determination of the Acceleration due to Gravity (g)

Abstract

The aim of the experiment: is to determine the acceleration of a falling object under the influence of earth gravity.

The method used: is by measuring the time of fall of a metal weight over a measured distance.

The main result: is $g = 970 \pm 30\ cm/s^2$

Theory

An object acted on only by its weight falls with constant acceleration (g). If the object starts from rest initially, then within a time interval t the object would fall a distance given by

$$y = \frac{1}{2} g t^2$$

Hence

$$g = \frac{2y}{t^2}$$

and the uncertainty in g can obtained from

$$\frac{\Delta g}{g} = \frac{\Delta y}{y} + 2\frac{\Delta t}{t}$$

PROCEDURE

A solenoid was connected to an alternating voltage of $50\ Hz$, so the core frequency is $50\ Hz$ as well. A falling weight was connected to a narrow strip of paper, when the object is released, the core, which hits periodically the strip and leaves marks on it every 0.02 seconds. Initially, the core is moved to the left, making the zero point on the paper strip, and the weight is hung on the core by means of a hook. When the solenoid is turned on, the core starts to oscillate allowing the weight to fall.

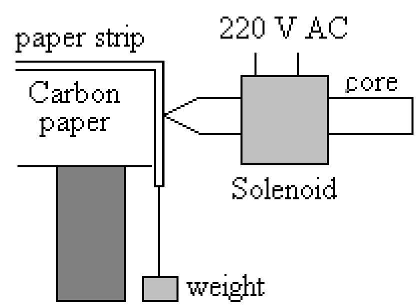


Figure F. 1: Determination of g Experiment Setup.

Data

The frequency of the solenoid $f = 50.0 \pm 0.1\ Hz$

Time between marks $T = \frac{1}{50.0} = 0.020\ seconds$

Estimated uncertainty in y, $\Delta y = \pm 0.2\ cm$

Mark Number	Distance $\boldsymbol{y}$ ($\boldsymbol{cm}$)	Time $\boldsymbol{t}$ ($\mathbf{sec}$)	$\boldsymbol{t^2}$ ($\mathbf{sec^2}$)
0	0.0	0.000	0.000
1	0.4	0.020	0.0004
2	0.7	0.040	0.0016
3	1.9	0.060	0.0036
4	3.1	0.080	0.0064
5	4.9	0.100	0.0100
6	6.9	0.120	0.0144
7	~~19.5~~ 9.5	0.140	0.0196
8	12.6	0.160	0.0256
9	15.6	0.180	0.0324

CALCULATIONS

- $f = 50.0\ Hz$ and therefore $\mathrm{T} = \frac{1}{50.0} = 0.020\ sec$. Time for the n^{th} mark is $nT = 0.020\ n$.
- Centroid for the graph $\left(\bar{y}, \overline{t^2}\right) = (7.5\ cm, 0.0154\ \sec^2)$.
- Slope of the graph $m = \frac{\Delta y}{\Delta t^2} = 487 \frac{cm}{\sec^2}$ (see the graph paper).
- The equation$y = \frac{1}{2} g t^2$ is similar to the straight line equation $y = mx$ with slope m, which means that $m = \frac{1}{2} g$ or $g = 2m = 967.8 \frac{cm}{\sec^2}$.
- Calculation of Δg
  Using the equation derived in the theory;
  $$\frac{\Delta g}{g} = \frac{\Delta y}{y} + 2\frac{\Delta t}{t}$$
  But
  $$T = \frac{1}{f}$$
  Hence
  $$\frac{\Delta T}{T} = \frac{\Delta f}{f} = \frac{0.1}{50.0} = 0.002$$

Note also $t = nT$ so $\frac{\Delta t}{t} = \frac{\Delta T}{T} = 0.002$ and $\Delta y = 0.2\ cm$. Clearly $\frac{\Delta y}{y}$ will be different for different values of y, so instead we use the average value of y, $\bar{y} = 7.5\ cm$.

Therefore,

$$\frac{\Delta y}{\bar{y}} = \frac{0.2}{7.5} = 0.027$$

Subsequently, we get

$$\frac{\Delta g}{g} = 0.027 + 2(0.002) = 0.031$$

Finally, we obtain the uncertainty in g

$$\Delta g = 0.031g = 0.031(967.8) = 30.0\frac{cm}{sec^2}$$

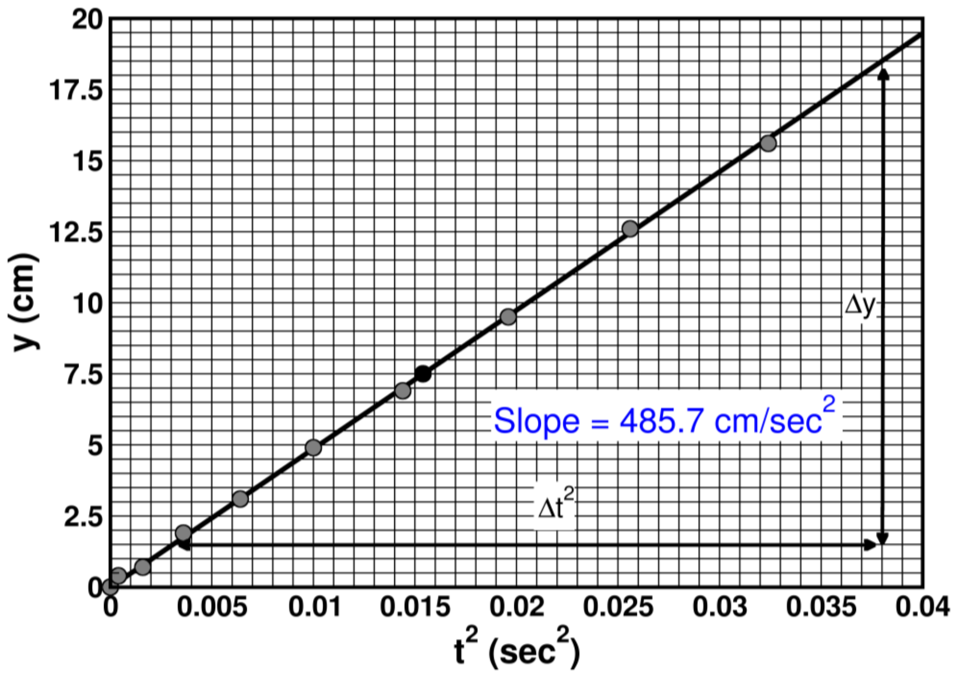


Figure F. 2: Determination of g; slope calculation.

Results and Conclusion

$$g = 970 \pm 30\ cm/sec^2$$

The accepted value of g is 980 $\frac{cm}{\text{sec}^2}$. But at the altitude of Birzeit, g is expected to be lower than that value. Our result agrees with the accepted value of g within the experimental error estimated.

There was a systematic error from the fact that the weight was not allowed to fall freely and because of friction as well as the blows from the solenoid on the paper strip. These effects would result in a lower value of g than the actual one.

Appendix G: Some of The Instruments Used in The Physics 111 Lab.

In this appendix, we briefly introduce some of the instruments used in experiments during this Laboratory.

1) **Meter Stick:** A part of the meter stick is shown below, as you can see, the smallest division in the meter stick is 1mm, so you can use the stick to measure a length as small as 1mm.

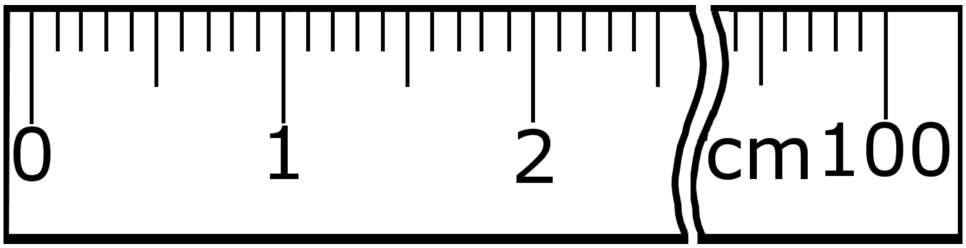


Figure G. 1: Meter Stick.

2) **Vernier Caliper:** A schematic representation for a vernier caliper is shown below. The caliper can measure lengths as small as 1/20 mm (0.05 mm).

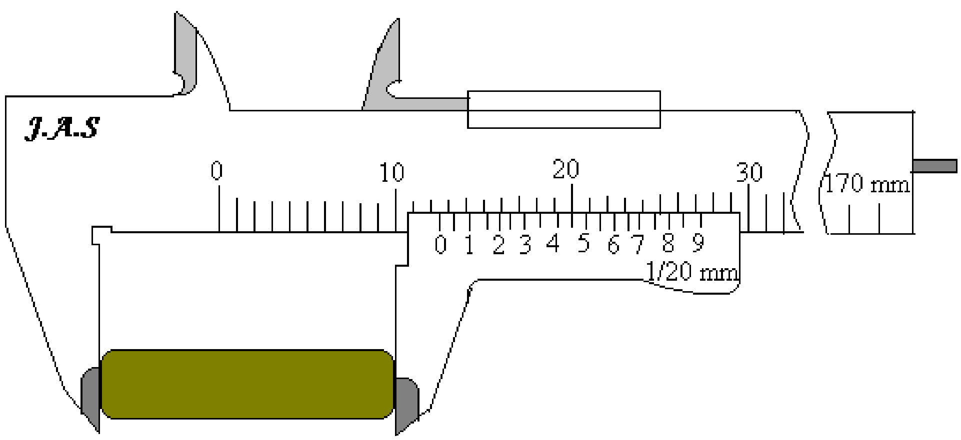


Figure G. 2: Vernier Caliper.

Let us show you how one can read the caliper shown in Figure G.2. Notice that the caliper has two readings, the lower reading and the upper reading, notice that the zero from the lower reading points between 12 and 13 of the upper reading (nearly midway between 12 and 13). Notice also the complete alignment between the vertical lines in the two readings occur only at one place, 4.5 mm the lower reading and 20 from the upper reading.

The reading of the caliper is: 12 mm +0.45 mm=12.45 mm

3) **The Micrometer:** A schematic representation for a micrometer is shown below. The micrometer can measure lengths as small as 0.01 mm; the smallest division in it is 0.01 mm.

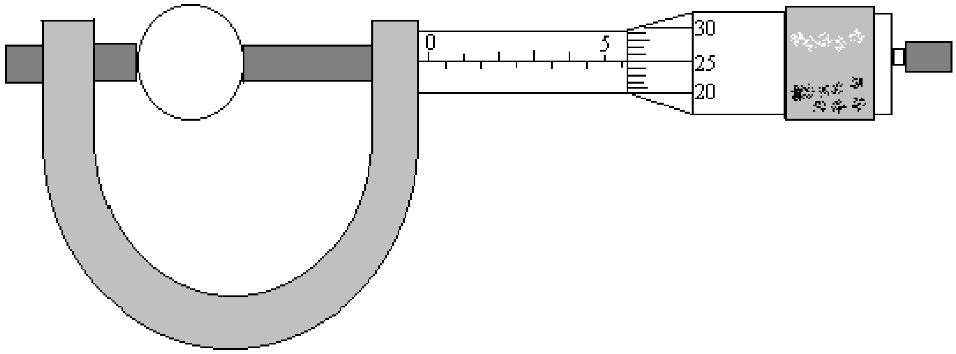


Figure G. 3: Micrometer.

Let us show you how one can read the micrometer shown above in Figure G.3. Notice that on the micrometer, there are two readings, a horizontal reading on the cylindrical metallic solid piece, and a vertical reading on a rotating metallic piece.

The Micrometer Reading is: 5.5 mm +0.25 mm=5.75 mm

4) **Scale Voltmeters and Ammeter:** A schematic representation a scale voltmeter with a 15 volt scale is shown in Figure G.4. The scale voltmeter shown has three scales; 3 volt, 15 volt and 150 volts. When you use the three volt scale, you cannot measure voltages greater than three volts, and the same applies for the 15 volt scale and the 150 volt scale. For demonstration purpose only the 15 volt scale is shown in Figure B.4. Note that the red input in the scale voltmeter is the positive input while the black input is the negative input.

Figure G. 4: Scale Voltmeter.

When you want to measure the voltage across a resistance, you have to make sure that the scale voltmeter is connected correctly; that is the positive input should end at the positive socket of the voltage source and the negative input should also end at the negative of the battery.

For example, suppose that you have a circuit consisting of two resistances connected in series to a three volts power supply. Now, suppose that you would like to measure the voltage across the resistance R_2 using the scale voltmeter shown in the figure below, then he will connect the scale voltmeter as shown in Figure D.5, from this figure the reading of the voltmeter is near 1.9 volts (to the nearest 0.1 volt).

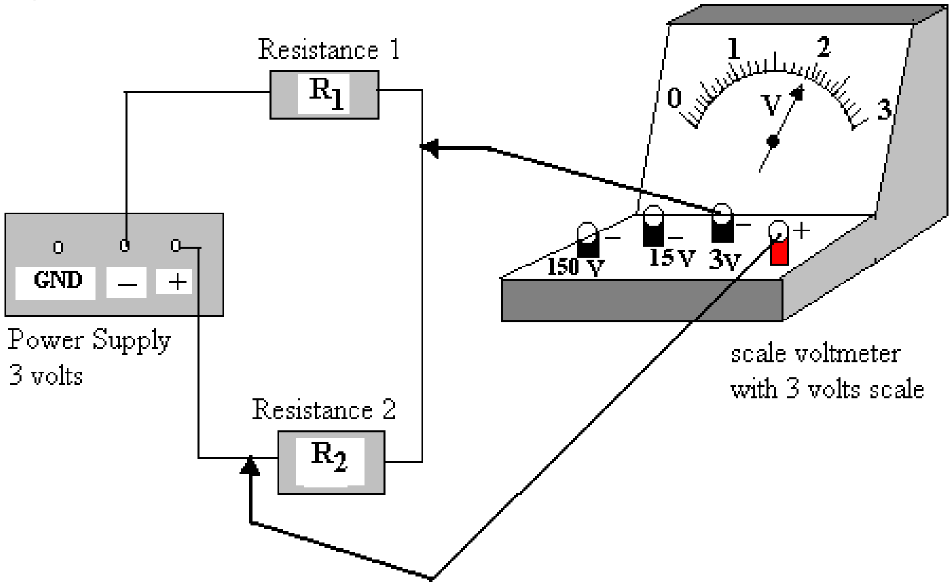


Figure D. 5: Example for Using a Voltmeter in an Electric Circuit.

A picture for the scale ammeter is shown in Figure D.6. The ammeter has three scales; 50 mA, 500 mA and 5A Scales. The 50 mA scale can measure a maximum current of 50 mA, and the same is for the 500 mA and 5 A scales.

We note here that when you connect the scale ammeter in a circuit to measure a current, you have to connect the ammeter such that its direction is in the direction of the current, positive to negative; remember that as we mentioned earlier the positive has the red color and the negative has the black color.

Figure D. 6: An Ammeter

5) **Digital multi-meter:** A picture for the digital multi-meter is shown in Figure D.7. The digital multi-meter can measure DC and AC current. DC and AC voltages and resistance. Let us first show you how one can use the digital multi-meter as an ammeter. Suppose that you want to measure a DC current through a resistance R using the multi-meter, then you have to use two wires; you have to connect one end of the first wire to the COM input,

connect one end of the second wire to mA input and push in mA button (button #3 in Figure B. 7), you then connect the ammeter in series with the resistance. Note that you can select the range that you need using the buttons #6 to #9, if you push in button #5, then the number you read from the digital multi-meter screen is in mA (note that $1mA = 10^{-3}A$) and the maximum current that you can measure using this button is $200\ mA$.

Suppose that you want measure a current near 1 mA (0.001 A), then you have to select a range greater than 1 mA so that you can measure that current, otherwise the multi-meter will not show you the reading.

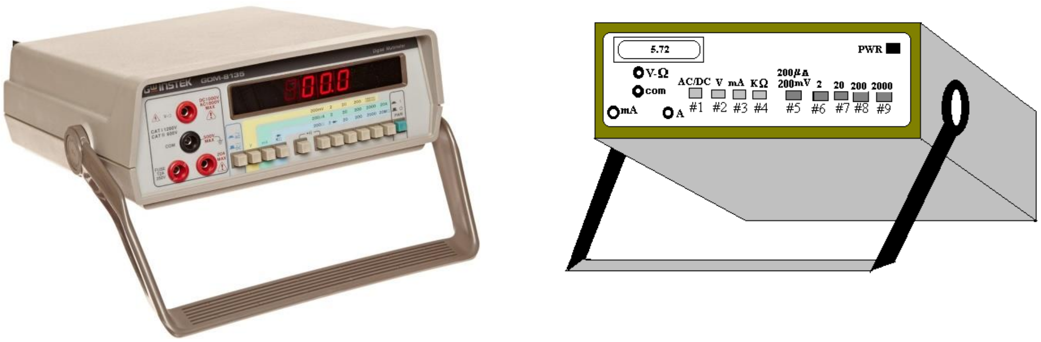


Figure D. 7: Digital Multimeter.

If you want to use the multi-meter to measure the voltage on a resistance R, then you have to use two wires, connect one end of the first wire to the COM input, connect one end of the second wire to v-n input, push in the v button (button #2 in the picture) and connect the other ends of the wires in parallel with the resistance. You can select the range you need using the buttons #5 to #9, if you push button #5, then the reading you see in the digital multi-meter screen _is -in mV ($1\ mV\ =\ 10^{-3}\ V$) and the maximum voltage you can measure using this button is 200 mV.

We note here that if you want to measure AC current or AC voltage using the multi-meter them you have to push in button #1 shown in Figure D.7.

Suppose that you want to measure the value of an unknown resistance R using the multimeter. You can do that by connecting one end of the resistance to the COM input, the other end of the resistance to the $V - \Omega$ input and pushing in the $K\Omega$ button. Note that you should select the suitable range using the buttons #5 to #6.

Figure D. 8: Signal Generator.

6) **Signal Generator**: Consider the picture for the signal generator shown in Figure D.8, this signal generator provides three kinds of electrical waveforms with any frequency you need from 0.1 Hz to 100 kHz. Suppose that you need a square wave input to your circuit with a frequency of 2 kHz, the signal generator can provide you a square wave, this is possible by connecting a cable from the 50Ω output and connect its other end to your circuit, as you see from the figure the signal generator can provide you with three kinds of waves: (1), sinusoidal waveform (2) square wave form (3) triangular wave form; you can select the square waveform by pushing the square wave button shown in the figure. To obtain a frequency of 2 kHz, you can push the kHz button and rotate the rotating piece such that the reading of that piece is 2, now the frequency you obtain in this case is 1 kHz multiplied by 2.

You can also obtain a frequency of 2 kHz by pushing the 10 kHz button and then rotate that piece such that its reading is 0.2, then the frequency which you will obtain in this case is 10 kHz multiplied by 0.2 which equals to 2 kHz. We note here that there are signal generators which you may use and provides the reading of the frequency of the wave on a screen instead of the rotating piece used in this signal generator.

7) **Balance Scale**: Figure D.9 is a picture for a balance scal1, notice from the picture that the balance has four scales. Each scale has a movable metallic piece on it which you can move along the rod to the right. We note here that the first scale has only three positions; 0,100 and 200 grams. You should not place the movable piece in a position between 0 gram and 100 gram. The second scale has 11 positions 0, 10, 20,.., 100 gram. You should only place the movable metallic piece in one of these readings from 0 to 100 gram. The third scale has also 11 readings from 0 to 10 grams. The fourth scale has readings from 0 to 1 gram, notice also that on the fourth scale each 0.1 division is divided into 10 divisions. You can read the scale balance shown below:

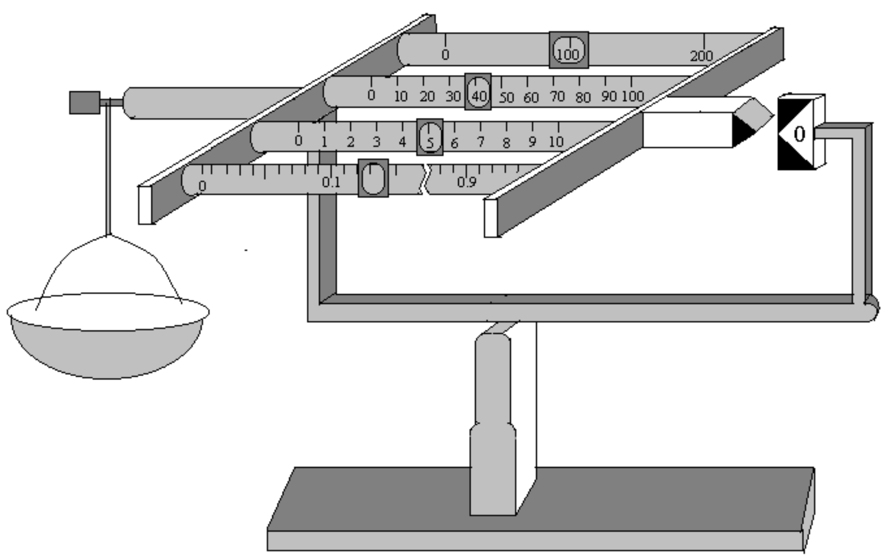


Figure D. 9: Balance Scale.

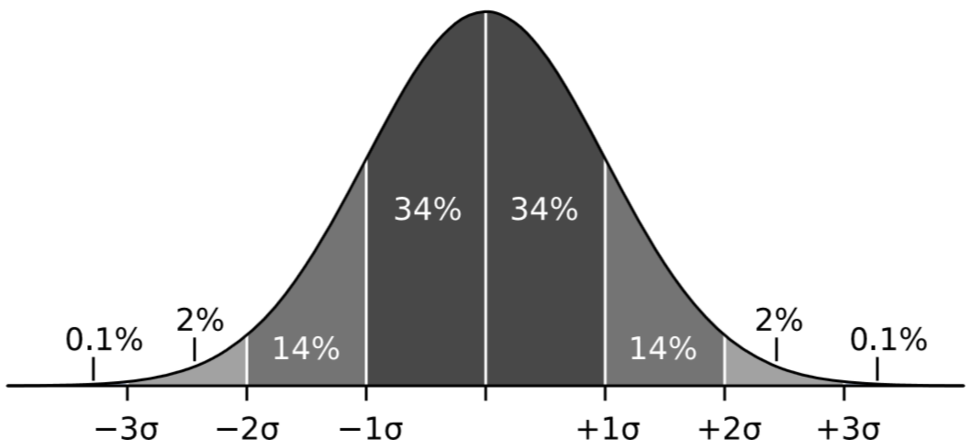
The Balance Reading is: 100+40+5+0.13=145.13g

But we note here that the balance we have in the physics 111 Lab. has an uncertainty much greater than 0.01 gram; ask your instructor about the uncertainty of the balance you are using.

Appendix H: Gaussian Distribution

Suppose that you have N measurments $x_1, x_2, \ldots, x_N$, the probability that a measurement x_i will lie between $\bar{x} - n\sigma_s$ and $\bar{x} + n\sigma_s$ is given by

$$P(\bar{x} - n\sigma_s \leq x_i \leq \bar{x} + n\sigma_s) = \int_{\bar{x}-n\sigma_s}^{\bar{x}+n\sigma_s} e^{-(x-\bar{x})^2/2\sigma_s} dx$$



- About 68% of the measurements are within ± 1 standard deviation of the mean
- About 95% of the measurements are within ± 2 standard deviations of the mean.

The value percentage probability for several values of n is summarized in the table below.

n	$P(\bar{x} - n\sigma_s \leq x_i \leq \bar{x} + n\sigma_s) \times 100\%$
1	68.27
2	95.45
3	99.73
4	99.994
5	99.99994

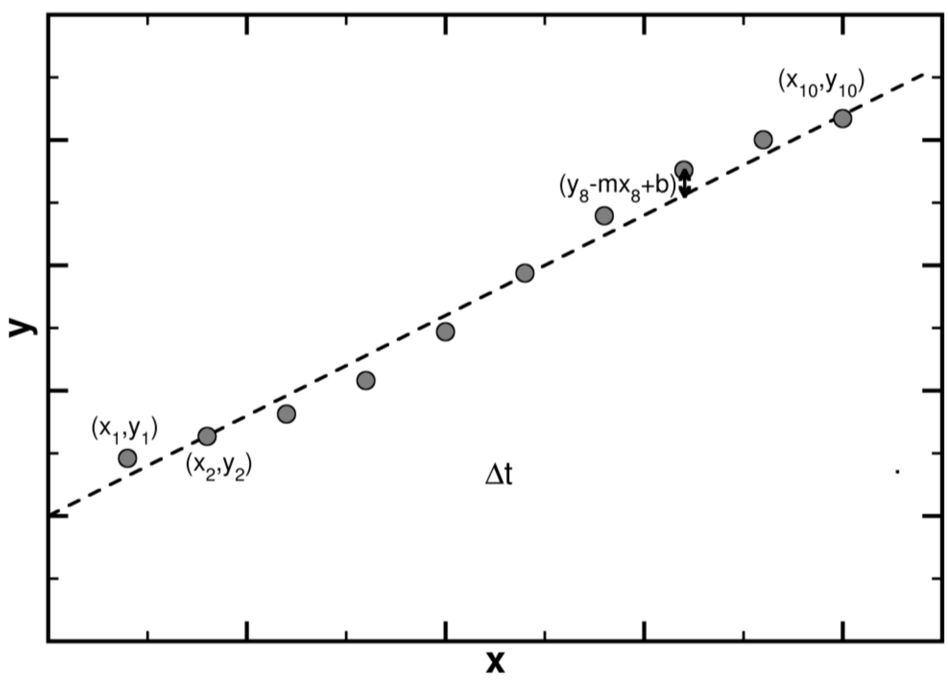
Appendix I: Method of Least Square

Suppose that you measured two quantities x and y, and you plotted y vs. x on a linear graph paper. If the relationship between y and x is a linear one, then all points will lie on a straight line. We mentioned earlier is the "average" one which passes as close as possible to as many points as of the points. Drawing that line was a matter of judgment, not any more. In this section we want to show you how to plot the *best line* for your data points; as you already know the equation of the straight line is:

$$y = mx + b \tag{J.1}$$

In the method of least square fit, you will find the best slope "m" for your data points, the best y-intercept "b", and the errors: Δm and Δb.

Consider the graph of y versus x for a set of N data points $\{(X_1, Y_1), (X_2, Y_2), \cdots (X_N, Y_N)\}$, as an example this is shown in Figure J.1.



Let us now find the best straight line of the form $y = mx + b$, that fits the set of measurements $\{(X_1, Y_1), (X_2, Y_2), \cdots (X_N, Y_N)\}$. Assuming that we knew the best slope "m" and the best y-intercept "b", then for each value of X_i , in addition to the measured value Y_i , we can find the corresponding value which lies on the best line, (the calculated y) y_i

such that $y_i = mX_i + b$. The best line is the one which makes the quantity $|Y_i - y_i|$ as small as possible for all points. This expression is expresses as

$$\delta^2 = \sum_{i=1}^{N}(Y_i - y_i)^2 \tag{J.2}$$

is minimum.

Notice that δ^2 is a function of the best slope (m) and the best y-intercept (b). Now let us find the values of m and b which makes δ^2a minimum. δ^2 will be a minimum when

$$\frac{\partial(\delta^2)}{\partial m} = 0 \text{ and } \frac{\partial(\delta^2)}{\partial b} = 0 \tag{J.3}$$

Substituting equation J.1 into equation J.2, we get :

$$\begin{aligned}\delta^2 &= \sum_{i=1}^{N}(Y_i - mX_i - b)^2 \\ &= \sum_{i=1}^{N}(Y_i^2 - 2mX_iY_i - 2bY_i + m^2X_i^2 + 2mbX_i + b^2) \\ &= \sum_{i=1}^{N}Y_i^2 - 2m\sum_{i=1}^{N}X_iY_i - 2b\sum_{i=1}^{N}Y_i + m^2\sum_{i=1}^{N}X_i^2 + 2mb\sum_{i=1}^{N}X_i \\ &+ Nb^2\end{aligned} \tag{J.4}$$

$$\frac{\partial(\delta^2)}{\partial m} = -2\sum_{i=1}^{N}X_iY_i + 2m\sum_{i=1}^{N}X_i^2 + 2b\sum_{i=1}^{N}X_i = 0 \tag{J.5}$$

and

$$\frac{\partial(\delta^2)}{\partial b} = -2\sum_{i=1}^{N}Y_i + 2m\sum_{i=1}^{N}X_i + 2Nb = 0 \tag{J.6}$$

Solving equations J.6 and J.7 for m and b yields

$$m = \frac{N(\sum_{i=1}^{N}X_iY_i) - (\sum_{i=1}^{N}X_i)(\sum_{i=1}^{N}Y_i)}{D} \tag{J.7}$$

and

$$b = \frac{(\sum_{i=1}^{N} X_i^2)(\sum_{i=1}^{N} Y_i) - (\sum_{i=1}^{N} X_i)(\sum_{i=1}^{N} X_i Y_i)}{D} \quad \text{(J.8)}$$

where

$$D = N\left(\sum_{i=1}^{N} X_i^2\right) - \left(\sum_{i=1}^{N} X_i\right)^2 \quad \text{(J.9)}$$

The results; equations J.7 and J.8 give the best estimates for the constants m and b of the straight line $y = mx + b$, based on the experimental data points $\{(X_1, Y_1), (X_2, Y_2), \cdots\ (X_N, Y_N)\}$.

The next step is to find the uncertainties in m and b, $\Delta m\ and\ \Delta b$, the estimated errors in the best slope and the y-intercept. To do this, we must first estimate the uncertainty σ_y in the original measurements $(Y_1, Y_2, Y_3 \cdots Y_N)$. One must remember that these measurements are not N m$e$asurements of the same quantity. Nevertheless we can estimate the uncertainty by considering the distribution of the deviation of the experimental points from their best values, i.e. the values that fit the straight line. This distribution is normal. $Y_i - mX_i - b$ are normally distributed all with the same central value zero and same width σ_y. This suggests that a good estimate for σ_y would be given by a sum of squares with the form:

$$\sigma_y^2 = \frac{1}{N-2}\sum_{i=1}^{N}(Y_i - mX_i - b)^2 \quad \text{(J.10)}$$

To understand the factor $N - 2$ in the denominator, consider the case where we measure just two pairs of data points (X_1, Y_1) and (X_2, Y_2), with only two points we can find a line that passes exactly through both points, and one can easily check that the least square fit gives this line. The last equation gives $\sigma_y = 0/0$ indicating correctly that σ_y is undetermined after two measurements, Using N in the denominator gives $\sigma_y = 0$, which is not the case.

Having found that the uncertainty σ_y, one can find the uncertainties in m and b by doing error propagation in terms of those in $Y_1, Y_2\ \ldots . Y_N$ (we assume that the errors in X_i, are negligible), we will not do it here but it is easy to show that

$$(\Delta m)^2 = \sigma_m^2 = \frac{N\sigma_y^2}{D} \text{ and } (\Delta b)^2 = \sigma_b^2 = \frac{\sigma_y^2}{D}\sum_{i=1}^{N} X_i^2 \quad \text{(J.11)}$$

Example:

Consider an idea gas whose volume is kept constant, then its temperature is a linear function of its pressure P. That is

$$T = aP + b$$

Here the constant b is the temperature at which the pressure would drop to zero. This constant depends on the nature of the gas, its mass and its volume. By measuring a series of values for T and P, we can find the best estimation of the constants $a\ and\ b$.

Suppose that you got the following measurements of $P\ and\ T$. Shown in the table below

Pressure ($mmHg$)	65	75	85	95	105
Temperature (℃)	-20	17	42	94	127

We want to find the constants $a\ and\ b$ using least square fit method as follows

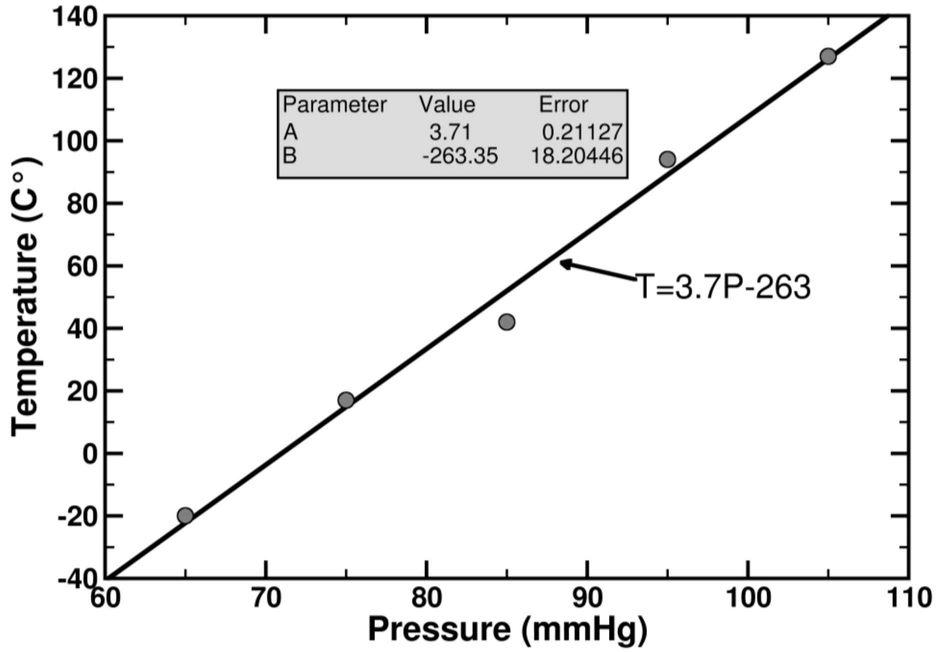
$$\sum_{i=1}^{5} P_i = 425, \sum_{i=1}^{5} P_i^2 = 37125, \sum_{i=1}^{5} T_i = 260, \sum_{i=1}^{5} T_i^2 = 27418\ and\ \sum_{i=1}^{5} P_i T_i = 25810$$

Hence

$$D = 5 \times 37125 - 425^2 = 5000, a = \frac{5 \times 25810 - 425 \times 260}{5000} = 3.71$$

$$and\ b = \frac{37125 \times 260 - 425 \times 25810}{5000} = -263.35$$

Also we can find the uncertainty in m and b using equations J.11. We get $\Delta m = 0.21\ and\ \Delta b = 18$. Therefore the final result is $a = 3.7 \pm 0.2$ ℃$/mmHg$ and $b = -260 \pm 20$ ℃ .


Parameter Value Error
A 3.71 0.21127
B -263.35 18.20446
T=3.7P-263
Temperature (C°)
Pressure (mmHg)
140
120
100
80
60
40
20
0
-20
-40
60
70
80
90
100
110