Birzeit University
Mathematics Department

Chapter 3 Math 234 2017/2018

Name.................................. Number................. Section ....

(Q1) Fill the blanks with true (T) or false (F).

[T] (1) If A is an $n \times n$ singular matrix, then $rank(A) \leq n - 1$.

[F] (2) Any set of vectors containing the zero vector is linearly independent.

//[F] (3) Every nonzero subspace of P_3 contains an infinite number of polynomials.

[T] (4) If A is a 5×3 matrix, then the row space of A can equal $\mathbb{R}^{1\times 3}$.

[T] (5) Any subset of linearly dependent vectors is linearly dependent.

[T] (6) $Span(u,v) = Span(u)$ iff v is a scalar multiple of u.

[F] (7) If A is a square matrix with linearly independent rows, then A is nonsingular.

[F] (8) The rank of a matrix is the number of the nonzero rows of A.

[F] (9) If the set $\{v_1,...,v_k\}$ spans P_4, then $k = 4$.

[F] (10) If the set $\{v_1,...,v_k\}$ is linearly independent in P_4, then $k = 4$.

[T] (11) If A is a nonsingular matrix, then $rank(A) = rank(A^{-1})$.

[F] (12) If S is a subspace of a vector space V and S is finite-dimensional, then V is finite-dimensional.

[T] (13) If S is a subspace of $\mathbb{R}^3$ containing the vectors e_1,e_2,e_3, then $S = \mathbb{R}^3$.

[F] (14) There exists a 5×4 matrix A with $CS(A) = \mathbb{R}^5$.

[T] (15) If A is an $m \times n$ matrix and $b \in \mathbb{R}^m$, then $CS(A)$ is the solution set of the system $Ax = b$.

[T] (16) A minimal spanning set of $\mathbb{R}^{m\times n}$ is a basis of $\mathbb{R}^{m\times n}$.

[T] (17) The vector space $C[-1,1]$ has no spanning set.

//[T] (18) If A is an 5×7 matrix, then $rank(A) = rank(A^T)$.

//[T] (19) If A is an 5×7 matrix, then $nullity(A) = nullity(A^T)$.

[F] (20) If A and B are 6×6 singular matrices, then $rank(A) = rank(B)$.

//[T] (21) If A and B are 5×5 matrices with $rank(AB) = 4$, then $rank(BA) < 5$.

[F] (22) If A and B are 3×3 matrices with $rank(A) = rank(B) = 2$, then $rank(AB) = 2$.

[T] (23) If f,g are vectors in P_n, then $2g \in span(f,g)$.

[F] (24) If u,v,w are nonzero vectors in $\mathbb{R}^2$, then $w \in span(u,v)$.

//[T] (25) If A is an $m \times n$ matrix with $N(A)$ 6= $\{0\}$, then the system $Ax = b$ cannot have a unique solution.

[F] (26) The column space of a matrix A is the set of the solutions of $Ax = 0$.

[T] (27) The set of all solutions of an $m \times n$ homogeneous linear system is a subspace of R^m.

//what?! 9*1[F] (28) If A is a 9×3 matrix with $nullity(A) = 0$,then $Ax = (1,2,3)^T$ has infinite number of solutions.

[F] (29) If A is an $m \times n$ matrix, then $rank(A) \le n$.

[T] (30) The set $\{1,\sin^2 x,\cos^2 x\}$ is linearly dependent in $C[0,\pi]$.

[T] (31) If S,T are subspaces of P_5, then $0 \in S \cap T$.

[T] (32) If S is a subspace of a finite-dimensional vector space V with $dim(S) = dim(V)$, then $S = V$.

[T] (33) Any basis of $\mathrm{R}^{2\times4}$ must contain exactly eight vectors.

[T] (34) The solutions of the equation $x_1 + x_2 - x_3 + 2x_4 = 0$ form a subspace of R^4.

//[T] (35) If A is a 4×4 matrix with $a_2 = -a_4$, then $N(A)$ =6 $\{0\}$.

[T] (36) If the vectors v_1,v_2,v_3,v_4 span $\mathrm{R}^{2\times2}$, then they are linearly independent.

[F] (37) $rank(A)$ = number of columns of A − number of rows of A.

[F] (38) If V is a vector space with dimension $n > 0$, then any set of n or more vectors is linearly dependent.

[F] (39) If three vectors span a vector space V, then any collection of six vectors in V spans V.

[T] (40) If the vectors $v_1,...,v_n$ span a vector space V and v_1 is a linear combination of $v_2,...,v_n$, then $V = span(v_2,...,v_n)$.

[T] (41) The set $B = \{v_1,...,v_n\}$ is a spanning set for a vector space V if every vector in V is a linear combination of the vectors of B.

[F] (42) The set $B = \{v_1,...,v_n\}$ is a basis of a vector space V if every vector in V is a linear combination of the vectors of B.

[T] (43) If S is a subset of a vector space V that does not contain 0, then S is not a subspace of V.

[T] (44) Every set of vectors spanning R^3 has at least three vectors.

[F] (45) If A is an $n \times n$ symmetric matrix, then $rank(A) = n$.

[T] (46) If A is a 4×7 matrix with $rank(A) = 4$, then the system $Ax = 0$ has a nontrivial solution.

//[T] (47) If $rank(A) = rank(A|b)$, then the system $Ax = b$ is consistent.

[F] (48) If $U = \{(x,y)^T; y = x + 1\}$, then U is a subspace of R^2.

[F] (49) If A is a 3×7 matrix, then it is possible that the dimension of $CS(A)$ is 6.

[T] (50) If A is a nonzero matrix of the form $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then $rank(A) = 2$.

[T] (51) If S is a subspace of R^4 with $dim(S) = 2$, then S can have a spanning set of three vectors.

[T] (52) The columns of a nonsingular 10×10 matrix form a basis for R^{10}.

[T] (53) The set $\{(x_1,x_2,x_3,x_4)^T \mid x_1 + x_3 = x_2 - x_4 = 0\}$ is a subspace of R^4.

[F] (54) If E is an elementary matrix, then it has linearly independent columns.

[T] (55) If v_1,v_2 are nonzero vectors in R^5 with $v_1 + v_2 = 2v_1 - v_2$, then v_1,v_2 are linearly dependent.

[T] (56) The set of all 8×8 elementary matrices forms a subspace of $R^{8\times 8}$.

//[F] (57) The set $B = \{1, x^2 - x\}$ is a basis for the subspace $S = \{ f \in P_3 \mid f \text{ is even } \}$.

[F] (58) The interval $[0,\infty)$ is a subspace of R.

[T] (59) If v_1,v_2,v_3 are linearly independent in R^3, then $span(v_1,v_2,v_3) = R^3$.

[T] (60) The functions $f(x) = 3x$ and $g(x) = |-3x|$ are linearly independent in $C[-4,4]$.

[F] (61) The functions $f(x) = 3x$ and $g(x) = |-3x|$ are linearly independent in $C[-4,0]$.

[F] (62) $dim(\ span(2,\cos(2x),\sin(2x))\ ) = 2$.

[T] (63) $dim(\ span(2,\cos^2 x,\sin^2 x)\ ) = 2$.

//[F] (64) $S = \{f(x) \in P_5 \mid f(-2) = 0 \text{ or } f(2) = 0\}$ is a subspace of P_5.

[T] (65) $S = \{f(x) \in P_5 \mid f(-2) = 0 \text{ and } f(2) = 0\}$ is a subspace of P_5.

[F] (66) If $nullity(A) = 0$ and $Ax = b$ has a solution, then $Ax = b$ has infinitely many solutions.

[T] (67) If the system $Ax = b$ has infinite number of solutions, then $N(A)$ 6= {0}.

[T] (68) If $b \in CS(A)$, then the system $Ax = b$ is consistent.

[T] (69) Two row equivalent matrices have the same nullity.

[T] (70) Two row equivalent matrices have the same rank.

[T] (71) Two row equivalent matrices have the same null space.

[F] (72) Two row equivalent matrices have the same column space.

[T] (73) Two row equivalent matrices have the same row space.

[F] (74) The coordinate vector of $12 + 6x$ with respect to the basis $\{x,4\}$ is $(3,6)^T$.

[F] (75) If three vectors span a vector space V, then $dim(V) = 3$.

// [T] (76) If $u,v \in V$ and B is a basis of V, then $[\alpha u + \beta v]_B = \alpha[u]_B + \beta[v]_B$ for any scalars α,β.

[T] (77) The transition matrix of two bases is always nonsingular.

[T] (78) If S is a subspace of a vector space V, then $0 \in S$.

[T] (79) If $dim(V) = n > 0$, then any set of $m > n$ vectors in V is linearly dependent.

[T] (80) If $dim(V) = n > 0$, then any set of $m < n$ vectors in V doesn't span V.

[T] (81) $rank(A) = \text{number of columns of } A - nullity(A)$.

[T] (82) If six vectors span V, then a collection of seven vectors in V is linearly dependent.

[T] (83) If two vectors are linearly dependent, then each of them is a scalar multiple of the other.

[T] (84) If the system $Ax = b$ is inconsistent, then b 6∈ $CS(A)$.

[T] (85) The vectors $(4,2,3)^T,(2,3,1)^T,(2,5,3)^T,(2,0,3)^T$ are linearly dependent.

[F] (86) Any subset of V that contains the zero vector is a subspace of V.

[T] (87) If $dim(V) = 4$ and v_1,v_2,v_3,v_4 are distinct vectors in V, then $span(v_1,v_2,v_3,v_4) = V$.

[F] (88) The vector space R has infinitely many subspaces.

[F] (89) If the columns of A are linearly independent, then $Ax = b$ is always consistent.

/[T] (90) The set of all $n \times n$ nonsingular matrices is a subspace of $R^{n\times n}$.

[F] (91) If $\{v_1,...,v_n\}$ is a spanning set of V, then $dim(V) \geq n$.

[F] (92) The dimension of $C^n[a,b]$ is n.

[T] (93) All solutions of the $m \times n$ system $Ax = b$ is a subspace of R^n.

/[T] (94) If x_1,x_2 are two distinct solutions of $Ax = b$, then x_1 and x_2 are linearly independent.

[T] (95) The set $\{(1,2)^T,(2,0)^T,(0,0)^T\}$ is a spanning set of R^2.

//مش شرط بس الزيرو صح ؟[ ] (96) If S_1,S_2 are two subspaces of R^2, then $S_1 \cap S_2 = \{0\}$.

[F] (97) If A is a 4×3 matrix with $rank(A) = 3$, then $Ax = 0$ has a nontrivial solution.

[T] (98) If U,W are subspace of V, then $U \cap W$ is a subspace of V.

[F] (99) If U,W are subspace of V, then $U \cup W$ is a subspace of V.

[ ] (100) If U,W are subspace of V, then $U + W$ is a subspace of V.

[F] (101) If A is a 5×4 matrix and $Ax = 0$ has only the trivial solution, then $rank(A) = 4$.

[F] (102) If $\{v_1,...,v_2\}$ is a spanning set of V and $v_{n+1} \in V$, then the set $\{v_1,...,v_n,v_{n+1}\}$ doesn't span V.

[T] (103) The vectors $(1,-1,1)^T,(1,-3,2)^T,(1,-2,0)^T$ form a basis for R^3.

[T] (104) If A is a 5×7 matrix, then $nullity(A) \geq 2$.

[T] (105) If A is a 4×4 matrix with $rank(A) = 4$, then A is row equivalent to I.

[T] (106) If A is a 4×4 matrix with $rank(A) = 3$, then A^T is singular.

[T] (107) If A is a 4×4 matrix with $rank(A) = 0$, then $adjA = 0$.

[T] (108) It is possible to find a matrix A of size 3×5 such that $nullity(A) = 1$.

[T] (109) If $f_1,...,f_n \in C^{n-1}[a,b]$ and $W[f_1,...,f_n](x)$ = 06 $\forall x \in [a,b]$, then $f_1,...,f_n$ are linearly independent in $C[a,b]$.

[T] (110) The set $\{x-1, x^2+2x+1, x^2+x-2\}$ forms a basis of P_3.

[T] (111) If A is an $n \times n$ matrix and $RS(A) = R^{1\times n}$, then $CS(A) = R^n$.

[F] (112) If $f,g,h \in C^2[a,b]$ and $W[f,g,h](x) = 0 \ \forall x \in [a,b]$, then f,g,h are linearly dependent in $C[a,b]$.

[T] (113) The vectors x,e^x,xe^x are linearly independent in $C[0,1]$.

[T] (114) If v_1,v_2 are linearly independent in R^3, then $\exists\ v_3 \in R^3$ such that $span(v_1,v_2,v_3) = R^3$.

[F] (115) If the vectors $v_1,...,v_n$ are linearly independent in V, then V is finite-dimensional.

[T] (116) If $dim(V) = n > 0$, then any $n + 1$ vectors in V are linearly dependent.

[F] (117) Every linearly independent set of vectors in P_n must contain n polynomials.

[F] (118) If V is an infinite-dimensional vector space, then any subspace of V is infinite-dimensional.

[T] (119) If $dim(V) = n$ and S is a nonzero subspace of V, then $0 < dim(S) \le n$.

[T] (120) If A is an $m \times n$ matrix such that the system $Ax = b$ is consistent for every $b \in \mathbb{R}^m$, then the reduced row echelon form of A has m nonzero rows.

[F] (121) The set of all polynomials of degree 3 under the usual addition and scalar multiplication is a vector space.

[T] (122) Any set of vectors which contains the zero vector is linearly dependent.

/[T] (123) If the set $\{v_1,v_2,v_3\}$ is linearly independent, then $\{v_1,v_1+v_2,v_1+v_2+v_3\}$ is linearly independent.

[T] (124) Any subspace of a vector space is also a vector space.

[ ] (125) The dimension of the subspace $\{A \in \mathbb{R}^{2\times2} \mid A \text{ is symmetric}\}$ is 2.

[ ] (126) If A is an $m \times n$ matrix and B a nonsingular $m \times m$ matrix, then $N(BA) = N(A)$.

[T] (127) If A is a 4×3 matrix and $Ax = 0$ has only the zero solution, then $dim(\ RS(A)\ ) = 3$.

[T] (128) If A is a 3×3 matrix, then A is nonsingular iff $N(A) = \{0\}$.

[F] (129) If A is a 3×5 matrix, then A can have four linearly independent columns.

[F] (130) If V is a vector space such that $span(v_1,v_2,v_3) = V$, then $dim(V) = 3$.

//[F] (131) If U,W are subspaces of a finite-dimensional vector space with U 6= W, then $dim(U)$ 6= $dim(W)$.

[T] (132) If A is an $n \times n$ matrix and $Ax = b$ has more than one solution for some $b \in \mathbb{R}^n$, then $rank(A)$ 6= n.

[T] (133) The dimension of the subspace $\{A \in \mathbb{R}^{2\times2} \mid A \text{ is diagonal}\}$ is 2.

/[T] (134) If A is an $m \times n$ matrix and $Ax = b$ is consistent $\forall\, b \in \mathbb{R}^m$, then $n \ge m$.

[F] (135) If the set $\{v_1,v_2,v_3\}$ is a basis of V, then any four vectors span V.

[T] (136) If A is an $n \times n$ matrix and $Ax = b$ is consistent $\forall\, b \in \mathbb{R}^n$, then A is nonsingular.

[ ] (137) If S is a set of linearly independent vectors, then any nonempty subset of S is linearly independent.

[F] (138) If S is set of linearly independent vectors in a vector space V, then any subset of V containing S is linearly independent.

// [T] (139) If B is a basis for a vector space V, then B spans any subspace of V.

[T] (140) $span(x + 1, x - 1)$ is a subspace of P_2.

[T] (141) If $U = RREF(A)$, then U and A have the same null space.

[T] (142) If $v_1,v_2,v_3 \in V$ and $span(v_1,v_2,v_3) = span(v_1,v_3)$, then v_1,v_2,v_3 are linearly dependent.

[T] (143) If the columns of a square matrix A are linearly independent, then $det(A)$ 6= 0.

[F] (144) If A is a singular matrix, then $nullity(A) = 0$.

[F] (145) If $dim(V) = 4$ and $\{v_1,v_2,v_3,v_4\} \subseteq V$, then $V = span(v_1,v_2,v_3,v_4)$.

[T] (146) In any vector space, $\alpha 0 = 0$.

[T?!] (147) If $u \in V$ and α a nonzero scalar with $\alpha u = 0$, then $u = 0$.

[T] (148) Every spanning set of $R^{2\times3}$ contains at least six vectors.

[T] (149) The set $\{p(x) \in P_5 \mid p(x)$ is even$\}$ is a subspace of P_5.

[ ] (150) The vector $(3, -1, 0)^T \in span(\ (2, -1, 3)^T, (-1, 1, 1)^T, (1, 1, 9)^T\ )$.

[T] (151) If A is a nonzero 3 × 2 matrix and $Ax = 0$ has a nonzero solution, then $rank(A) = 1$.

[F] (152) In R^3, every set with more than three vectors can be reduced to a basis of R^3.

/ [T] (153) If A is a nonsingular matrix, then $RS(A) = RS(A^T)$.

[F] (154) R^2 is a subspace of R^4.

[T] (155) P_2 is a subspace of P_4.

// [ ] (156) It is possible to find a pair of two-dimensional subspaces S and T of R^3 such that $S \cap T = \{0\}$.

//[T] (157) If A and B are $n \times n$ nonsingular matrices, then $rank(AB) = rank(BA)$.

[T] (158) If $\hat{x}$ is a solution of the system $Ax = b$ and $\hat{y} \in N(A)$, then $\hat{x} - 5\hat{y}$ is a solution of $Ax = b$.

[T] (159) We cannot find a 7 × 7 matrix with $rank(A) = nullity(A)$.

/[T] (160) If A is a 5 × 4 matrix with linearly independent columns, then $nullity(A^T) = 1$.