APPLICATION OF THE LAPLACE TRANSFORM TO CIRCUIT ANALYSIS

LEARNING GOALS

Laplace circuit solutions
Showing the usefulness of the Laplace transform

Circuit Element Models
Transforming circuits into the Laplace domain

Analysis Techniques
All standard analysis techniques, KVL, KCL, node, loop analysis, Thevenin's theorem are applicable

Transfer Function
The concept is revisited and given a formal meaning

Chapter 13 The Laplace Transform in Circuit Analysis

- Circuit Elements in the s Domain
- Circuit Analysis in the s Domain
- The Transfer Function and the Steady- State Sinusoidal Response

Why Another Technique?

1) We wish to consider transient behavior of circuits containing more than a single node voltage or mesh current
2) Transient response of circuits containing complicated signal sources
3) Laplace is used to consider/define transfer functions
4) Relate time domain to frequency domain behavior

Definition of Laplace transform

$$\mathcal{L}[f(t)] = f(s) = \int_0^\infty f(t)e^{-st}dt$$

F(s) = £{ f(t)}

Laplace transform of some elementary functions

TABLE 12.1 An Abbreviated List of Laplace Transform Pairs

Type	$f(t)$ $(t > 0-)$	$F(s)$
(impulse)	$\delta(t)$	1
(step)	$u(t)$	$\frac{1}{s}$
(ramp)	t	$\frac{1}{s^2}$
(exponential)	e^{-at}	$\frac{1}{s+a}$
(sine)	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
(cosine)	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$

TABLE 12.1 An Abbreviated List of Laplace Transform Pairs

Type	$f(t)$ $(t > 0-)$	$F(s)$
(damped ramp)	te^{-at}	$\frac{1}{(s+a)^2}$
(damped sine)	$e^{-at}\sin\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
(damped cosine)	$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

CIRCUIT ELEMENT MODELS

For a more efficient approach:
1. Develop s-domain models for circuit elements
2. Draw the "Laplace equivalent circuit" keeping the interconnections and replacing the elements by their s-domain models
3. Analyze the Laplace equivalent circuit. All usual circuit tools are applicable and all equations are algebraic.

Independent sources

$$v_S(t) \rightarrow V_S(s)$$
$$i_S(t) \rightarrow I_S(s)$$

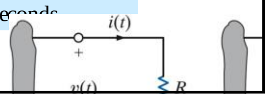
Time domain ⇒ **Frequency domain**

Dependent sources

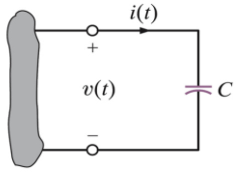
$$v_D(t) = Ai_C(t) \rightarrow V_D(s) = AI_C(s)$$
$$i_D(t) = Bv_C(t) \rightarrow I_D(s) = BV_C(s)$$
...

Resistor

$$v(t) = Ri(t) \Rightarrow V(s) = RI(s)$$
$I(s)$ ampere.seconds
$V(s)$ volt.se

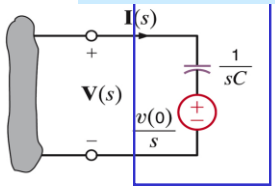


Capacitor: Model 1



$v(t)=\frac{1}{C}\int_0^t i(x)dx+v(0^-)$

Source transformation



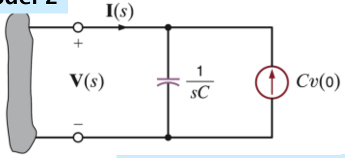
$V(s)=\frac{1}{Cs}I(s)+\frac{v(0^-)}{s}$

Impedance in series with voltage source

$\mathcal{L}\left[\int_0^t i(x)dx\right]=\frac{I(s)}{s}$

$I_{eq}=\frac{\frac{v(0^-)}{s}}{\frac{1}{Cs}}=Cv(0^-)$

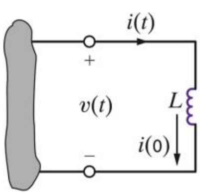
Capacitor: Model 2

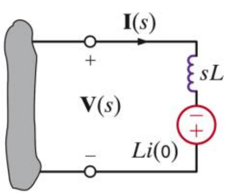


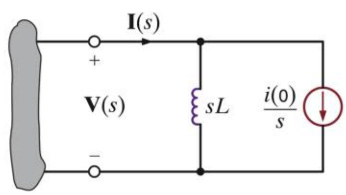
$I(s)=CsV(s)-Cv(0^-)$

Impedance in parallel with current source

Inductor Models







$v(t)=L\frac{di}{dt}(t)\Rightarrow V(s)=L(sI(s)-i(0^-))$

$I(s)=\frac{V(s)}{Ls}+\frac{i(0^-)}{s}$

$\mathcal{L}\left[\frac{di}{dt}\right]=sI(s)-i(0^-)$

Without Initial Conditions

$$v_c(0^-) = 0$$

$$i_L(0^-) = 0$$

		Z	G
C	$\rightarrow$	$\frac{1}{sC}$	sC
L	$\rightarrow$	sL	$\frac{1}{sL}$
R	$\rightarrow$	R	$\frac{1}{R}$

- **All Previous series, parallel, Δ-Y conversions are applicable in s-domain**
- **KCL, KVL are also applicable**
- **All other techniques can be used in s -domain**
- **Only with energy stored in s-domain, ohm's law is modified to add sources related to initial energy in L and C**

How to analyze a circuit in the s-domain?

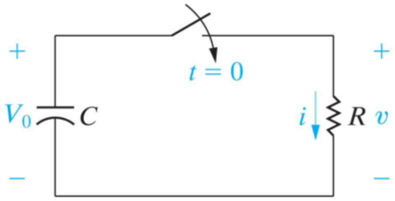
- Replacing each circuit element with its s-domain equivalent. The initial energy in L or C is taken into account by adding independent source in series or parallel with the element impedance.
- Writing & solving algebraic equations by the same circuit analysis techniques developed for resistive networks.
- Obtaining the t-domain solutions by inverse Laplace transform.

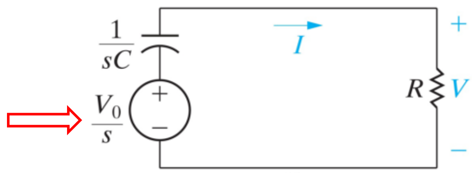
Why to operate in the s-domain?

- It is convenient in solving transient responses of linear, lumped parameter circuits, for the initial conditions have been incorporated into the equivalent circuit.
- It is also useful for circuits with multiple essential nodes and meshes, for the simultaneous ODEs have been reduced to simultaneous algebraic equations.
- It can correctly predict the impulsive response, which is more difficult in the t-domain (Sec. 13.8).

Natural response of an RC circuit (1)

- Q: *i(t), v(t)=?*





- Replacing the charged capacitor by a Thévenin equivalent circuit in the s-domain.
- ➔KVL, & algebraic equation & solution of *I(s):*

$$\frac{V_0}{s} = \frac{I}{sC} + IR, \Rightarrow I(s) = \frac{CV_0}{1+RCs} = \frac{V_0/R}{s+(RC)^{-1}}.$$

Natural response of an RC circuit (2)

- The t-domain solution is obtained by inverse Laplace transform:

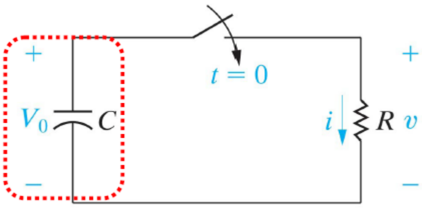
$$i(t) = L^{-1}\left\{\frac{V_0/R}{s+(RC)^{-1}}\right\} = \frac{V_0}{R}e^{-t/(RC)}L^{-1}\left\{\frac{1}{s}\right\}$$

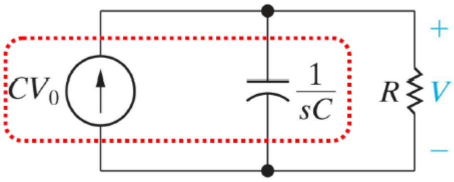
$$i(t) = \frac{V_o}{R}\left(e^{-\frac{t}{\tau}}\right)u(t);\ t>0$$

- $i(0^+) = V_0/R$, which is true for $v_C(0^+) = v_C(0^-) = V_0$.
- $i(\infty) = 0$, which is true for capacitor becomes open (no loop current) in steady state.

Natural response of an RC circuit (3)

- To directly solve $v(t)$, replacing the charged capacitor by a Norton equivalent in the s-domain.





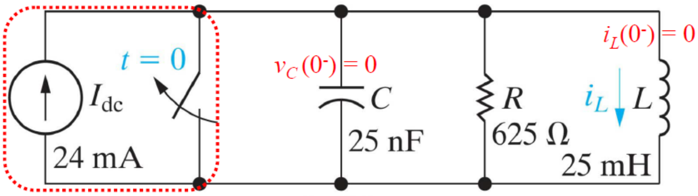
- Solve $V(s)$, perform inverse Laplace transform:

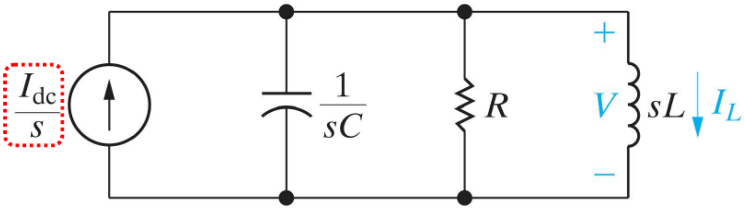
$$CV_0 = sCV + \frac{V}{R}, \Rightarrow V(s) = \frac{V_0}{s + (RC)^{-1}}.$$

$$\Rightarrow v(t) = L^{-1}\left\{V_0 / \left[s + (RC)^{-1}\right]\right\} = V_0 e^{-t/(RC)} u(t) = Ri(t).$$

Step response of a parallel RLC (1)

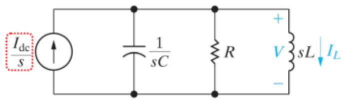
- Q: $i_L(t) = ?$





Step response of a parallel RLC (2)

- KCL, ⇒ algebraic equation & solution of $V(s)$:



$$\frac{I_{dc}}{s} = sCV + \frac{V}{R} + \frac{V}{sL},$$

$$\Rightarrow V(s) = \frac{I_{dc}/C}{s^2 + (RC)^{-1}s + (LC)^{-1}}.$$

- Solve $I_L(s)$: $$I_L(s) = \frac{V(s)}{sL} = \frac{I_{dc}(LC)^{-1}}{s\left[s^2 + (RC)^{-1}s + (LC)^{-1}\right]}$$

$$= \frac{3.84 \times 10^7}{s\left[s^2 + (6.4 \times 10^4)s + (1.6 \times 10^9)\right]}.$$

Step response of a parallel RLC (3)

- Perform partial fraction expansion and inverse Laplace transform:

$$= \frac{3.84 \times 10^7}{s\left[s^2 + (6.4 \times 10^4)s + (1.6 \times 10^9)\right]}.$$

$$I_L(s) = \frac{24}{s} + \frac{20\angle 127^\circ}{s - (-32\text{k} + j24\text{k})} + \frac{20\angle -127^\circ}{s - (-32\text{k} - j24\text{k})} \ (\text{mA} \cdot \text{s}).$$

Pair Number	Nature of Roots	$F(s)$	$f(t)$
1	Distinct real	$\frac{K}{s+a}$	$Ke^{-at}u(t)$
2	Repeated real	$\frac{K}{(s+a)^2}$	$Kte^{-at}u(t)$
3	Distinct complex	$\frac{K}{s+\alpha-j\beta}+\frac{K^*}{s+\alpha+j\beta}$	$2\|K\|e^{-\alpha t}\cos(\beta t+\theta)u(t)$
4	Repeated complex	$\frac{K}{(s+\alpha-j\beta)^2}+\frac{K^*}{(s+\alpha+j\beta)^2}$	$2t\|K\|e^{-\alpha t}\cos(\beta t+\theta)u(t)$

Note: In pairs 1 and 2, K is a real quantity, whereas in pairs 3 and 4, K is the complex quantity $|K|\angle\theta$.

Step response of a parallel RLC (3)

- Perform partial fraction expansion and inverse Laplace transform:

$$I_L(s)=\frac{24}{s}+\frac{20\angle 127^\circ}{s-(-32\text{k}+j24\text{k})}+\frac{20\angle -127^\circ}{s-(-32\text{k}-j24\text{k})}\,(\text{mA}\cdot\text{s}).$$

$$i_L(t)=24u(t)+\left[20e^{j127^\circ}e^{-(32\text{k})t}e^{j(24\text{k})t}u(t)+c.c.\right]$$

$$=\left\{24+40e^{-(32\text{k})t}\cos\left[(24\text{k})t+127^\circ\right]\right\}\,u(t)\,(\text{mA})$$

Transient response due to a sinusoidal source (1)

- For a parallel RLC circuit, replace the current source by a sinusoidal one: $i_g(t) = I_m \cos\omega t \cdot u(t)$.

  The algebraic equation changes:

$$sCV + \frac{V}{R} + \frac{V}{sL} = I_g = \frac{sI_m}{s^2 + \omega^2},$$

$$\Rightarrow V(s) = \frac{(I_m/C)s^2}{(s^2+\omega^2)\left[s^2 + (RC)^{-1}s + (LC)^{-1}\right]},$$

$$\Rightarrow I_L(s) = \frac{V}{sL} = \frac{I_m (LC)^{-1} s}{(s^2+\omega^2)\left[s^2 + (RC)^{-1}s + (LC)^{-1}\right]}.$$

Transient response due to a sinusoidal source (2)

- Perform partial fraction expansion and inverse Laplace transform:

$$I_L(s) = \frac{K_1}{s - j\omega} + \frac{K_1^*}{s + j\omega} + \frac{K_2}{s - (-\alpha + j\beta)} + \frac{K_2^*}{s - (-\alpha - j\beta)}.$$

Driving frequency (ω) — Neper frequency (α) — Damped frequency (β)

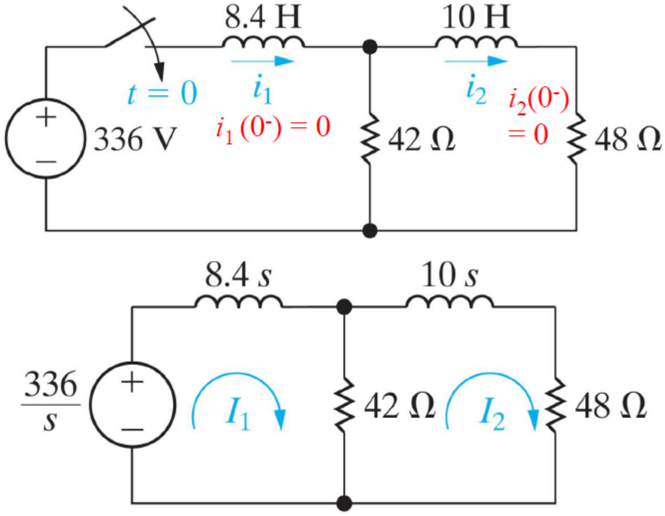
$$i_L(t) = \left\{ 2|K_1|\cos(\omega t + \angle K_1) + 2|K_2|e^{-\alpha t}\cos(\beta t + \angle K_2) \right\} u(t).$$

Steady-state response (source): $2|K_1|\cos(\omega t + \angle K_1)$

Natural response (RLC parameters): $2|K_2|e^{-\alpha t}\cos(\beta t + \angle K_2)$

Step response of a 2-mesh circuit (1)

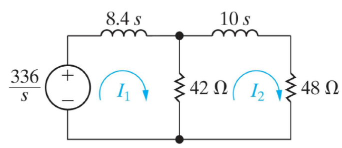
- Q: $i_1(t), i_2(t) = ?$



Step response of a 2-mesh circuit (2)

$\Rightarrow$ 2 algebraic equations & solutions:

$$\begin{cases} 8.4sI_1 + 42(I_1 - I_2) = \dfrac{336}{s} \cdots (1) \\ 42(I_2 - I_1) + (10s + 48)I_2 = 0 \cdots (2) \end{cases}$$

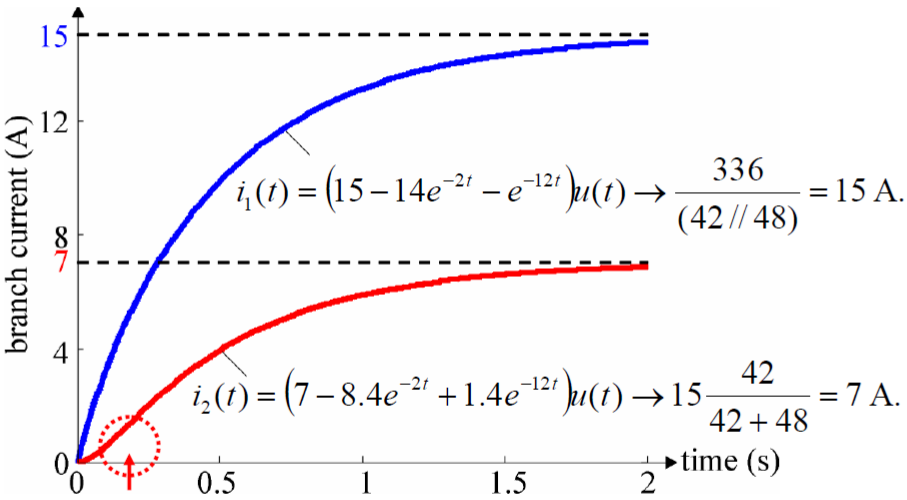


$$\Rightarrow \begin{bmatrix} 42 + 8.4s & -42 \\ -42 & 90 + 10s \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 336/s \\ 0 \end{bmatrix}.$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 42 + 8.4s & -42 \\ -42 & 90 + 10s \end{bmatrix}^{-1} \times \begin{bmatrix} 336/s \\ 0 \end{bmatrix} = \begin{bmatrix} \dfrac{15}{s} - \dfrac{14}{s+2} - \dfrac{1}{s+12} \\ \dfrac{7}{s} - \dfrac{8.4}{s+2} + \dfrac{1.4}{s+12} \end{bmatrix}.$$

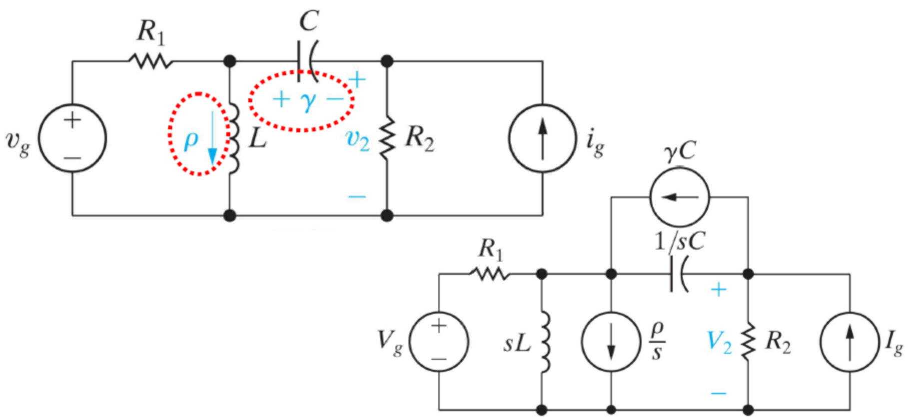
Step response of a 2-mesh circuit (3)

- Perform inverse Laplace transform:

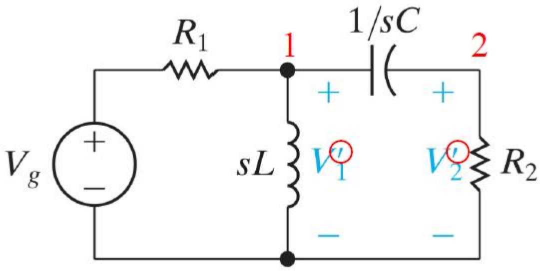


Use of superposition (1)

- Given 2 independent sources v_g, i_g and initially charged C, L, $\Rightarrow v_2(t) = ?$



Use of superposition: V_g acts alone (2)



$$\begin{cases} \dfrac{V_1'-V_g}{R_1}+\dfrac{V_1'}{sL}+\dfrac{V_1'-V_2'}{(sC)^{-1}}=0, \\ \dfrac{V_2'-V_1'}{(sC)^{-1}}+\dfrac{V_2'}{R_2}=0. \end{cases} \Rightarrow \begin{cases} \left(\dfrac{1}{R_1}+\dfrac{1}{sL}+sC\right)V_1'-sCV_2'=\dfrac{V_g}{R_1}, \\ -sCV_1'+\left(\dfrac{1}{R_2}+sC\right)V_2'=0. \end{cases}$$

Use of superposition (3)

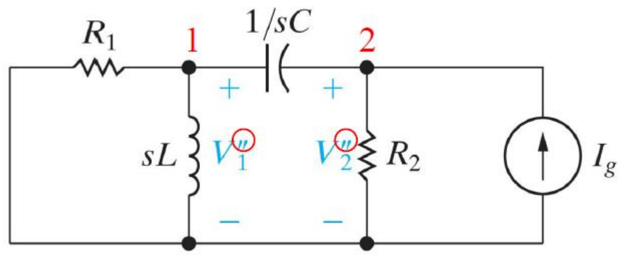
- For convenience, define admittance matrix:

$$\begin{bmatrix} \dfrac{1}{R_1}+\dfrac{1}{sL}+sC & -sC \\ -sC & \dfrac{1}{R_2}+sC \end{bmatrix} \times \begin{bmatrix} V_1' \\ V_2' \end{bmatrix}$$

$$= \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12} & Y_{22} \end{bmatrix} \times \begin{bmatrix} V_1' \\ V_2' \end{bmatrix} = \begin{bmatrix} V_g/R_1 \\ 0 \end{bmatrix}.$$

$$\Rightarrow V_2' = \frac{-Y_{12}/R_1}{Y_{11}Y_{22}-Y_{12}^2} V_g.$$

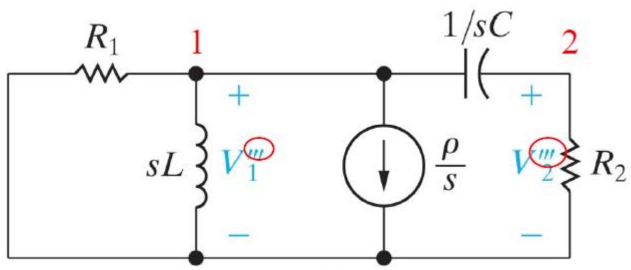
Use of superposition: I_g acts alone (4)



$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12} & Y_{22} \end{bmatrix} \times \begin{bmatrix} V_1'' \\ V_2'' \end{bmatrix} = \begin{bmatrix} 0 \\ I_g \end{bmatrix}, \Rightarrow V_2'' = \frac{Y_{11}}{Y_{11}Y_{22} - Y_{12}^2} I_g$$

Same matrix — Same denominator

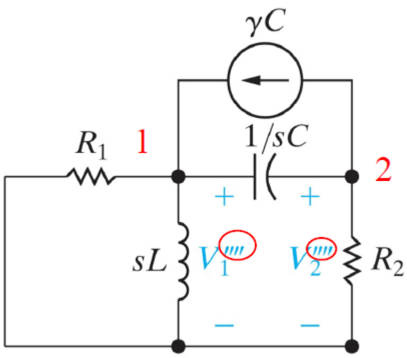
Use of superposition: Energized L acts alone (5)



$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12} & Y_{22} \end{bmatrix} \times \begin{bmatrix} V_1''' \\ V_2''' \end{bmatrix} = \begin{bmatrix} -\rho/s \\ 0 \end{bmatrix}, \Rightarrow V_2''' = \frac{Y_{12}/s}{Y_{11}Y_{22} - Y_{12}^2} \rho$$

Same matrix — Same denominator

Use of superposition: Energized C acts alone (6)



$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12} & Y_{22} \end{bmatrix} \times \begin{bmatrix} V_1'''' \\ V_2'''' \end{bmatrix} = \begin{bmatrix} \gamma C \\ -\gamma C \end{bmatrix},$$

$$\Rightarrow V_2'''' = \frac{-(Y_{11}+Y_{12})C}{Y_{11}Y_{22}-Y_{12}^2}\gamma.$$

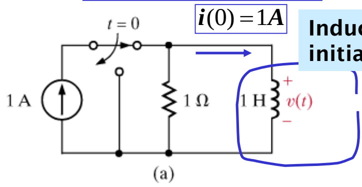
- The total voltage is: $V_2 = V_2' + V_2'' + V_2''' + V_2''''.$

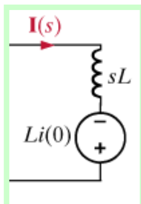
LEARNING BY DOING — **Determine the model in the s-domain and the expression for the voltage across the inductor**

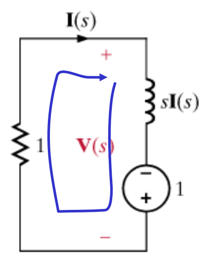
Steady state for t<0

$i(0) = 1A$

Inductor with initial current







KVL: $1 = (1+s)I(s)$

Ohm's Law

$V(s) = -1 \times I(s) \Rightarrow V(s) = -\frac{1}{s+1}$

$v_S(t) = -e^{-t}u(t)$ V

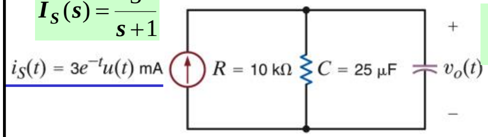
Equivalent circuit in s-domain

ANALYSIS TECHNIQUES

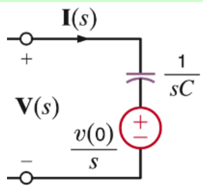
All the analysis techniques are applicable in the s-domain

LEARNING EXAMPLE Draw the s-domain equivalent and find the voltage in both s-domain and time domain

$I_S(s)=\frac{3}{s+1}$

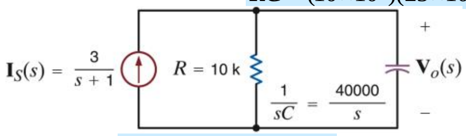


One needs to determine the initial voltage across the capacitor



$i_S(t)=0,\ t<0 \Rightarrow v_o(0)=0$

$RC=(10\times10^3)(25\times10^{-6})=0.25$



$$V_o(s)=\left(R\,\|\,\frac{1}{Cs}\right)I_S(s)$$

$$V_o(s)=\frac{\frac{R}{Cs}}{R+\frac{1}{Cs}}I_S(s)=\frac{1/C}{s+1/RC}\times\frac{3\times10^{-3}}{s+1}$$

$$V_o(s)=\frac{120}{(s+4)(s+1)}=\frac{K_1}{s+4}+\frac{K_2}{s+1}$$

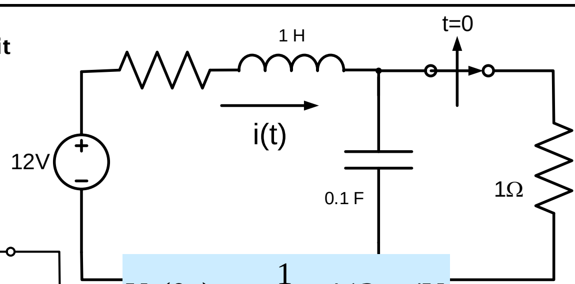
$$K_1=(s+4)V_o(s)|_{s=-4}=-40$$

$$K_2=(s+1)V_o(s)|_{s=-1}=40$$

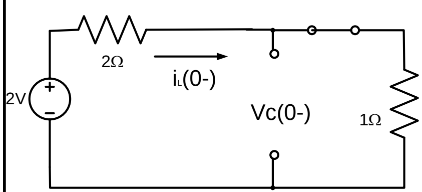
$$v_o(t)=40\left[e^{-t}-e^{-4t}\right]u(t)$$

Given the following Circuit

Find i(t) for t >0



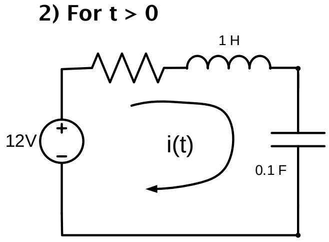
1) For t <0



$$V_C(0^-)=\frac{1}{(1+2)}*12=4V$$

$$i_L(0^-)=\frac{12}{(1+2)}=4\,A$$

2) For t > 0



$$12=2i(t)+1\frac{di}{dt}(t)+\frac{1}{0.1}\int_0^t i(t)dt+V_C(0^-$$

$$\frac{12}{s}=2I(s)+sI(s)-i_L(0^-)+\frac{10}{s}I(s)+\frac{V_C(0^-)}{s}$$

$$\frac{12}{s}=I(s)\left(2+s+\frac{10}{s}\right)-4+\frac{4}{s}$$

$$\frac{12}{s} + 4 - \frac{4}{s} = I(s)\left(\frac{2s + s^2 + 10}{s}\right)$$

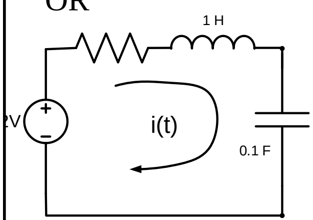
$$\frac{12 + 4s - 4}{s} = I(s)\left(\frac{2s + s^2 + 10}{s}\right)$$

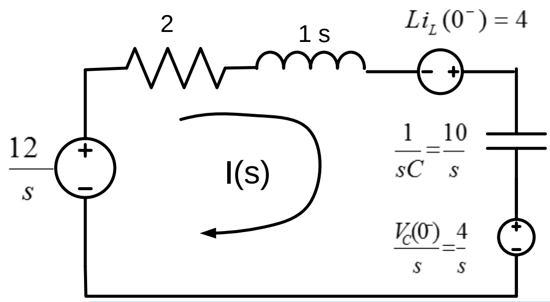
$$4(2 + s) = I(s)\left(2s + s^2 + 10\right) \rightarrow$$

$$I(s) = \frac{4(s+2)}{\left(s^2 + 2s + 10\right)}$$

we need $L^{-1}[I(s)]$ to get i(t)

OR





$$\frac{12}{s} + 4 - \frac{4}{s} = I(s)\left(2 + s + \frac{10}{s}\right)$$

$$I(s) = \frac{4(s+2)}{(s+1-j3)(s+1+j3)}$$

$$I(s) = \frac{K_1}{(s+1-j3)} + \frac{K_1^*}{(s+1+j3)}$$

$$K_1 = \frac{4(s+2)}{(s+1+j3)} \mapsto @\, s = -1 + j3$$

$$= \frac{4(-1+j3+2)}{-1+j3+1+j3} = \frac{4(1+j3)}{j6}$$

$$= \frac{(1+j3)\text{x}(-j6)}{j6\text{x}(-j6)}.4 = \frac{(-j6+18)}{36}.4$$

$K_1 = 2.11\angle -18.4^o$ $\quad K_1^* = 2.11\angle 18.4^o$

$$i(t) = 2|K_1|e^{-t}\cos\left(3t - 18.4^o\right)$$

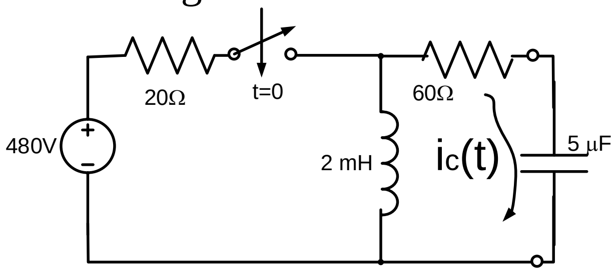
$$= 4.22.e^{-t}\cos\left(3t - 18.4^o\right)$$

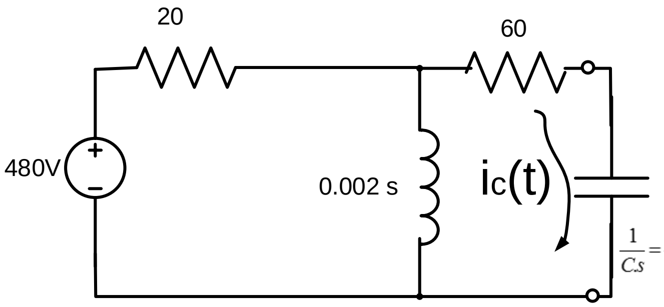
Thevenin's

Find ic(t) for t >0 using TEC

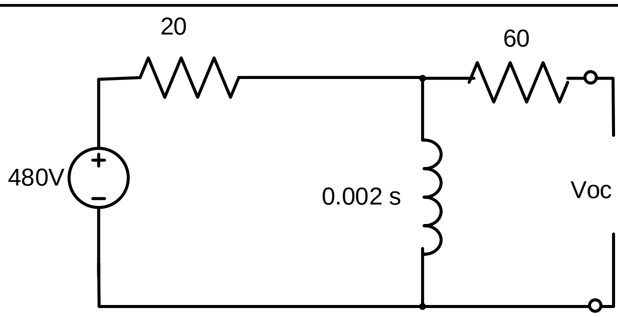
$V_C(0^-) = 0$

$i_L(0^-) = 0$





$$\frac{1}{Cs} = \frac{1}{5.10^{-6}.s} = \frac{2.10^5}{s}$$



$$V_{OC} = V_{th} = \frac{0.002s}{0.002s + 20} * \frac{480}{s}$$
$$= \frac{480}{s + 10^4}$$

$$Z_{th} = 60 + (20 // 0.002s)$$
$$= 60 + \frac{20x(0.002s)}{20 + 0.002s}$$
$$= \frac{60(20 + 0.002s) + 0.04s}{20 + 0.002s}$$

$$I(s) = \frac{V_{th}}{Z_{th} + \frac{2.10^5}{s}} \Rightarrow I(s) = \frac{6s}{(s + 5000)^2}$$

$$I(s)=\frac{6s}{(s+5000)^2}$$

$$=\frac{K_1}{(s+5000)^2}+\frac{K_2}{(s+5000)}$$

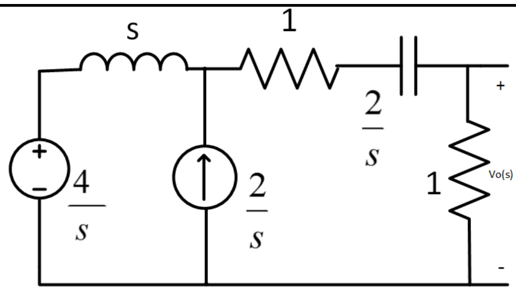
$$K_1=6s\ @s=-50000$$
$$=-30{,}000$$
$$K_2=d/ds(6s)\ \ @s=-50000$$
$$=6$$

$$\text{or}\quad K_1+K_2(s+5000)=6s$$
$$==>K_2s=6s==>K_2=6$$
$$\&\ K_1+K_2(5000)=0\ \ ==>K_1=-30{,}000$$

⇩

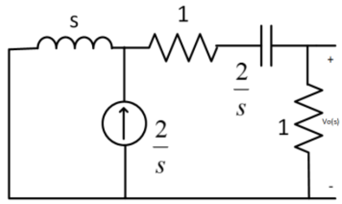
$$ic(t)=\left(-30000te^{-5000t}+6e^{-5000t}\right)u(t)\quad \text{A}$$

Superposition



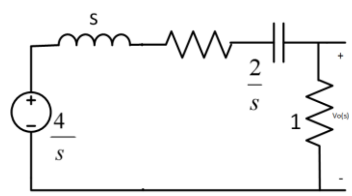
1) Let 4/s=0

$$V_O{}'(s)=\frac{s}{s+\frac{2}{s}+2}*\frac{2}{s}*1$$
$$=\frac{2s}{s^2+2s+2}$$



2) Let 2/s=0

$$V_O{}''(s)=\frac{1}{s+\frac{2}{s}+2}*\frac{4}{s}$$
$$=\frac{4}{s^2+2s+2}$$



Superposition

3) $V_O(s) = V_O{}'(s) + V_O{}''(s)$ ⇒

$$V_O(s) = \frac{2s}{s^2+2s+2} + \frac{4}{s^2+2s+2}$$
$$= \frac{2s+4}{s^2+2s+2}$$
$$= \frac{K_1}{s+1-j} + \frac{K_1^*}{s+1+j}$$

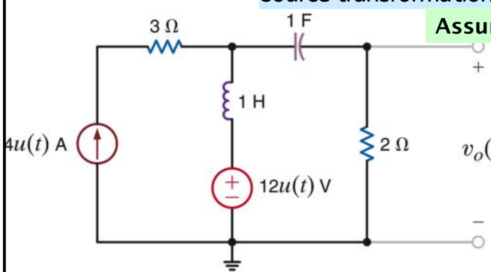
$$K_1 = \frac{2s+4}{s+1+j1} @ \mathrm{s} = \text{-}1+\mathrm{j}1$$
$$= \frac{2(\text{-}1+\mathrm{j}1)+4}{(\text{-}1+\mathrm{j}1)+1+j1} = \frac{2+\mathrm{j}2}{j2}$$
$$= 1 - \mathrm{j}$$
$$= \sqrt{2} \angle -45^o$$

⇒ $L^{-1}[V_o(s)]$ to get $V_o(t)$

$$V_O(t) = 2|K_1|e^{-t}\cos(t-45^o)$$
$$= 2\sqrt{2}\, e^{-t}\cos(t-45^o) \ \mathrm{V}$$

LEARNING EXAMPLE Find $v_o(t)$ using node analysis, loop analysis, superposition, source transformation, Thevenin's and Norton's theorem.

Assume all initial conditions are zero



KCL @ V_1

$$-\frac{4}{s} + \frac{V_1(s) - \frac{12}{s}}{s} + \frac{V_1(s) - V_o(s)}{\frac{1}{s}} = 0 \quad \times s$$

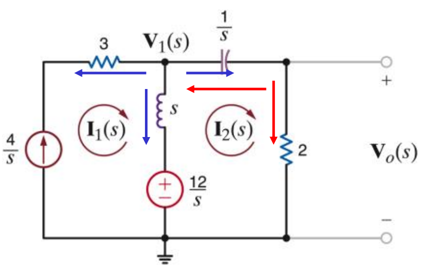
$\frac{V_1(s)}{2+\frac{1}{s}}$

KCL@V_o

$$\frac{V_o(s)}{2} + \frac{V_o(s) - V_1(s)}{\frac{1}{s}} = 0 \quad \times 2$$

Could have used voltage divider here

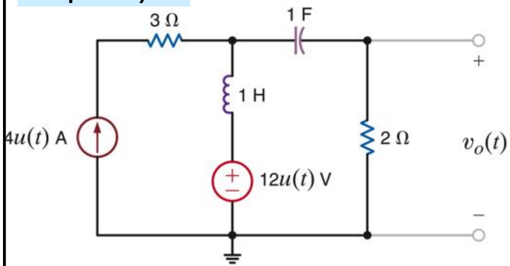
Node Analysis

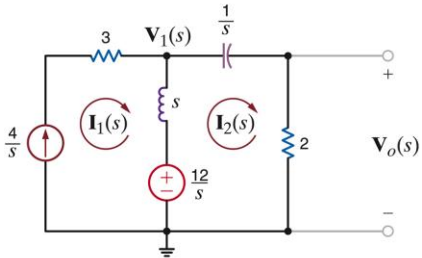


$$(1+s^2)V_1(s) - s^2 V_o(s) = \frac{4s+12}{s} \quad \times 2s$$
$$-2sV_1(s) + (1+2s)V_o(s) = 0 \quad \times (1+s^2)$$

$$V_o(s) = \frac{8(s+3)}{(1+s)^2}$$

Loop Analysis





Loop 1

$$I_1(s) = \frac{4}{s}$$

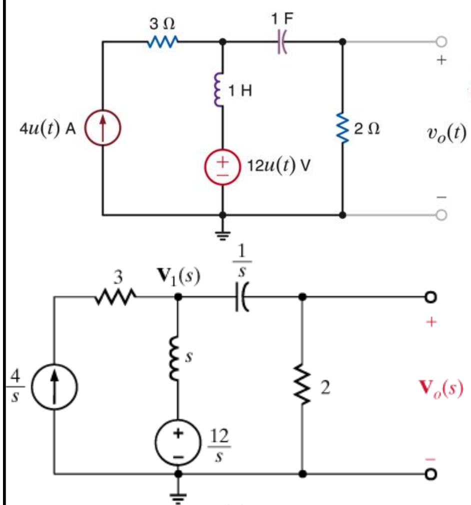
Loop 2

$$v_o(t) \quad s(I_2(s) - I_1(s)) + \frac{1}{s} I_2(s) + 2I_2(s) = \frac{12}{s}$$

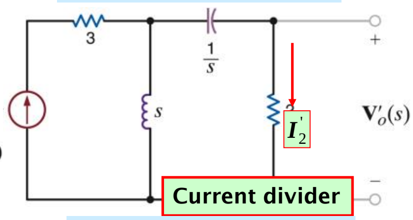
$$I_2(s) = \frac{4(s+3)}{(s+1)^2}$$

$$V_o(s) = 2I_2(s) = \frac{8(s+3)}{(s+1)^2}$$

Source Superposition



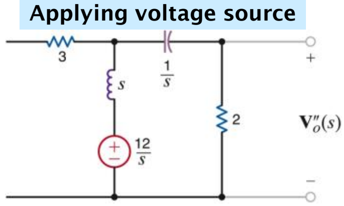
Applying current source



Current divider

$$V_o^{'}(s) = 2 \times \frac{s}{2 + \frac{1}{s} + s} \times \frac{4}{s}$$

Applying voltage source

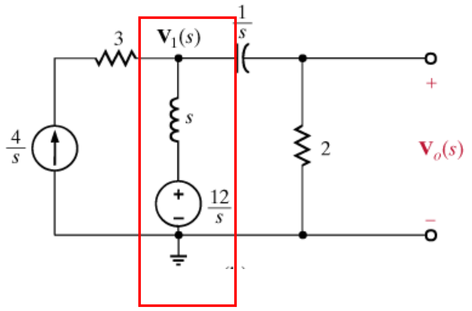


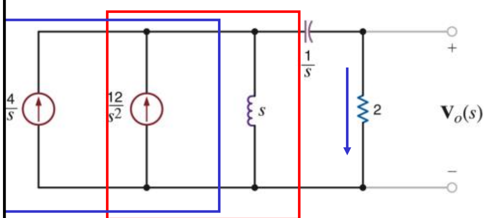
Voltage divider

$$V_o^{''}(s) = \frac{2}{2 + \frac{1}{s} + s} \times \frac{12}{s}$$

$$V_o(s) = V_o^{'}(s) + V_o^{''}(s) = \frac{8(s+3)}{(s+1)^2}$$

Source Transformation





The resistance is redundant

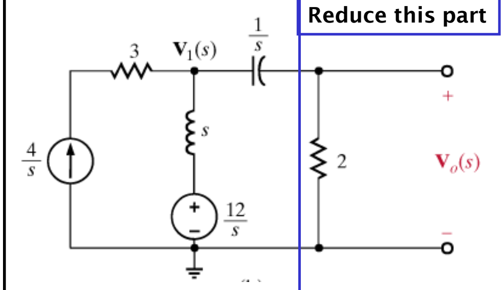
Combine the sources and use current divider

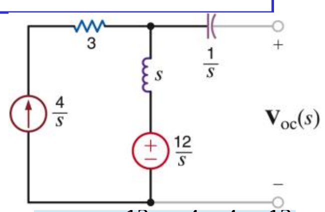
$$V_o(s) = 2 \times \frac{s}{s + \frac{1}{s} + 2}\left(\frac{4}{s} + \frac{12}{s^2}\right)$$

$$V_o(s) = \frac{8(s+3)}{(s+1)^2}$$

Using Thevenin's Theorem

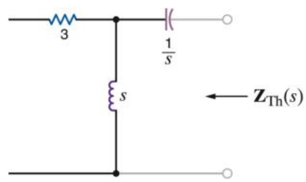
Reduce this part



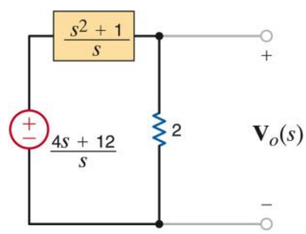


$$V_{OC}(s) = \frac{12}{s} + s\frac{4}{s} = \frac{4s+12}{s}$$

Only independent sources



$$Z_{Th} = \frac{1}{s} + s = \frac{s^2+1}{s}$$



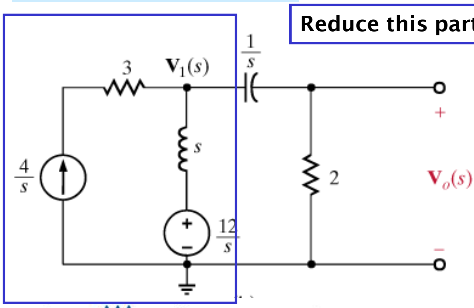
Voltage divider

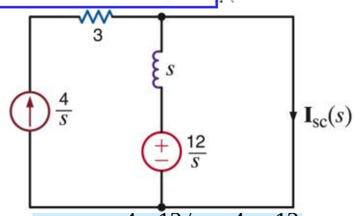
$$V_o(s) = \frac{2}{2 + \frac{s^2+1}{s}} \frac{4s+12}{s}$$

$$V_o(s) = \frac{8(s+3)}{(s+1)^2}$$

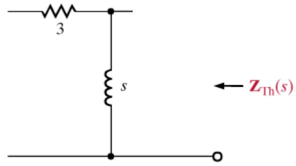
Using Norton's Theorem

Reduce this part



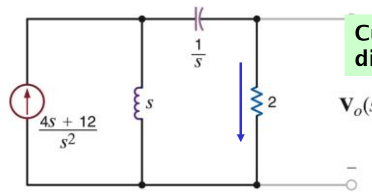


$$I_{SC}(s)=\frac{4}{s}+\frac{12/s}{s}=\frac{4s+12}{s^2}$$



$Z_{Th}=s$

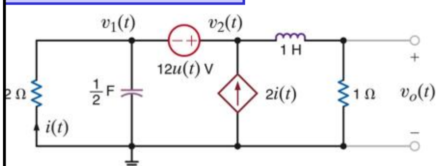
Current division



$$V_o(s)=2\times\frac{s}{s+\frac{1}{s}+2}\frac{4s+12}{s^2}$$

$$V_o(s)=\frac{8(s+3)}{(s+1)^2}$$

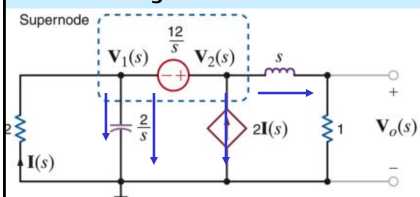
LEARNING EXAMPLE Determine the voltage $v_o(t)$. Assume all initial conditions to be zero



Selecting the analysis technique:

- . Three loops, three non-reference nodes
- . One voltage source between non-reference nodes - supernode
- . One current source. One loop current known or supermesh
- . If v_2 is known, v_o can be obtained with a voltage divider

Transforming the circuit to s-domain



Supernode constraint : $V_2(s)-V_1(s)=\frac{12}{s}$

KCL@ supernode : $\frac{V_1(s)}{2}+\frac{V_1(s)}{2/s}-2I(s)+\frac{V_2(s)}{s+1}=0$

Controlling variable : $I(s)=-\frac{V_1(s)}{2}$

Voltage divider : $V_0(s)=\frac{1}{s+1}V_2(s)$

Doing the algebra : $V_1(s)=V_2(s)-12/s$

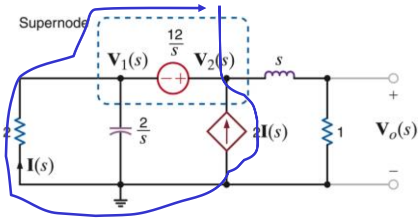
$I(s)=-V_2(s)/2+6/s$

$(1/2)(s+1)(V_2(s)-12/s)-2(-V_2(s)/2+6/s)$
$+V_2(s)/(s+1)=0$

$$V_2(s)=\frac{12(s+1)(s+3)}{s(s^2+4s+5)}$$

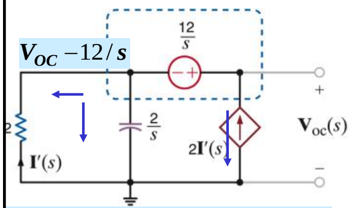
$$V_o(s)=\frac{12(s+3)}{s(s^2+4s+5)}$$

Continued ... Compute $V_o(s)$ using Thevenin's theorem



- -keep dependent source and controlling variable in the same sub-circuit
- -Make sub-circuit to be reduced as simple as possible
- -Try to leave a simple voltage divider after reduction to Thevenin equivalent

$V_{OC} - 12/s$

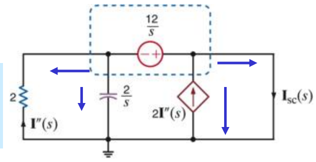


$$\frac{V_{OC}-12/s}{2}+\frac{V_{OC}-12/s}{2/s}-2I'=0$$

$$I'=-\frac{V_{OC}-12/s}{2} \qquad V_{OC}(s)=\frac{12}{s}$$

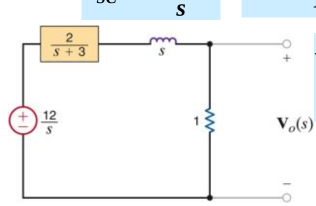
$$I'=0$$

$-$
$12/s$
$+$



$$I_{SC}-2I''-I''-2I''/(2/s)=0 \qquad I''=6/s$$

$$I_{SC}=\frac{6(s+3)}{s} \qquad Z_{TH}=\frac{V_{OC}(s)}{I_{SC}(s)}=\frac{2}{s+3}$$



$$V_o(s)=\frac{1}{1+s+\dfrac{2}{s+3}}\times\frac{12}{s}$$

Continued ... **Computing the inverse Laplace transform**

Analysis in the s-domain has established that the Laplace transform of the output voltage is

$$V_o(s)=\frac{12(s+3)}{s(s^2+4s+5)} \qquad s^2+4s+5=(s+2-j1)(s+2+j1)=(s+2)^2+1$$

$$V_o(s)=\frac{12(s+3)}{s(s+2-j1)(s+2+j1)}=\frac{K_o}{s}+\frac{K_1}{(s+2-j1)}+\frac{K_1^*}{(s+2+j1)}$$

$$K_o=sV_o(s)|_{s=0}=36/5 \qquad \frac{K_1}{(s+\alpha-j\beta)}+\frac{K_1^*}{(s+\alpha+j\beta)}\leftrightarrow 2|K_1|e^{-\alpha t}\cos(\beta t+\angle K_1)u(t)$$

$$K_1=(s+2-j1)V_o(s)|_{s=-2+j1}=\frac{12(1+j1)}{(-2+j1)(j2)}=\frac{12\sqrt{2}\angle 45°}{\sqrt{5}\angle 153.43°(2\angle 90°)}$$
$$=3.79\angle -198.43°=3.79\angle 161.57°$$

One can also use quadratic factors...

$$V_o(s)=\frac{12(s+3)}{s\left[(s+2)^2+1\right]}=\frac{C_o}{s}+\frac{C_1(s+2)}{(s+2)^2+1}+\frac{C_2}{(s+2)^2+1} \qquad v_o(t)=\left(\frac{36}{5}+7.59e^{-2t}\cos(t+161.57°)\right)u(t)$$

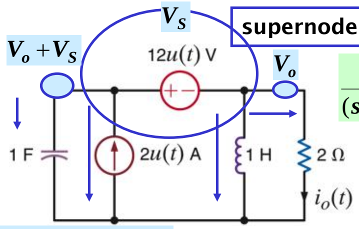
$$C_o=sV_o(s)|_{s=0}=36/5 \qquad \frac{C_1(s+\alpha)}{(s+\alpha)^2+\beta^2}+\frac{C_2\beta}{(s+\alpha)^2+\beta^2}\leftrightarrow e^{-\alpha t}[C_1\cos\beta t+C_2\sin\beta t]u(t)$$

$$12(s+3)=C_o((s+2)^2+1)+s[C_1(s+2)+C_2] \qquad s=-2\Rightarrow 12=C_o-2C_2\Rightarrow C_2=36/10-6=-12/5$$

Equating coefficients of $s^2: 0=C_o+C_1\Rightarrow C_1=-36/5$

$$v_o(t)=\left[\frac{36}{5}(1-e^{-2t}\cos t)-\frac{12}{5}e^{-2t}\sin t\right]u(t)$$

LEARNING EXTENSION Find $i_o(t)$ using node equations



Assume zero initial conditions
Implicit circuit transformation to s-domain

$$\frac{K_1}{(s+\alpha-j\beta)} + \frac{K_1^*}{(s+\alpha+j\beta)} \leftrightarrow 2|K_1|e^{-\alpha t}\cos(\beta t+\angle K_1)u(t)$$

$$K_1 = \left(s+\frac{1}{4}-j\frac{\sqrt{15}}{4}\right)I_o(s)\Big|_{s=-\frac{1}{4}+j\frac{\sqrt{15}}{4}} = \frac{1-6\left(-\frac{1}{4}+j\frac{\sqrt{15}}{4}\right)}{2j\frac{\sqrt{15}}{4}}$$

KCL at supernode

$$Cs(V_o(s)+V_S(s)) - \frac{2}{s} + \frac{V_o(s)}{s} + \frac{V_o(s)}{2} = 0$$

$$V_S(s) = \frac{12}{s}, \quad I_o(s) = \frac{V_o(s)}{2}$$

Doing the algebra

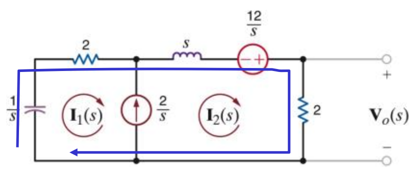
$$I_o(s) = \frac{1-6s}{s^2+0.5s+1} = \frac{1-6s}{\left(s+\frac{1}{4}\right)^2+\frac{15}{16}}$$

$$I_o(s) = \frac{1-6s}{\left(s+\frac{1}{4}-j\frac{\sqrt{15}}{4}\right)\left(s+\frac{1}{4}+j\frac{\sqrt{15}}{4}\right)} = \frac{K_1}{\left(s+\frac{1}{4}-j\frac{\sqrt{15}}{4}\right)} + \frac{K_1^*}{\left(s+\frac{1}{4}+j\frac{\sqrt{15}}{4}\right)}$$

$$K_1 = \frac{6.33\angle -66.72^\circ}{0.97\angle 90^\circ} = 6.53\angle -156.72^\circ$$

$$i_o(t) = 13.06e^{-\frac{t}{4}}\cos\left(\frac{\sqrt{15}}{4}t - 156.72^\circ\right)$$

LEARNING EXTENSION Find $v_o(t)$ using loop equations



supermesh

constraint due to source

$$\frac{2}{s} = I_2 - I_1$$

KVL on supermesh

$$\frac{1}{s}I_1 + 2I_1 + sI_2 - \frac{12}{s} + 2I_2 = 0$$

$$I_2(s) = \frac{16s+2}{s(s^2+4s+1)} = \frac{16s+2}{s(s+0.27)(s+3.73)}$$

Determine inverse transform

$$I_2(s) = \frac{K_0}{s} + \frac{K_1}{s+0.27} + \frac{K_2}{s+3.73}$$

$$K_0 = sI_2(s)|_{s=0} = 2$$

$$K_1 = (s+0.27)I_2(s)|_{s=-0.27} = \frac{16(-0.27)+2}{(-0.27)(-0.27+3.73)} = 2.48$$

$$K_2 = (s+3.73)I_2(s)|_{s=-3.73} = \frac{16(-3.73)+2}{(-3.73)(-3.73+0.27)} = -4.47$$

$$i_2(t) = \left(2+2.48e^{-0.27t} - 4.47e^{-3.73t}\right)u(t)$$

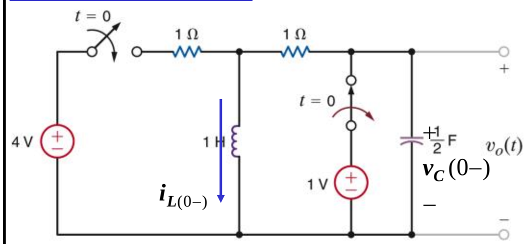
$$v_o(t) = 2i_2(t)$$

TRANSIENT CIRCUIT ANALYSIS USING LAPLACE TRANSFORM

For the study of transients, especially transients due to switching, it is important to determine initial conditions. For this determination, one relies on the properties:

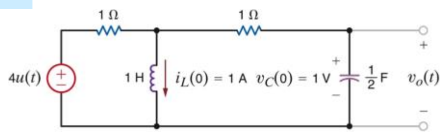
1. **Voltage across capacitors cannot change discontinuously**
2. **Current through inductors cannot change discontinuously**

LEARNING EXAMPLE Determine $v_o(t), t > 0$

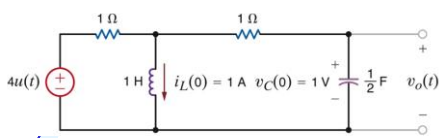


Assume steady state for t<0 and determine voltage across capacitors and currents through inductors

For DC case capacitors are open circuit
inductors are shortcircuit
$v_C(0-) = 1V,\ i_L(0-) = 1A$

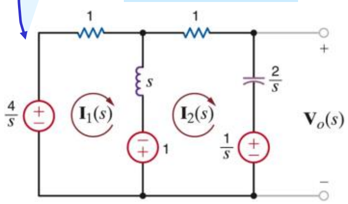


Circuit for t>0



Laplace

Circuit for t>0



Use mesh analysis

$$(s+1)I_1 - sI_2 = \frac{4}{s} + 1$$

$$-sI_1 + (s+1+\frac{2}{s})I_2 = -\frac{1}{s} - 1$$

$$I_2(s) = \frac{2s-1}{2s^2+3s+2}$$

$$V_o(s) = \frac{2}{s}I_2(s) + \frac{1}{s}$$

$$V_o(s) = \frac{2s+7}{2s^2+3s+2}$$

Now determine the inverse transform

$b^2 - 4ac < 0 \Rightarrow$ complex conjugate roots

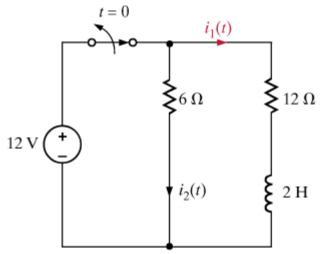
$$V_o(s) = \frac{K_1}{s + \frac{3}{4} - j\frac{\sqrt{7}}{4}} + \frac{K_1^*}{s + \frac{3}{4} + j\frac{\sqrt{7}}{4}}$$

$$K_1 = \left(s + \frac{3}{4} - j\frac{\sqrt{7}}{4}\right)V_o(s)\Big|_{s=-\frac{3}{4}+j\frac{\sqrt{7}}{4}} = 2.14\angle -76.5°$$

$$\frac{K_1}{(s+\alpha-j\beta)} + \frac{K_1^*}{(s+\alpha+j\beta)} \leftrightarrow 2|K_1|e^{-\alpha t}\cos(\beta t + \angle K_1)u(t)$$

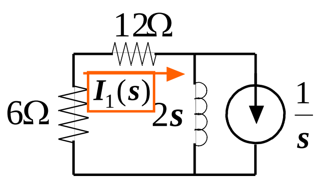
$$v_o(t) = 4.28e^{-\frac{3}{4}t}\cos(\frac{\sqrt{7}}{4}t - 76.5°)$$

LEARNING EXTENSION Determine $i_1(t),\ t>0$



Initial current through inductor

$i_L(0-)=i_L(0+)=1A$

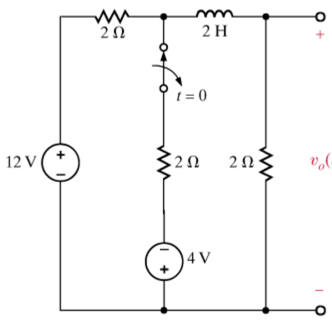


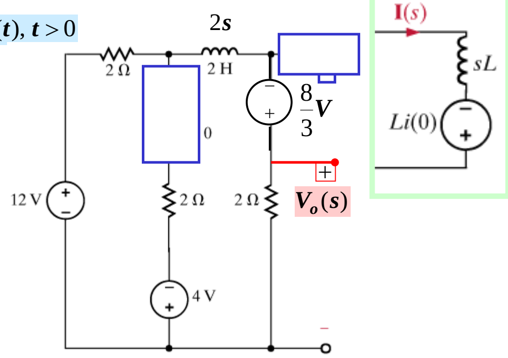
$$I_1(s)=\frac{2s}{2s+18}\times\frac{1}{s}$$

Current divider

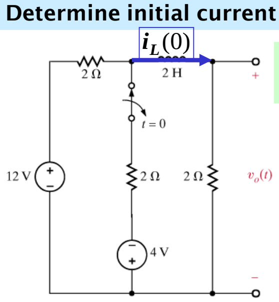
$$I_1(s)=\frac{1}{s+9}\rightarrow i_1(t)=e^{-9t}u(t)$$

LEARNING EXTENSION Determine $v_o(t),\ t>0$





Determine initial current through inductor



Use source superposition

$i_{12V}=2A$

$i_{4V}=-\frac{2}{3}A$

$i_L(0)=\frac{4}{3}A$

$$V_o(s)=\frac{2}{4+2s}\times\left(\frac{12}{s}+\frac{8}{3}\right)\ \text{(voltage divider)}$$

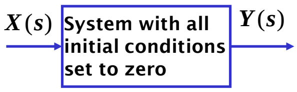
$$V_o(s)=\frac{(8s+36)}{3s(s+2)}=\frac{K_1}{s}+\frac{K_2}{s+2}$$

$$K_1=sV_o(s)|_{s=0}=6$$

$$K_2=(s+2)V_o(s)|_{s=-2}=-\frac{10}{3}$$

$$v_o(t)=\left(6-\frac{8}{3}e^{-2t}\right)u(t)$$

TRANSFER FUNCTION



$$H(s)=\frac{Y(s)}{X(s)}$$

If the model for the system is a differential equation

$$b_n\frac{d^n y}{dt^n}+b_{n-1}\frac{d^{n-1}y}{dt^{n-1}}+\ldots+b_1\frac{dy}{dt}+b_o y$$
$$=a_m\frac{d^m x}{dt^m}+a_{m-1}\frac{d^{m-1}x}{dt^{m-1}}+\ldots+a_1\frac{dx}{dt}+a_o x$$

If all initial conditions are zero

$$\mathcal{L}\left[\frac{d^k y}{dt^k}\right]=s^k Y(s)$$

$$b_n s^n Y(s)+\ldots+b_1 sY(s)+b_0 Y(s)$$
$$=a_m s^m X(s)+\ldots+a_1 sX(s)+a_0 X(s)$$

$$Y(s)=\frac{b_n s^n+\ldots+b_1 s+b_0}{a_m s^m+\ldots+a_1 s+a_0}X(s)$$

$$H(s)=\frac{b_n s^n+\ldots+b_1 s+b_0}{a_m s^m+\ldots+a_1 s+a_0}$$

For the impulse function

$$x(t)=\delta(t)\Rightarrow X(s)=1$$

H(s) can also be interpreted as the Laplace transform of the output when the input is an impulse and all initial conditions are zero

The inverse transform of H(s) is also called the impulse response of the system

If the impulse response is known then one can determine the response of the system to ANY other input

LEARNING EXAMPLE

A network has impulse response $h(t)=e^{-t}u(t)$

Determine the response, $v_o(t)$, for the input $v_i(t)=10e^{-2t}u(t)$

In the Laplace domain, *Y(s)=H(s)X(s)*

$$\therefore V_o(s)=H(s)V_i(s)$$

$$h(t)=e^{-t}u(t)\Rightarrow H(s)=\frac{1}{s+1}$$

$$v_i(t)=10e^{-2t}u(t)\Rightarrow V_i(s)=\frac{10}{s+2}$$

$$V_o(s)=\frac{10}{(s+1)(s+2)}\quad=\frac{K_1}{s+1}+\frac{K_2}{s+2}$$

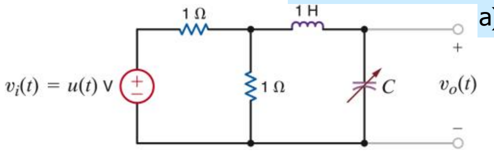
$$K_1=(s+1)V_o(s)|_{s=-1}=10$$

$$K_2=(s+2)V_o(s)|_{s=-2}=-10$$

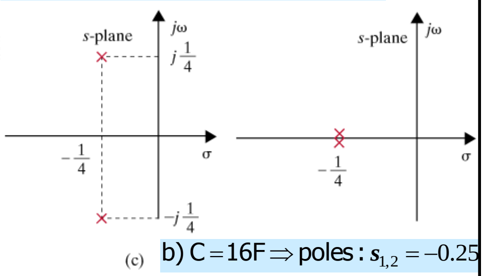
$$v_o(t)=10\left(e^{-t}-e^{-2t}\right)u(t)$$

LEARNING EXAMPLE

Determine the transfer function $H(s)=\dfrac{V_o(s)}{V_i(s)}$

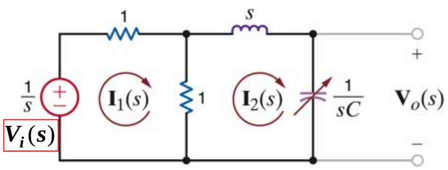


a) $C = 8F \Rightarrow$ poles : $s_{1,2} = -0.25 \pm j0.25$

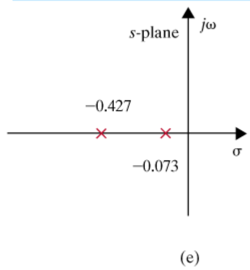


b) $C = 16F \Rightarrow$ poles : $s_{1,2} = -0.25$

Transform the circuit to the Laplace domain. All initial conditions set to zero



c) $C = 32F \Rightarrow$ poles : $s_{1,2} = -0.427, -0.073$



Mesh analysis

$V_i(s) = 2I_1 - I_2$

$0 = -I_1 + \left(1 + s + \dfrac{1}{sC}\right) I_2$

$V_o(s) = \dfrac{1}{sC} I_2(s)$

$V_o(s) = \dfrac{(1/2C)}{s^2 + (1/2)s + 1/C}$

LEARNING EXTENSION

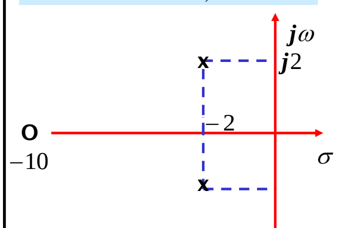
Determine the pole-zero plot, the type of damping and the unit step response

$H(s) = \dfrac{s+10}{s^2+4s+8}$

zero : $z = -10$

poles :

$s^2 + 4s + 8 = 0 \Rightarrow s_{1,2} = -2 \pm j2$



$s^2 + 4s + 8$ ($4 = 2\varsigma\omega_o$, $8 = \omega_o^2$) $\Rightarrow \varsigma = \dfrac{\sqrt{2}}{2}$

$Y(s) = H(s)\dfrac{1}{s} = \dfrac{s+10}{s(s^2+4s+8)}$

$s^2 + 4s + 8 = (s + 2 - j2)(s + 2 + j2)$

$Y(s) = \dfrac{K_1}{s} + \dfrac{K_2}{s+2-j2} + \dfrac{K_2^*}{s+2+j2}$

$\dfrac{K_1}{(s+\alpha-j\beta)} + \dfrac{K_1^*}{(s+\alpha+j\beta)} \leftrightarrow 2|K_1| e^{-\alpha t}\cos(\beta t + \angle K_1)u(t)$

$K_1 = sY(s)|_{s=0} = \dfrac{10}{8}$

$K_2 = (s+2-j2)V_o(s)|_{s=-2+j2} = \dfrac{8+j2}{(-2+j2)(j4)}$

$K_2 = \dfrac{8.25\angle 14}{2.83\angle 135^\circ \times 4\angle 90^\circ} = 0.73\angle -211^\circ$

$v_o(t) = \left(\dfrac{10}{8} + 1.46e^{-2t}\cos(2t - 211^\circ)\right)u(t)$

STEADY STATE RESPONSE

$Y(s) = H(s)X(s)$ Response when all initial conditions are zero

Laplace uses positive time functions. Even for sinusoids the response contains transitory terms

EXAMPLE $H(s) = \frac{1}{s+1}, X(s) = \frac{s}{s^2+\omega^2} \ (\Rightarrow x(t) = [\cos\omega t]u(t))$

$$Y(s) = \frac{s}{(s+1)(s+j\omega)(s-j\omega)} = \frac{K_1}{s+1} + \frac{K_2}{s+j\omega} + \frac{K_2^*}{s-j\omega}$$

If interested in the steady state response only, then don't determine residues associated with transient terms

$$y(t) = \left(Ke^{-t} + 2|K_2|\cos(\omega t + \angle K_2°)\right)u(t)$$

transient — Steady state response

If $x(t) = X_M \cos(\omega_o t + \theta)u(t)$

$y_{ss}(t) = X_M |H(j\omega_o)|\cos(\omega_o t + \angle H(j\omega_o) + \theta)$

For the general case

$$X_M \cos\omega t\, u(t) = \frac{X_M}{2}\left(e^{j\omega t} + e^{-j\omega t}\right) \Rightarrow X(s) = \frac{1}{2}\left(\frac{X_M}{s-j\omega_o} + \frac{X_M}{s+j\omega_o}\right)$$

$$Y(s) = H(s)\left[\frac{1}{2}\left(\frac{X_M}{s-j\omega_o} + \frac{X_M}{s+j\omega_o}\right)\right] = \frac{K_x}{s-j\omega_o} + \frac{K_x^*}{s+j\omega_o} + \text{transient terms}$$

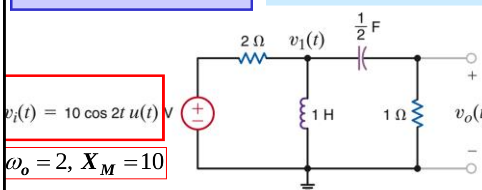
$$K_x = (s-j\omega_o)Y(s)|_{s=j\omega_o} = \frac{1}{2}X_M H(j\omega_o)$$

$$y(t) = 2|K_x|\cos(\omega_o t + \angle K_2) + \text{transient terms}$$

$$y_{ss}(t) = X_M |H(j\omega_o)|\cos(\omega_o t + \angle H(j\omega_o))$$

LEARNING EXAMPLE

Determine the steady state response



$\omega_o = 2, X_M = 10$

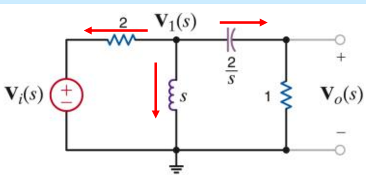
If $x(t) = X_M \cos(\omega_o t + \theta)u(t)$

$y_{ss}(t) = X_M |H(j\omega_o)|\cos(\omega_o t + \angle H(j\omega_o) + \theta)$

$$V_o(s) = \frac{s^2}{3s^2+4s+4}V_i(s) \Rightarrow H(s) = \frac{s^2}{3s^2+4s+4}$$

Transform the circuit to the Laplace domain. Assume all initial conditions are zero

$$H(j2) = \frac{(j2)^2}{3(j2)^2 + 4(j2) + 4} = 0.354\angle 45°$$

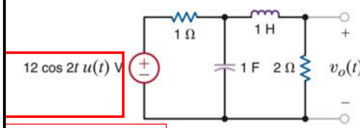


$$\therefore y_s(t) = 3.54\cos(2t + 45°)V$$

$$\text{KCL@V}_1: \frac{V_1 - V_i}{2} + \frac{V_1}{s} + \frac{V_1}{\frac{2}{s}+1} = 0$$

$$\text{Voltage divider}: V_o = \frac{1}{\frac{2}{s}+1}V_1$$

LEARNING EXTENSION Determine $v_{oss}(t), t > 0$



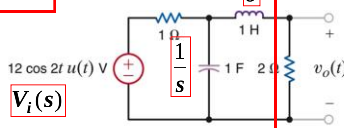
$\omega_o = 2,\ X_M = 12$

If $x(t) = X_M \cos(\omega_o t + \theta) u(t)$

$y_{ss}(t) = X_M \,|H(j\omega_o)|\cos(\omega_o t + \angle H(j\omega_o) + \theta)$

Transform circuit to Laplace domain. Assume all initial conditions are zero

Thevenin



$$V_{OC}(s) = \frac{\frac{1}{s}}{1+\frac{1}{s}} V_i(s) = \frac{1}{s+1} V_i(s)$$

$$Z_{Th}(s) = s + \|1, \frac{1}{s}\| = s + \frac{1}{s+1} = \frac{s^2+s+1}{s+1}$$

$$V_o(s) = \frac{2}{2+Z_{Th}(s)} V_{OC}(s)$$

$$V_o(s) = \frac{2}{2+\dfrac{s^2+s+1}{s+1}} \times \frac{1}{s+1} V_i(s)$$

$$V_o(s) = \frac{2}{s^2+3s+3} V_i(s)$$

$H(s)$

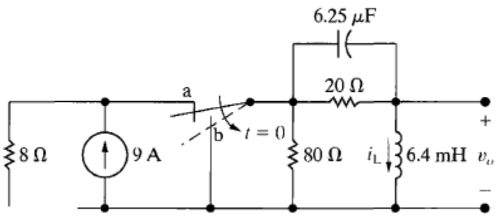
$$H(j2) = \frac{2}{-4+6j+3} = \frac{2}{-1+6j} = \frac{2}{6.08\angle 99.46°}$$

$$v_{oss}(t) = 12 \times \frac{2}{6.08} \cos(2t - 99.46°)$$

STOP

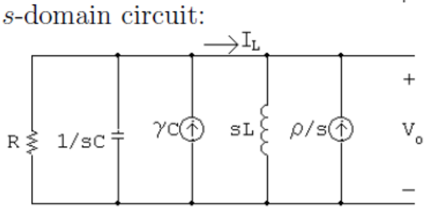
13.12 PSPICE MULTISIM The switch in the circuit in Fig. P13.12 has been in position a for a long time. At $t = 0$, the switch moves instantaneously to position b.

a) Construct the s-domain circuit for $t > 0$.
b) Find V_o.
c) Find I_L.
d) Find v_o for $t > 0$.
e) Find i_L for $t \geq 0$.



Answer

s-domain circuit:



$$V_o = \frac{-48(s+8000)}{s^2+8000s+25\times 10^6}$$

$$I_L = \frac{2.4(s+4875)}{(s^2+8000s+25\times 10^6)}$$

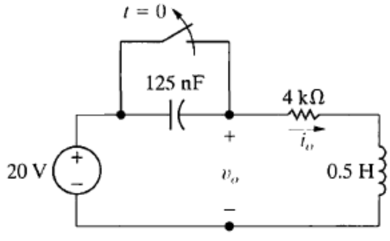
$$v_o(t) = [80e^{-4000t}\cos(3000t + 126.87°)]u(t)\ \text{V}$$

$$i_L(t) = [2.5e^{-4000t}\cos(3000t - 16.26°)]u(t)\ \text{A}$$

13.13 PSPICE MULTISIM The switch in the circuit in Fig. P13.13 has been closed for a long time. At $t = 0$, the switch is opened.

a) Find v_o for $t \geq 0$.

b) Find i_o for $t \geq 0$.

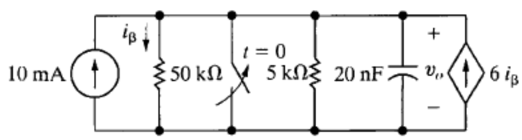


Answer

$$v_o(t) = [20te^{-4000t} + 40{,}000e^{-4000t}]u(t)\text{ V}$$

$$i_o(t) = [20te^{-4000t} + 0.005e^{-4000t}]u(t)\text{ A}$$

13.19 PSPICE MULTISIM The switch in the circuit in Fig. P13.19 has been closed for a long time before opening at $t = 0$. Find v_o for $t \geq 0$.



Answer

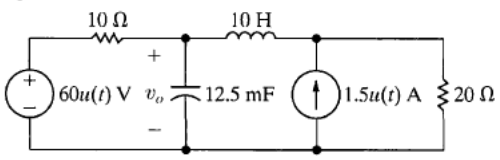
$$v_o(t) = [100 - 100e^{-5000t}]u(t)\text{ V}$$

13.42 There is no energy stored in the circuit seen in Fig. P13.42 at the time the two sources are energized.

PSPICE MULTISIM

a) Use the principle of superposition to find V_o.

b) Find v_o for $t > 0$.



Answer

$$V_{01} = \frac{480(s+2)}{s(s+4)(s+6)}$$

$$V_{02} = \frac{240}{s(s+4)(s+6)}$$

$$V_o = V_{o1} + V_{o2} = \frac{480(s+2)+240}{s(s+4)(s+6)} = \frac{480(s+2.5)}{s(s+4)(s+6)}$$

$$v_o(t) = [50 + 90e^{-4t} - 140e^{-6t}]u(t)\ \mathrm{V}$$