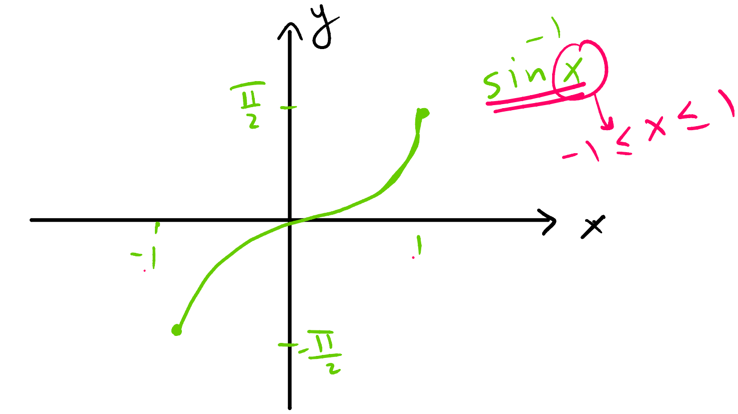
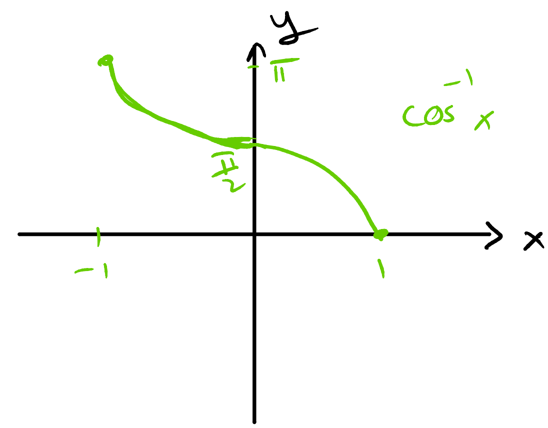
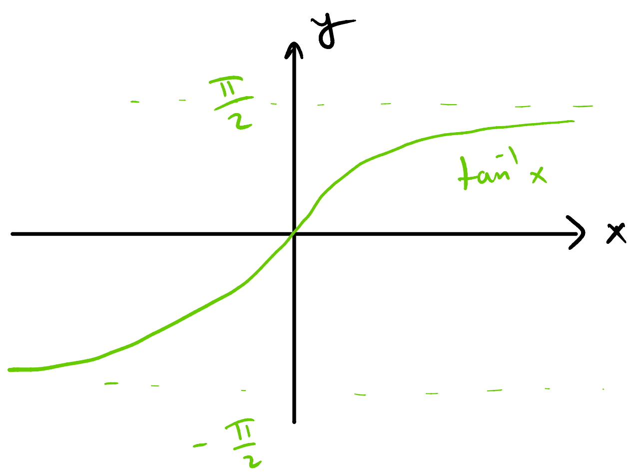
7.6 Part 2

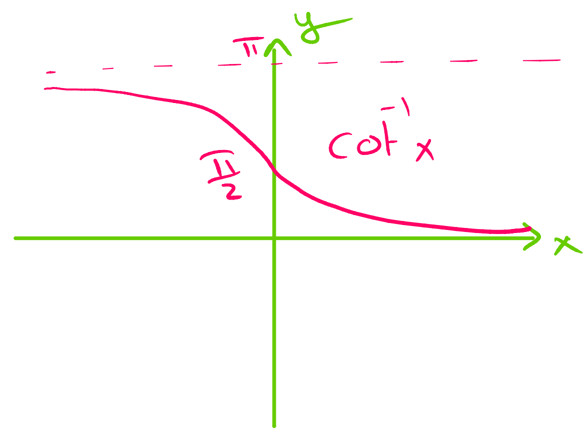
Wednesday, December 15, 2021 12:48 PM

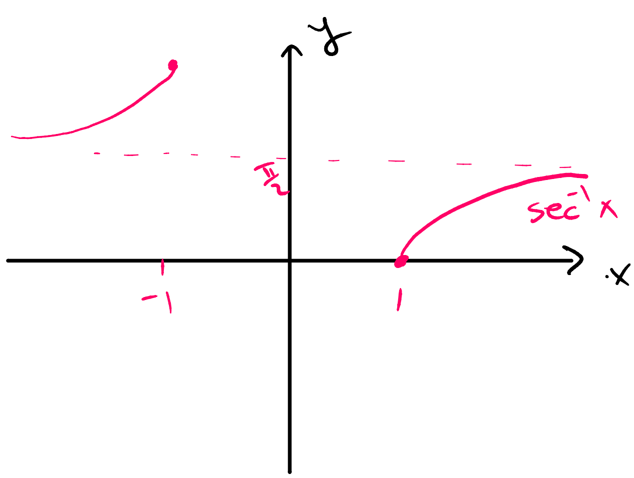
Inverse of Trigonometric functions

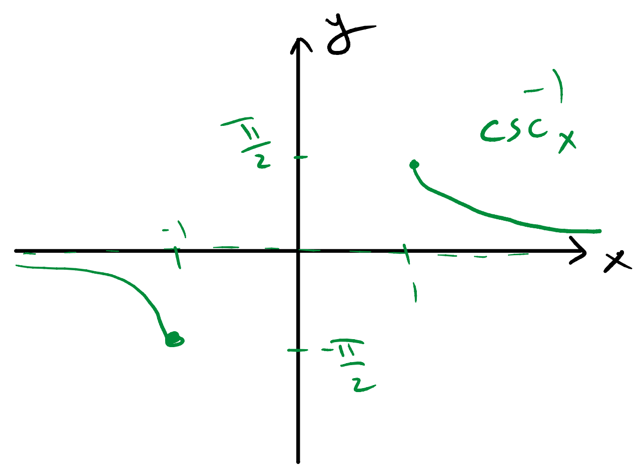












$f(x) = \sin^{-1} x \Rightarrow f'(x)$??

$\cos^{-1} x$

$f(x) = \sin^{-1} x \Rightarrow f'(x)$

$\cos^{-1} x$

$\tan^{-1} x$

$\sec^{-1} x$

$\csc^{-1} x$

$\cot^{-1} x$

If $u(x)$ is diff and

1. $f(x) = \sin^{-1} u \Rightarrow f'(x) = \dfrac{1}{\sqrt{1-u^2}} \cdot u'$

$1-u^2>0 \Rightarrow 1>u^2$

$|u|<1$

2. $f(x) = \cos^{-1} u \Rightarrow f'(x) = \dfrac{-1}{\sqrt{1-u^2}} \cdot u'$

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3. $f(x) = \tan^{-1} u \Rightarrow f'(x) = \dfrac{1}{1+u^2} \cdot u'$

4. $f(x) = \cot^{-1} u \Rightarrow f'(x) = \dfrac{-1}{1+u^2} \cdot u'$

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5. $f(x) = \sec^{-1} u \Rightarrow f'(x) = \dfrac{1}{|u|\sqrt{u^2-1}} \cdot u'$

$u^2-1>0 \Rightarrow u^2>1$

$\Rightarrow \boxed{|u|>1}$

6. $f(x) = \csc^{-1} u \Rightarrow f'(x) = \dfrac{-1}{|u|\sqrt{u^2-1}} \cdot u'$

16) $f(x) = \csc u \Rightarrow f'(x) = \frac{-1}{|u|\sqrt{u^2-1}} \cdot u'$

Exp $y = \sin^{-1} \underbrace{\sqrt{5}\,x}_{u}$ Find $y'(0)$

$u = \sqrt{5}\,x$

$u^2 = 5x^2$

$u' = \sqrt{5}$

$$y' = \frac{1}{\sqrt{1-u^2}} \cdot u' = \frac{1}{\sqrt{1-5x^2}} \cdot \sqrt{5}$$

$$= \frac{\sqrt{5}}{\sqrt{1-5x^2}}$$

$$y'(0) = \frac{\sqrt{5}}{\sqrt{1-5(0)^2}} = \frac{\sqrt{5}}{\sqrt{1}} = \sqrt{5}$$

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2) $y = \ln\left(\tan^{-1} x\right)$ Find $y'(1)$

$u = x$

$u' = 1$

$$y' = \frac{\frac{1}{1+u^2} \cdot u'}{\tan^{-1} x}$$

$$= \frac{\frac{1}{1+x^2}}{\tan^{-1} x}$$

$$y'(1) = \frac{\frac{1}{1+1^2}}{} = \frac{\frac{1}{2}}{\frac{\pi}{4}} = \frac{1}{2} \cdot \frac{4}{\pi} = \frac{2}{\pi}$$

$$y'(1) = \frac{\frac{1}{1+1^2}}{\tan^{-1} 1} = \frac{\frac{1}{2}}{\frac{\pi}{4}} = \frac{1}{2} \cdot \frac{4}{\pi} = \frac{2}{\pi}$$

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(3) $y = \sec^{-1}(2x+1)$ Find $y'(1)$

$u = 2x+1$

$u' = 2$

$$y' = \frac{1}{|u|\sqrt{u^2-1}} \cdot u'$$

$$= \frac{1}{|2x+1|\sqrt{(2x+1)^2-1}} \cdot 2$$

$$y'(1) = \frac{2}{|2(1)+1|\sqrt{(2(1)+1)^2-1}} = \frac{2}{3\sqrt{9-1}} = \frac{2}{3\sqrt{8}}$$

$$= \frac{\cancel{2}}{3\cancel{(2)}\sqrt{2}}$$

$$= \frac{1}{3\sqrt{2}}$$

$$\sqrt{8} = \sqrt{2 \cdot 4} = \sqrt{2}\sqrt{4} = 2\sqrt{2}$$

Exp Show that $(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$

$f(x) = \sin x \Rightarrow f^{-1}(x) = \sin^{-1} x$

$f'(x) = \cos x$

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$\cos x = \sqrt{1-\sin^2 x}$

df^{-1} 1 1

$\cos x = \sqrt{1-\sin^2 x}$

$$\frac{df^{-1}}{dx} = \frac{1}{f'\left(f^{-1}(x)\right)} = \frac{1}{\cos\left(\sin^{-1}x\right)}$$

$$= \frac{1}{\sqrt{1-\sin^2\left(\sin^{-1}x\right)}} = \frac{1}{\sqrt{1-x^2}}$$

$$\sin^2\left(\sin^{-1}x\right) = \left[\cancel{\sin}\left(\cancel{\sin^{-1}}x\right)\right]^2 = x^2$$

$\cos x$ similar

Exp show that $\frac{d}{dx}\left(\sec^{-1}u\right) = \frac{1}{|u|\sqrt{u^2-1}} \cdot u'$

find y'

$y = \sec^{-1}x$

$\sec y = \cancel{\sec}\left(\cancel{\sec^{-1}}x\right) = x$

$\boxed{\sec y = x}$

$\sec y \tan y \frac{dy}{dx} = 1$

$1 + \tan^2 y = \sec^2 y$

$$\frac{dy}{dx} = \frac{1}{\ldots}$$

$\tan y = \pm\sqrt{\sec^2 y - 1}$

$$\frac{dy}{dx} = \frac{1}{\sec y \tan y} \qquad \tan y = \pm\sqrt{\sec^2 y - 1}$$

$$= \frac{1}{\pm\, x \sqrt{\sec^2 y - 1}}$$

$$= \pm \frac{1}{x\sqrt{x^2-1}}$$

$$= \frac{1}{|x|\sqrt{x^2-1}}$$

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for any $a \neq 0 \Rightarrow$

1. $$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C \quad , \quad a^2 > u^2$$

2. $$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\boxed{3}\quad \int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + c \quad , \quad u^2 > a^2$$

جواب

مهم

$$\text{(63)}\quad \int_0^{\ln\sqrt{3}} \frac{e^x\,dx}{1+e^{2x}}$$

$u = e^x$

$du = e^x\,dx$

$u^2 = (e^x)^2 = e^{2x}$

$x = 0 \Rightarrow u = e^0 = 1$

$x = \ln\sqrt{3} \Rightarrow u = e^{\ln\sqrt{3}} = \sqrt{3}$

$$\int_1^{\sqrt{3}} \frac{du}{1+u^2}$$

$$\int_1^{\sqrt{3}} \frac{du}{1^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) = \frac{1}{1}\tan^{-1}\left(\frac{u}{1}\right)\Big|_1^{\sqrt{3}}$$

$a=1$

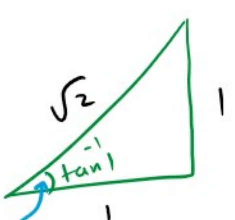
$$= \tan^{-1}u\Big|_1^{\sqrt{3}}$$

$$= \tan^{-1}\sqrt{3} - \tan^{-1}1$$

$$= \frac{\pi}{3} - \frac{\pi}{4}$$

$$= \frac{4\pi - 3\pi}{12}$$

$$= \frac{\pi}{12}$$

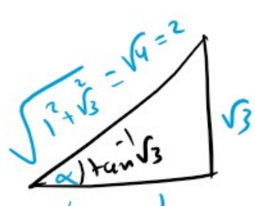


$\theta = \tan^{-1}1$

$\cos\theta = \frac{1}{\sqrt{2}}$

$\sin\theta = \frac{1}{\sqrt{2}}$

$\theta = \frac{\pi}{4}$



$\alpha = \tan^{-1}\sqrt{3}$

$\cos\alpha = \frac{1}{2}$

$\alpha = \frac{\pi}{3}$

$\sin\theta = \frac{1}{\sqrt{2}}$ ✓ $\qquad = \frac{1}{\sqrt{2}}$

75) $$\int \frac{\overset{A}{x} + \overset{B}{4}}{\underbrace{x^2+4}_{C}}\,dx \qquad \frac{A+B}{C} = \frac{A}{C} + \frac{B}{C}$$

$$\int \left(\frac{x}{x^2+4} + \frac{4}{x^2+4} \right) dx$$ ✓ $\tan^{-1}$

$$\frac{1}{2}\int \frac{2x}{x^2+4}\,dx + \int \frac{4}{x^2+4}\,dx$$

$$\frac{1}{2}\ln\left|x^2+4\right| + 4\int \frac{dx}{x^2+2^2}$$

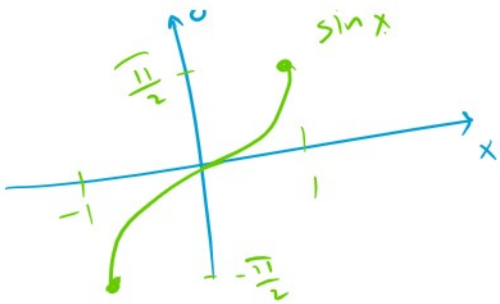
$u = x \qquad a = 2$

$$= \frac{1}{2}\ln(x^2+4) + 4\left(\frac{1}{2}\right)\tan^{-1}\left(\frac{x}{2}\right) + C$$

$$= \frac{1}{2}\ln(x^2+4) + 2\tan^{-1}\frac{x}{2} + C$$

Exp Find the domain of $f(x) = \sin^{-1}(\ln x)$

$$-1 \le \ln x \le 1$$



$$-1 \le \ln x \le 1$$

$$e^{-1} \le e^{\ln x} \le e^{1}$$

$$\boxed{\frac{1}{e} \le x \le e}$$

$$D = \left[\frac{1}{e}, e\right] \checkmark$$

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(59) $$\int \frac{dx}{(2x-1)\sqrt{(2x-1)^2 - 4}}$$

$\sec^{-1}$

$u = 2x-1$

$du = 2\,dx$

$\frac{du}{2} = dx$

$$\int \frac{\frac{du}{2}}{u\sqrt{u^2-4}} = \frac{1}{2}\int \frac{du}{u\sqrt{u^2-2^2}}$$

$a=2$

$$= \frac{1}{2}\,\frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C$$

$$= \frac{1}{2}\left(\frac{1}{2}\right)\sec^{-1}\left|\frac{2x-1}{2}\right| + C$$

$$= \frac{1}{4} \sec^{-1}\left|\frac{2x-1}{2}\right| + c$$

70 $\displaystyle\int_{\frac{1}{2}}^{1} \frac{6\,dt}{\sqrt{3+4t-4t^2}}$

$\sin^{-1}$

$3+4t-4t^2$

$= -[4t^2-4t-3]$

$= -[(2t-1)^2-4]$

$(2t)^2 = 4t^2$

$2(\text{ﻭﺍﺣﺪ})(\text{ﺍﻟﺜﺎﻧﻲ}) = 2(2t)(-1)$

$= -4t$

$(\text{ﺍﻟﺜﺎﻧﻲ})^2 = (-1)^2 = 1$

$= 4-(2t-1)^2$

$1t^2 - 6t + 3 = (t-3)^2 + \boxed{-6}$

$t^2 - 6t + 9$ $\quad 9 \to 3$

$\frac{-6}{2} = -3$

$1t^2 + 4t - 3 = (t+2)^2 + \boxed{-7}$

$4 \to -3$

$\frac{4}{2} = 2$

$4 - 7 = -3$

$$\int_{\frac{1}{2}}^{1} \frac{6\,dt}{\sqrt{3+4t-4t^2}} = \int_{\frac{1}{2}}^{1} \frac{6\,dt}{\sqrt{4-(2t-1)^2}}$$

$a^2 = 2^2$

$a = 2$

$u = 2t-1$

$du = 2\,dt$

$\frac{du}{2} = dt$

$$\int_0^1 \frac{\cancel{6}^{\,3}\,\frac{du}{\cancel{2}}}{\sqrt{2^2 - u^2}}$$

$a = 2$

$u = 2t - 1$

$t = \frac{1}{2} \Rightarrow u = 2\left(\frac{1}{2}\right) - 1 = 1 - 1 = 0$

$t = 1 \Rightarrow u\ 2(1) - 1 = 2 - 1 = 1$

$$3\int_0^1 \frac{du}{\sqrt{2^2 - u^2}} = 3\sin^{-1}\frac{u}{a}\Big|_0^1 = 3\sin^{-1}\frac{u}{2}\Big|_0^1$$

$-u^2 \rightarrow \sin^{-1}$

$$= 3\left[\sin^{-1}\frac{1}{2} - \sin^{-1}\frac{0}{2}\right]$$

$$= 3\left[\frac{\pi}{6} - 0\right]$$

$$= \cancel{3}^{\,1}\,\frac{\pi}{\cancel{6}_{\,2}} = \frac{\pi}{2}$$

Exp

$$\int_{\frac{1}{\sqrt{2}}}^{\frac{\sqrt{3}}{2}} \frac{dx}{\sqrt{1 - x^2}} = \sin^{-1}\frac{u}{a}\Big|_{\frac{1}{\sqrt{2}}}^{\frac{\sqrt{3}}{2}} = \sin^{-1}x\Big|_{\frac{1}{\sqrt{2}}}^{\frac{\sqrt{3}}{2}}$$

$a = 1$ $\quad u = x$ $\quad -x^2 \rightarrow \sin^{-1}$

$$= \sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}\frac{1}{\sqrt{2}}$$

$$= \frac{\pi}{3} - \frac{\pi}{4}$$

$$= \frac{\pi}{12}$$

Exp

$$\int \frac{dx}{\sqrt{4x - x^2}}$$

$\sin^{-1} x$

$4x - x^2 = -\left[x^2 - 4x\right]$

$= -\left[(x-2)^2 + \boxed{-4}\right]$

$+4$

$= 4 - (x-2)^2$

$u = x - 2$

$du = dx$

$$\int \frac{dx}{\sqrt{4 - (x-2)^2}}$$

u

2^2

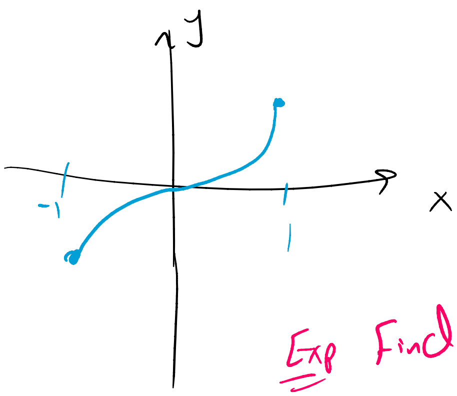
$a = 2$

$$\int \frac{du}{\sqrt{2^2 - u^2}} = \sin^{-1}\frac{u}{a} + C$$

$$= \sin^{-1}\left(\frac{x-2}{2}\right) + c$$

$= \sin^{-1}\left(\frac{\quad}{2}\right)$

$\sin^{-1}(x) \qquad -1 \le x \le 1$

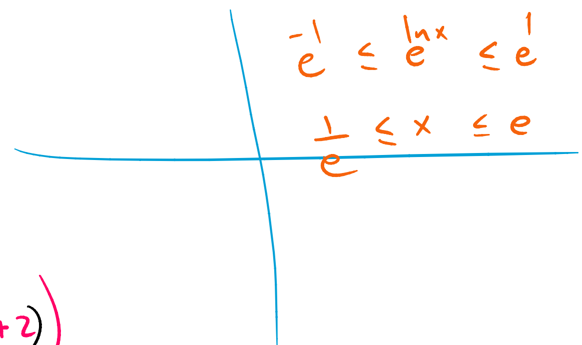


$\sin^{-1}(\ln x)$

$-1 \le \ln x \le 1$

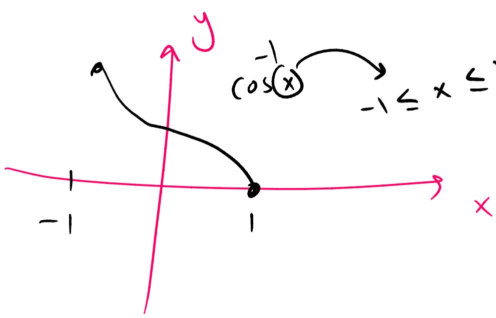
$e^{-1} \le e^{\ln x} \le e^{1}$

$\frac{1}{e} \le x \le e$



Exp Find D of $\cos^{-1}(\ln(x+2))$

D → Domain



$\cos^{-1}(x) \to -1 \le x \le 1$

$-1 \le \ln(x+2) \le 1$

$e^{-1} \le e^{\ln(x+2)} \le e^{1}$

$\frac{1}{e} \le x+2 \le e$

$\boxed{\frac{1}{e} - 2 \le x \le e - 2}$