# Chapter 10: Rotation

- $\theta = \frac{s}{r}$ , $s \equiv$ arc length , $r \equiv$ radius of the circular arc
- $1 \text{ rev} = 360^\circ = 2\pi \text{ rad}$
- Translational motion $\Longrightarrow$ Rotational motion

  $r, v, a$ $\qquad$ $\theta, \omega, \alpha$

$\Rightarrow$ • Angular displacement $\theta_2 - \theta_1 = \Delta\theta \Rightarrow \begin{cases} +ve, \text{ counterclock-wise} \\ \text{negative, clockwise} \end{cases}$

- Average angular velocity $\omega_{avg} = \frac{\Delta\theta}{\Delta t}$
- Angular velocity (Instantaneous) $\omega = \frac{d\theta}{dt}$
- Average Angular acceleration $\alpha_{avg} = \frac{\Delta\omega}{\Delta t}$
- Angular Acceleration $\alpha = \frac{d\omega}{dt}$ "Instantaneous"

$r \longrightarrow v \longrightarrow a$ differentiation

$\longleftarrow \quad \longleftarrow$ Integration

$\theta \longrightarrow \omega \longrightarrow \alpha$ differentiation

$\alpha \longrightarrow \omega \longrightarrow \theta$ Integration

* Rotation with ~~Angular~~ constant Angular Acceleration

$$\omega = \omega_0 + \alpha t$$

$$\Delta\theta = \theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\theta - \theta_0 = \frac{1}{2}(\omega + \omega_0)t$$

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- The linear and Angular variables

$\rightarrow s = \theta r$

$v = \omega r$

$a_t = \alpha r$

$a_r = \frac{v^2}{r} = \omega^2 r$

- Uniform circular motion $\Rightarrow T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$

$f = \frac{1}{T} \quad , \quad \omega = 2\pi f$

- Kinetic Energy $\Rightarrow K = \frac{1}{2} I \omega^2$

$I = \begin{cases} \sum m_i r_i^2 & \text{Discrete particles} \\ \int r^2 \, dm & \text{continuous distribution of mass} \end{cases}$

- The parallel axis theorem

$I = I_{com} + Mh^2$

$h$ = distance between the new axis and the COM axis

- Torque $= \vec{r} \times \vec{F}$

Newton's Second Law of Rotation $\tau_{net} = I\alpha$

- Rotational Equilibrium $\Rightarrow \tau_{net} = \text{Zero}$

$\Rightarrow$ Work $= \int_{\theta_i}^{\theta_f} \tau \, d\theta \quad , \quad P = \frac{dW}{dt} = \tau \omega$

Work – Kinetic energy Theorem for Rotating bodies

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10-2 What is the angular speed of (a) the second hand, (b) the minute hand, and (c) the hour hand of a smoothly running analog watch? Answers in radians per second.

- $\omega = \Delta\theta / \Delta t$

$\Rightarrow$ Second hand $\Rightarrow$ $\omega = \frac{2\pi}{60} = 0.105$ rad/sec

$\Rightarrow$ Minute hand $\Rightarrow$ $\omega = \frac{2\pi}{60 \times 60} = 1.75 \times 10^{-3} \frac{rad}{sec}$

$\Rightarrow$ Hour hand $\Rightarrow$ $2\pi$ in 12 hours

$$\omega = \frac{2\pi}{12 \times 60 \times 60} = 1.45 \times 10^{-4} \frac{rad}{sec}$$

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10-13 A flywheel turns through 40 rev as it slows from an angular speed of 1.5 $\frac{rad}{sec}$ to a stop. (a) Assuming a constant angular acceleration, find the time for it to come to rest. (b) What is its angular acceleration? (c) How much time is required for it to complete the first 20 of the 40 revolutions?

(a) $\theta = \frac{\omega_0 + \omega_f}{2} t$

$40(2\pi) = \frac{1.5 + 0}{2} t \Rightarrow t = 335.1 \text{ sec} \Rightarrow \boxed{3.4 \times 10^2 \text{ sec} = t}$

(b) $\omega_f = \omega_0 + \alpha t$

$0 = 1.5 + (3.4 \times 10^2)\alpha \Rightarrow \boxed{\alpha = -4.48 \times 10^{-3} \frac{rad}{sec^2}}$

(c) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$20(2\pi) = 1.5 t + \frac{1}{2}(-4.48 \times 10^{-3}) t^2$

$0 = -2.24 \times 10^{-3} t^2 + 1.5 t - 125.7$

$t = \cancel{571.44} \text{ sec}, \quad 98.2 \text{ sec}$

rejected

(greater than the time needed to complete 40 rev)

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10-23 A flywheel with a diameter of 1.20 m is rotating at an angular speed of 200 rev/min. (a) What is the angular speed of the flywheel in radians per second? (b) What is the linear speed of a point on the rim of the flywheel? (c) What constant angular acceleration (in revolutions per minute-squared) will increase the wheel's angular speed to 1000 rev/min in 60.0 sec? (d) How many revolutions does the wheel make during that 60.0 sec?

(a) The angular speed of the wheel

$$\omega_0 = \frac{\Delta\theta}{\Delta t} = 200\ \frac{\text{rev}}{\text{min}}\left[\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right]\left[\frac{1\ \text{min}}{60\ \text{sec}}\right] = 20.9\ \frac{\text{rad}}{\text{sec}}$$

(b) $v = \omega r$ ; $v \equiv$ the linear speed and $r \equiv$ the radius of the wheel

$$v = r\omega = \frac{1.20}{2}\text{m}\left[20.9\ \frac{\text{rad}}{\text{sec}}\right] = 12.5\ \text{m/s}$$

(c) $$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega - \omega_0}{\Delta t} = \frac{1000\ \frac{\text{rev}}{\text{min}} - 200\ \frac{\text{rev}}{\text{min}}}{1\ \text{min}}$$

$$\alpha = 800\ \frac{\text{rev}}{\text{min}^2}$$

(d) $$\theta = \frac{1}{2}(\omega + \omega_0)t = \frac{1}{2}\left[200\ \frac{\text{rev}}{\text{min}} + 1000\ \frac{\text{rev}}{\text{min}}\right]1\ \text{min}$$

$$\boxed{\theta = 600\ \text{rev}}$$

OR $$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$= 0 + \left(200\ \frac{\text{rev}}{\text{min}}\ 1\ \text{min}\right) + \left(\frac{1}{2}\ 800\ \frac{\text{rev}}{\text{min}^2}\right)1\ \text{min}^2$$

$$\boxed{\theta = 600\ \text{rev}}$$

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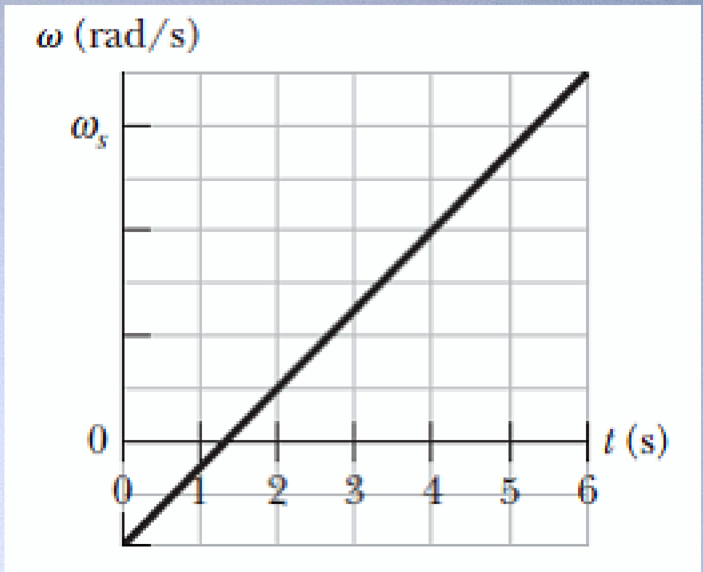
10-34 The below figure gives angular speed versus time for a thin rod that rotates around one end. The scale on the $\omega$ axis is set by $\omega_s = 6.0$ rad/sec. (a) What is the magnitude of the rod's angular acceleration? (b) At $t = 4.0$ sec, the rod has a rotational Kinetic energy of 1.60 J. What is its Kinetic energy at $t = 0$?




(a) $\alpha$ = Angular acceleration

$\alpha$ = slope of the $\omega$ vs. t graph

$$\alpha = 1.5 \ \frac{rad}{sec^2}$$

(b) at $t = 0$ ; $\omega = -2 \ \frac{rad}{sec}$

$$K = \frac{1}{2} I \omega^2$$

at $t = 4$ sec, $\omega = 4 \ \frac{rad}{sec}$ and $K = 1.60$ J

$$I = \frac{2K}{\omega^2} = \frac{2(1.6)}{16} = 0.2 \ Kg\,m^2$$

$$\Rightarrow K(t=0) = \frac{1}{2} I \omega^2$$

$$= \frac{1}{2}(0.2)(-2)^2$$

$$\boxed{K(t=0) = 0.4 J}$$

$$\frac{K_o}{K_4} = \frac{\omega_o^2}{\omega_4^2} = \frac{(-2)^2}{(4)^2} = 0.25$$

$$K_o = 0.25 K_4 = 0.25(1.6)$$

$$\boxed{K_o = 0.4 J}$$

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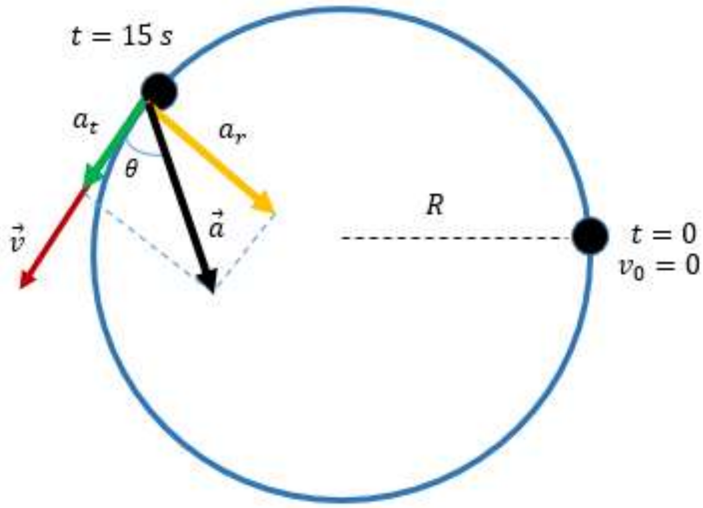
**10-32) A car starts from rest and moves around a circular track of radius 32.0 m. Its speed increases at the constant rate of 0.600 m/s$^2$. (a) What is the magnitude of its *net* linear acceleration 15.0 s later? (b) What angle does this net acceleration vector make with the car's velocity at this time?**

$a_t = 0.6\ m/s^2$

$R = 32\ m$




a) What is the magnitude of the net linear acceleration?

To find the linear acceleration of the car you must find the tangential acceleration $a_t$ (which is given in problem) and the radial acceleration $a_r$

At first, let us find the speed at $t = 15\ s$

$v = v_0 + a_t t$

$v = 0 + 0.6 * 15 = 9\ m/s$

Thus, the radial acceleration can be found as

$$a_r = \frac{v^2}{R} = \frac{9^2}{32} = 2.53\ m/s^2$$

thus, the magnitude of the linear acceleration is

$$a = \sqrt{a_t^2 + a_r^2}$$

$$a = \sqrt{0.6^2 + 2.53^2} = 2.6\ m/s^2$$

b) What angle does this net acceleration vector make with the car's velocity at this time?

$$\theta = \tan^{-1}\frac{2.53}{0.6} = 76.66°$$

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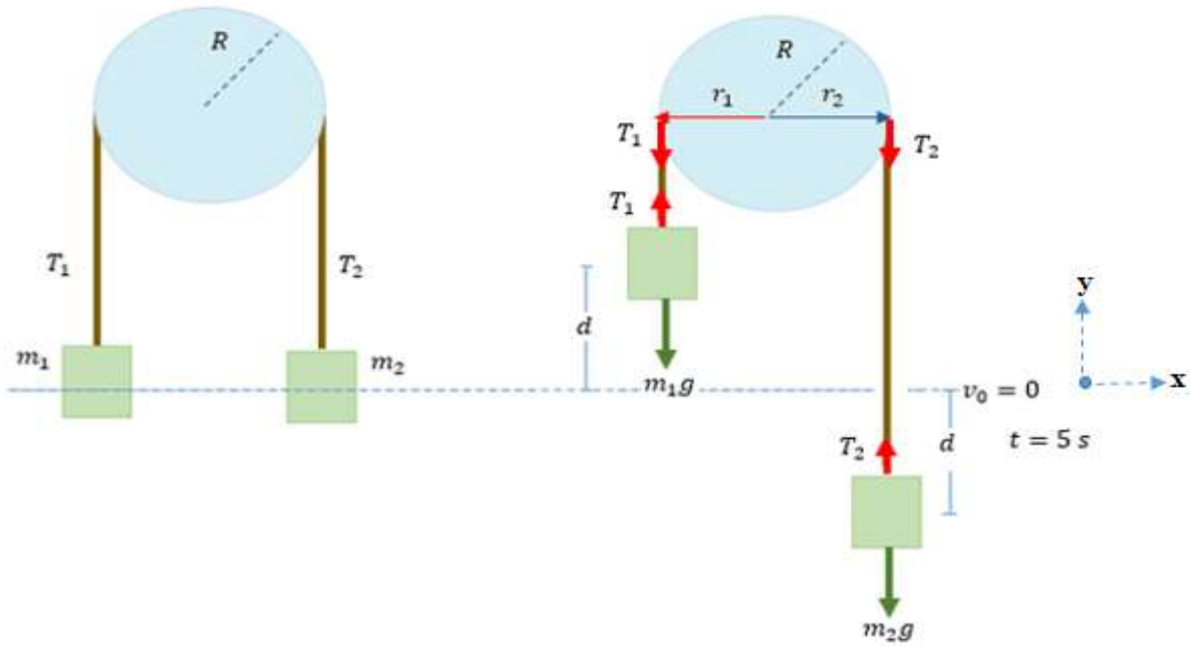
**10-51) In the below figure, block 1 has mass $m_1$=460 g, block 2 has mass $m_1$=500 g, and the pulley, which is mounted on a horizontal axle with negligible friction, has radius *R* =5.00 cm. When released from rest, block 2 falls 75.0 cm in 5.00 s without the cord slipping on the pulley. (a) What is the magnitude of the acceleration of the blocks? What are (b) tension $T_2$ and (c) tension $T_1$? (d) What is the magnitude of the pulley's angular acceleration? (e) What is its rotational inertia?**




$m_1 = 460g = 0.46\ Kg$

$m_2 = 500g = 0.5\ Kg$

$R = 5\ cm = 0.05\ m$

$d = 75\ cm = 0.75\ m$

$v_0 = 0$

$t = 5\ s$

a) The magnitude of the acceleration of the blocks

For block2

$y - y_0 = v_0 t + \frac{1}{2} g t^2$

$-0.75 = 0 + \frac{1}{2} a(5^2)$

$a_2 = -0.06\ m/s^2$

Note: the negative sign indicates that the acceleration of block2 is downward, thus

$\vec{a}_2 = -0.06j\ m/s^2$

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Block1 has the same acceleration but in the opposite direction of block2. This is because the two blocks moved the same distance $d$ without the cord slipping on the pulley. This means that

$\vec{a}_1 = +0.06j\ m/s^2$

The magnitude of acceleration of the two block is similar, it is $0.06\ m/s^2$

b) Tension $T_2$

Applying Newton's second low on bock2

$T_2 - m_2 g = m_2(-a_2)$

$T_2 = m_2 g - m_2 a_2$

$T_2 = 0.5 * 9.8 - 0.5 * 0.06 = 4.87\ N$

c) Tension $T_1$

Applying Newton's second low on bock1

$T_1 - m_1 g = m_1 a_1$

$T_1 = m_1 g + m_1 a_1$

$T_1 = 0.46 * 9.8 + 0.46 * 0.06 \approx 4.54N$

d) What is the magnitude of the pulley's angular acceleration

The angular acceleration $\alpha$ of the pulley is

$$\alpha = \frac{a}{R} = \frac{0.06}{0.05} = 1.2\ rad/s^2$$

e) What is the pulley's rotational inertia

Now applying Newton's second low on the pulley

$\vec{\tau}_{net} = I_{pulley}\vec{\alpha}$

$\vec{\tau}_1 + \vec{\tau}_2 = I\vec{\alpha}$ ............................ (1)

Note:

$|\vec{\tau}_1| = |\vec{r}_1 \times \vec{T}_1| = r_1 T_1 \sin 90 = RT_1 = 0.05 * 4.54 = 0.227\ N.m$

$|\vec{\tau}_2| = |\vec{r}_2 \times \vec{T}_1| = r_2 T_2 \sin 90 = RT_2 = 0.05 * 4.87 = 0.244N.m$

Using the right hand rule: The direction of $\vec{\tau}_1$ out of the page (counterclockwise (+))
The direction of $\vec{\tau}_2$ into the page (clockwise (-))

Note: the direction of the angular acceleration $\vec{\alpha}$ is the same as the direction of $\vec{\tau}_{net}$, thus , it is into the page also as $|\vec{\tau}_2| > |\vec{\tau}_1|$ (CLOCKWISE)

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Substituting in eq.(1) above you get

$$+0.2 - 0.2 = I(-1.2)$$

$$I = \frac{+0.227 - 0.244}{-1.2} = 0.0138\ Kg.m^2$$

**3** A force is applied to the rim of a disk that can rotate like a merry-go-round, so as to change its angular velocity. Its initial and final angular velocities, respectively, for four situations are: (a) −2 rad/s, 5 rad/s; (b) 2 rad/s, 5 rad/s; (c) −2 rad/s, −5 rad/s; and (d) 2 rad/s, −5 rad/s. Rank the situations according to the work done by the torque due to the force, greatest first.

**a)**
$$\begin{aligned} W &= \frac{1}{2}\, I\, \Delta\omega^2 \\ &= \frac{1}{2}\, I\, \left(\omega_f^2 - \omega_i^f\right) \\ &= \frac{1}{2} \times I \times \left((5\ \mathrm{rad/s})^2 - (-2\ \mathrm{rad/s})^2\right) \\ &= 10.5\ I \end{aligned}$$

**b)**
$$\begin{aligned} W &= \frac{1}{2}\, I\, \Delta\omega^2 \\ &= \frac{1}{2}\, I\, \left(\omega_f^2 - \omega_i^f\right) \\ &= \frac{1}{2} \times I \times \left((5\ \mathrm{rad/s})^2 - (2\ \mathrm{rad/s})^2\right) \\ &= 10.5\ I \end{aligned}$$

**c)**
$$\begin{aligned} W_c &= \frac{1}{2}\, I\, \Delta\omega^2 \\ &= \frac{1}{2}\, I\, \left(\omega_f^2 - \omega_i^f\right) \\ &= \frac{1}{2} \times I \times \left((-5\ \mathrm{rad/s})^2 - (-2\ \mathrm{rad/s})^2\right) \\ &= 10.5\ I \end{aligned}$$

**d)**
$$\begin{aligned} W_d &= \frac{1}{2}\, I\, \Delta\omega^2 \\ &= \frac{1}{2}\, I\, \left(\omega_f^2 - \omega_i^f\right) \\ &= \frac{1}{2} \times I \times \left((-5\ \mathrm{rad/s})^2 - (2\ \mathrm{rad/s})^2\right) \\ &= 10.5\ I \end{aligned}$$

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**Q5.** In Fig. 10-22, two forces and act on a disk that turns about its center like a merry-go-round. The forces maintain the indicated angles during the rotation, which is counterclockwise and at a constant rate. However, we are to decrease the angle $\theta$ of $\vec{F}_1$ without changing the magnitude of $\vec{F}_1$. (a) To keep the angular speed constant, should we increase, decrease, or maintain the magnitude of $\vec{F}_2$? Do forces (b) $\vec{F}_1$ and (c) $\vec{F}_2$ tend to rotate the disk clockwise or counterclockwise?

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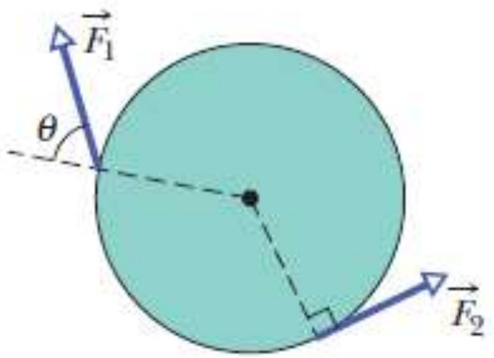
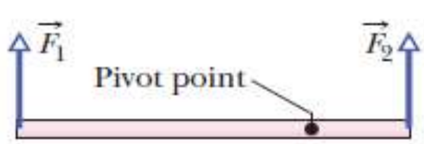
(a) **decrease**. Increasing $\theta$ reduces the magnitude of the torque caused by $\vec{F}_1$, so we must also reduce the magnitude of the torque caused by $\vec{F}_2$.
(b) **Clockwise**; a negative torque.
(c) **Counterlockwise**; a positive torque.




**7** Figure 10-25*a* is an overhead view of a horizontal bar that can pivot; two horizontal forces act on the bar, but it is stationary. If the angle between the bar and $\vec{F}_2$ is now decreased from 90° and the bar is still not to turn, should $F_2$ be made larger, made smaller, or left the same?




**made greater**. The torque $F_2$ exerts about the pivot must be maintained. The factor are $|\vec{r}|$ doesn't change, so if $\sin\theta$ is to decrease, $|\vec{F}|$ has to increase.

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