Mustafa Jarrar & Radi Jarrar: Lecture Notes in Discrete Mathematics.
Birzeit University, Palestine, 2021

Propositional Logic

2.1. Introduction and Basics

2.2 Conditional Statements

2.3 Inferencing

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Acknowledgement:
This lecture is based on (but not limited to) to chapter 2 in "Discrete Mathematics with Applications by Susanna S. Epp (3rd Edition)".

What is Discrete Math?

- Discrete mathematics is the branch of mathematics dealing with objects that can assume only distinct, separated values
- It is a branch of mathematics concerned with the study of objects that can be represented finitely (or countable)
- Discrete mathematics is the mathematical language of computer science

What is Discrete Math?

- Concepts and notations from discrete mathematics are useful in studying and describing objects and problems in all branches of computer science, such as algorithms, artificial intelligence, programming languages, security and cryptography, automated theorem proving, and software development in general
- Mathematical reasoning is interesting but also a great way to increase creative mathematical thinking. It helps in your personal life as much as your output as a software developer

Propositional Logic

What is Logic?

- Logic is concerned with the methods of reasoning
- Logic provides rules and techniques to determine whether a given argument is valid
- Logic is the language used for most formal specification languages
- Programs can be described with mathematics, and Propositional logic can be used to reason about their correctness (design and analysis of algorithms)

Proposition

- The basic building blocks of logic—propositions
- A **proposition** is a statement (that is, a sentence that declares a fact) that is either true or false, but not both

- Examples:

Amman is the capital of Jordan.
1 + 1 = 2.
2 + 2 = 3.

Proposition

- Consider the following examples, are they propositions?

- What time is it? a question not a statement.

- Read this carefully. not a statement.

- x + 1 = 2. not a statement.

- x + y = z. not a statement.

Negation

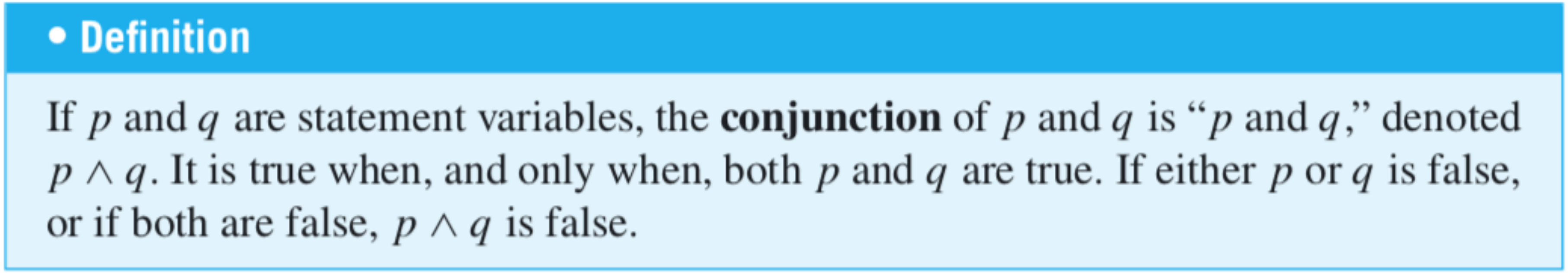
- **Definition**

If p is a statement variable, the **negation** of p is "not p" or "It is not the case that p" and is denoted $\sim p$. It has opposite truth value from p: if p is true, $\sim p$ is false; if p is false, $\sim p$ is true.

$$\left.\begin{array}{l}\sim p\\ \neg p\end{array}\right\}\text{not } p$$

p	$\sim p$
T	F
F	T

Conjunction → and (gate)

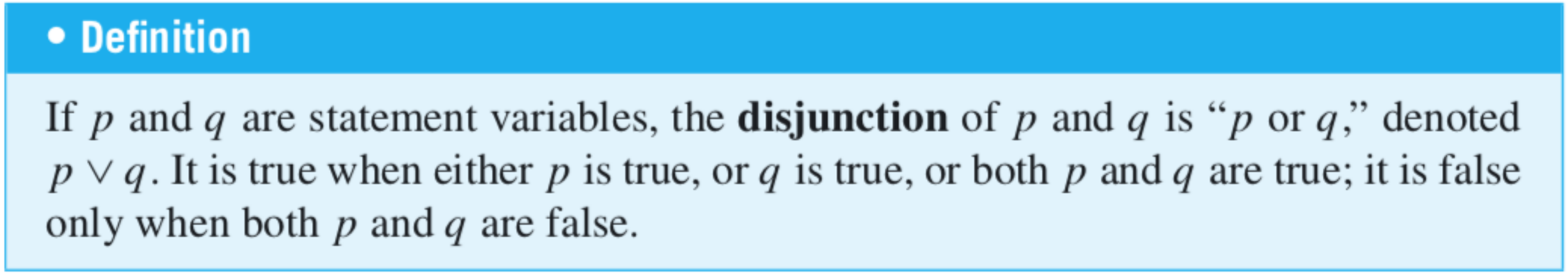


• Definition

If p and q are statement variables, the **conjunction** of p and q is "p and q," denoted $p \wedge q$. It is true when, and only when, both p and q are true. If either p or q is false, or if both are false, $p \wedge q$ is false.

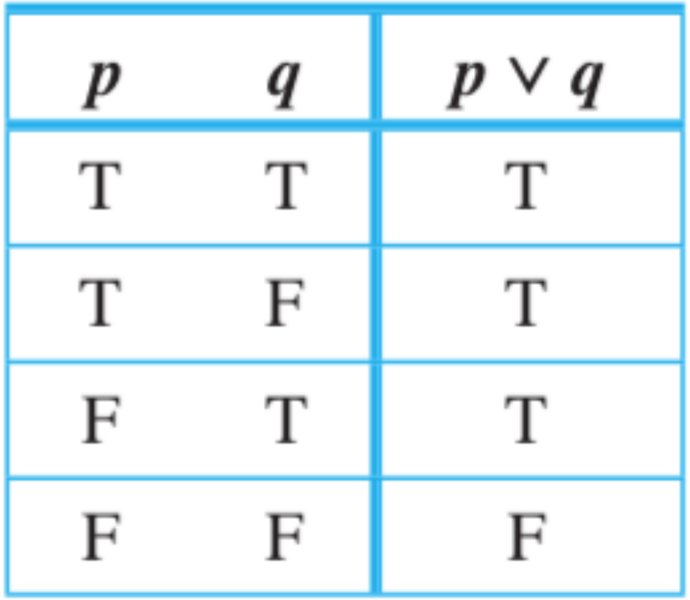
p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction → or



• Definition

If p and q are statement variables, the **disjunction** of p and q is "p or q," denoted $p \vee q$. It is true when either p is true, or q is true, or both p and q are true; it is false only when both p and q are false.

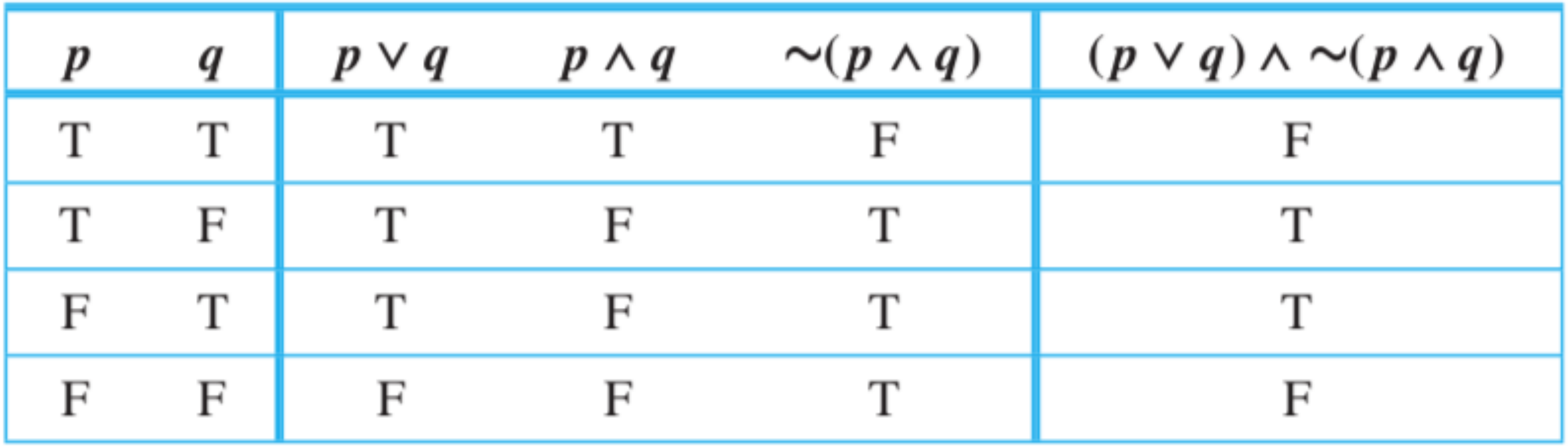


p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Truth Table for Exclusive Or

$$p \oplus q \quad \text{or} \quad p \text{ XOR } q$$

$$(p \vee q) \wedge \sim(p \wedge q)$$



p	q	$p \vee q$	$p \wedge q$	$\sim(p \wedge q)$	$(p \vee q) \wedge \sim(p \wedge q)$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

Logical Equivalence

- **Definition**

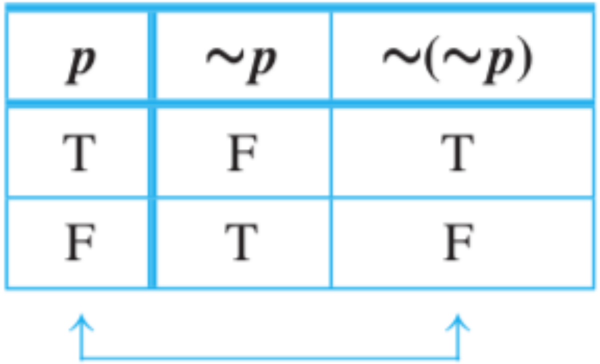
Two *statement forms* are called **logically equivalent** if, and only if, they have identical truth values for each possible substitution of statements for their statement variables. The logical equivalence of statement forms P and Q is denoted by writing $P \equiv Q$.

Two *statements* are called **logically equivalent** if, and only if, they have logically equivalent forms when identical component statement variables are used to replace identical component statements.

p	q	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

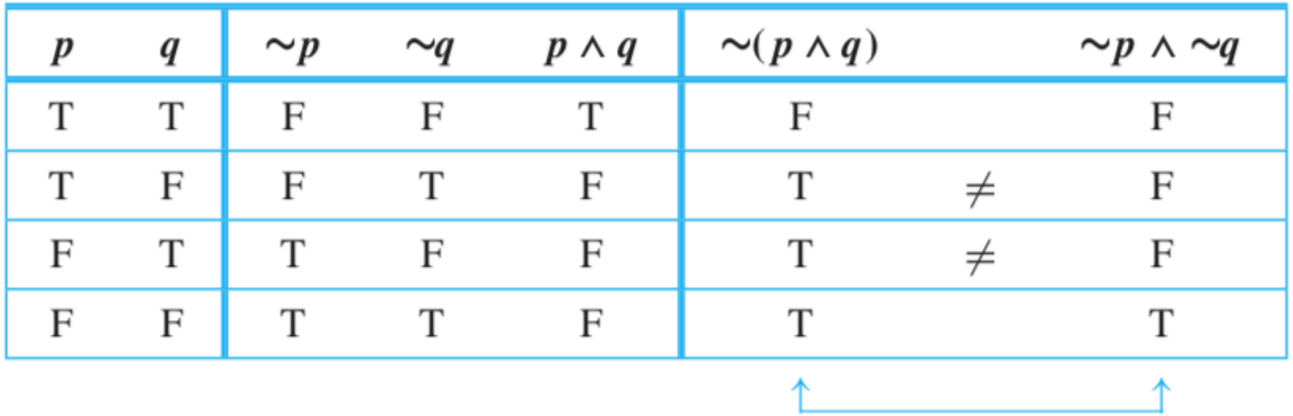
$p \wedge q$ and $q \wedge p$ always have the same truth values, so they are logically equivalent

Logical Equivalence



p	$\sim p$	$\sim(\sim p)$
T	F	T
F	T	F

p and $\sim(\sim p)$ always have the same truth values, so they are logically equivalent



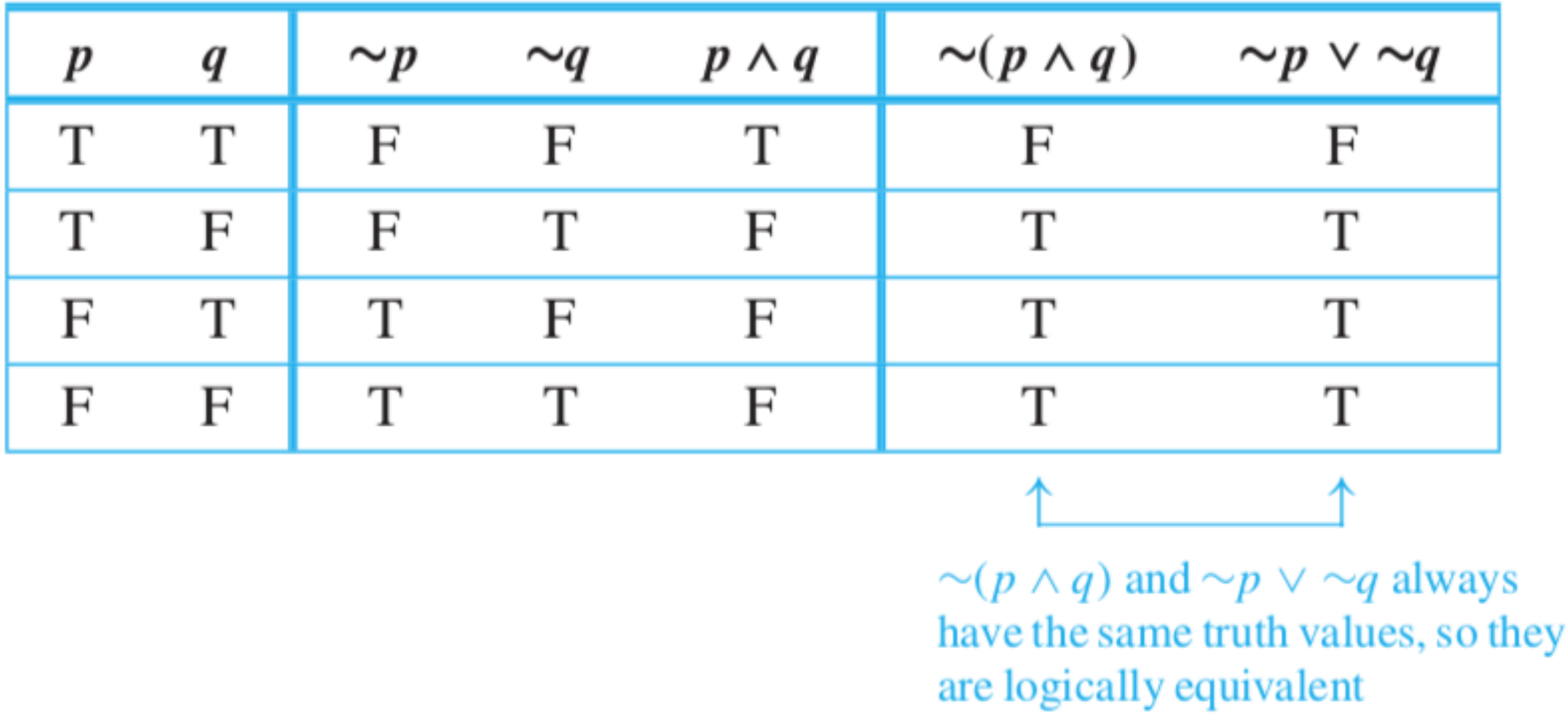
p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$		$\sim p \wedge \sim q$
T	T	F	F	T	F		F
T	F	F	T	F	T	$\neq$	F
F	T	T	F	F	T	$\neq$	F
F	F	T	T	F	T		T

$\sim(p \wedge q)$ and $\sim p \wedge \sim q$ have different truth values in rows 2 and 3, so they are not logically equivalent

Negations of And and Or: De Morgan's Laws

$$\sim(p \land q) \equiv \sim p \lor \sim q.$$

$$\sim(p \lor q) \equiv \sim p \land \sim q.$$



p	q	$\sim p$	$\sim q$	$p \land q$	$\sim(p \land q)$	$\sim p \lor \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

$\sim(p \land q)$ and $\sim p \lor \sim q$ always have the same truth values, so they are logically equivalent

Tautologies and Contradictions

- **Definition**

A **tautology** is a statement form that is always true regardless of the truth values of the individual statements substituted for its statement variables. A statement whose form is a tautology is a **tautological statement.**

A **contradication** is a statement form that is always false regardless of the truth values of the individual statements substituted for its statement variables. A statement whose form is a contradication is a **contradictory statement.**

p	$\mathbf{t}$	$p \wedge \mathbf{t}$	p	$\mathbf{c}$	$p \wedge \mathbf{c}$
T	T	T	T	F	F
F	T	F	F	F	F

same truth values, so $p \wedge \mathbf{t} \equiv p$

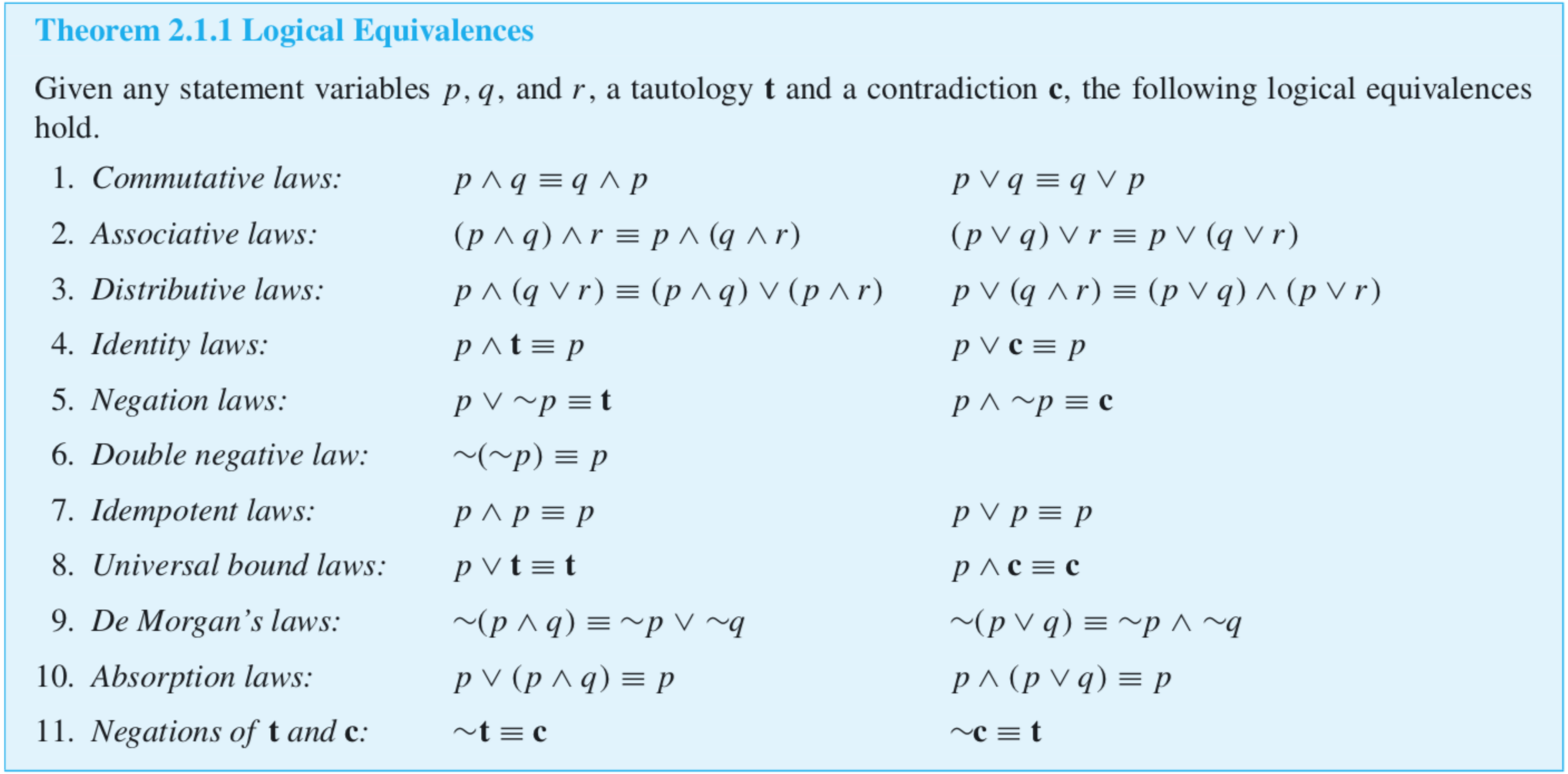
same truth values, so $p \wedge \mathbf{c} \equiv \mathbf{c}$

p	$\sim p$	$p \vee \sim p$	$p \wedge \sim p$
T	F	T	F
F	T	T	F

all T's so $p \vee \sim p$ is a tautology

all F's so $p \wedge \sim p$ is a contradiction

Summary of Logical Equivalences



Theorem 2.1.1 Logical Equivalences

Given any statement variables p, q, and r, a tautology $\mathbf{t}$ and a contradiction $\mathbf{c}$, the following logical equivalences hold.

1. *Commutative laws:*	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2. *Associative laws:*	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. *Distributive laws:*	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. *Identity laws:*	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
5. *Negation laws:*	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
6. *Double negative law:*	$\sim(\sim p) \equiv p$	
7. *Idempotent laws:*	$p \wedge p \equiv p$	$p \vee p \equiv p$
8. *Universal bound laws:*	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
9. *De Morgan's laws:*	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$
10. *Absorption laws:*	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11. *Negations of* **t** *and* **c**:	$\sim \mathbf{t} \equiv \mathbf{c}$	$\sim \mathbf{c} \equiv \mathbf{t}$

Simplifying Statement Forms

using 1) Truth Table.
2) Theorem of logical Equivalences.

$$\sim(\sim p \land q) \land (p \lor q) \equiv p$$

$$\begin{aligned}
\sim(\sim p \land q) \land (p \lor q) &\equiv (\sim(\sim p) \lor \sim q) \land (p \lor q) && \text{by De Morgan's laws} \\
&\equiv (p \lor \sim q) \land (p \lor q) && \text{by the double negative law} \\
&\equiv p \lor (\sim q \land q) && \text{by the distributive law} \\
&\equiv p \lor (q \land \sim q) && \text{by the commutative law for } \land \\
&\equiv p \lor \mathbf{c} && \text{by the negation law} \\
&\equiv p && \text{by the identity law.}
\end{aligned}$$

Propositional Logic

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If-Then Statements

$$\underbrace{\text{If 9 is divisible by 6,}}_{\text{hypothesis}}\ \text{then}\ \underbrace{\text{9 is divisible by 3}}_{\text{conclusion}}$$

$$\underbrace{\text{If you study,}}_{\text{hypothesis}}\ \text{then}\ \underbrace{\text{you pass}}_{\text{conclusion}}$$

If-Then Statements

Remark that this if-then is a logical (not a causal) condition

$p \to q \equiv \sim p \vee q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Your mum said {you study → you pass}, is it True?

you studied and you passed

you studied and you didn’t passed

you didn’t study and you passed

you didn’t study and you didn’t pass

If p and q are statement variables, the **conditional** of q by p is “If p then q” or “p implies q” and is denoted $p \to q$. It is false when p is true and q is false; otherwise it is true. We call p the **hypothesis** (or **antecedent**) of the conditional and q the **conclusion** (or **consequent**).

Conditional Statement with a False Hypothesis

If 0 = 1 then 1 = 2.

The statement as whole is true.

Notice that we don’t test the correctness of the conclusion

Truth Tables involving →

Construct a truth table for the statement form $p \vee \sim q \rightarrow \sim p$.

$\equiv \sim(p \vee \sim q) \vee \sim p$

$\equiv (\sim p \wedge q) \vee \sim p$

$\equiv \sim p \vee (\sim p \wedge q)$

		conclusion		hypothesis	
p	q	$\sim p$	$\sim q$	$p \vee \sim q$	$p \vee \sim q \rightarrow \sim p$
T	T	F	F	T	F
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

Logical Equivalences involving ⟶

Division into Cases: Showing that $p \vee q \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

عدد الاحتمالات = 2^n

n = عدد المتغيرات

	p	q	r	$p \vee q$	$p \rightarrow r$	$q \rightarrow r$	$p \vee q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$
1	T	T	T	T	T	T	T	T
2	T	T	F	T	F	F	F	F
3	T	F	T	T	T	T	T	T
4	T	F	F	T	F	T	F	F
5	F	T	T	T	T	T	T	T
6	F	T	F	T	T	F	F	F
7	F	F	T	F	T	T	T	T
8	F	F	F	F	T	T	T	T

$p \vee q \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$
always have the same truth values,
so they are logically equivalent

Representation of If-Then As Or

$$p \rightarrow q \equiv \sim p \vee q$$

Examples?

p	q	$\sim p$	$p \rightarrow q$	$\sim p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$\equiv$

The Negation of a Conditional Statement

The negation of "if *p* then *q*" is logically equivalent to "*p* and not *q*."

$$\sim(p \rightarrow q) \equiv p \wedge \sim q$$

$\sim(p \rightarrow q) \equiv \sim(\sim p \vee q)$

$\equiv \sim(\sim p) \wedge (\sim q)$ by De Morgan's laws

$\equiv p \wedge \sim q$ by the double negative law

The Negation of a Conditional Statement

Examples

If my lecture is at Masri109, then I cannot buy coffee

my lecture is at Masri109 and I can buy coffee

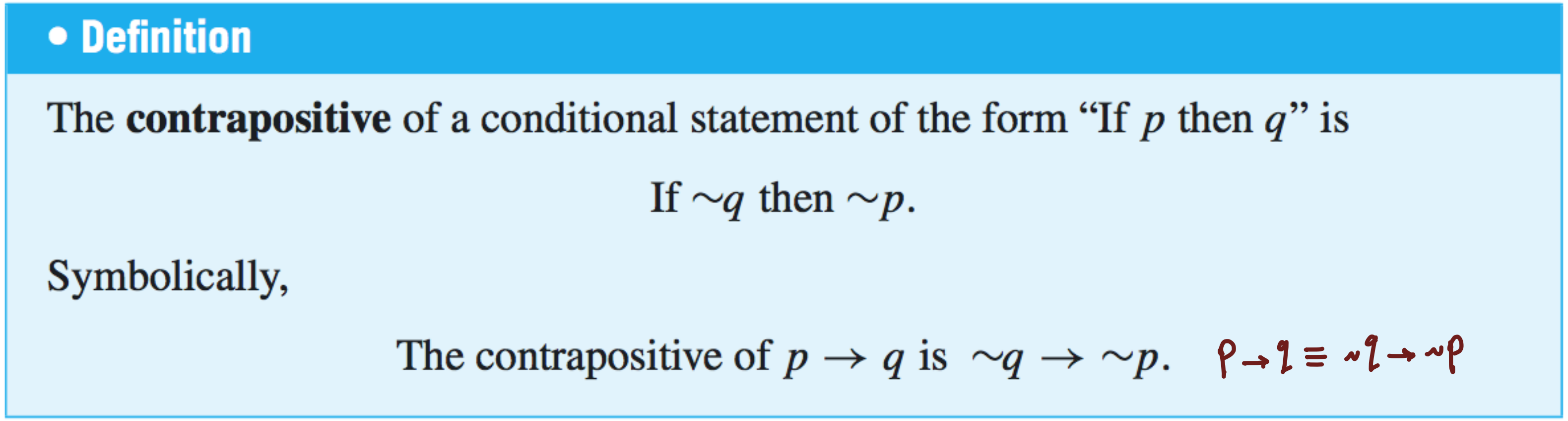
If Amjad loves Zatar, then Amjad is smart

Amjad loves Zatar and Amjad is not smart

Contrapositive Statements

Conditional statement = its contrapositive.

If you don't pass then you didn't study



- **Definition**

The **contrapositive** of a conditional statement of the form "If p then q" is

$$\text{If } \sim q \text{ then } \sim p.$$

Symbolically,

The contrapositive of $p \to q$ is $\sim q \to \sim p$. $p \to q \equiv \sim q \to \sim p$

if study then pass

if not pass then not study

Try the truth table

Contrapositive Statements

Examples

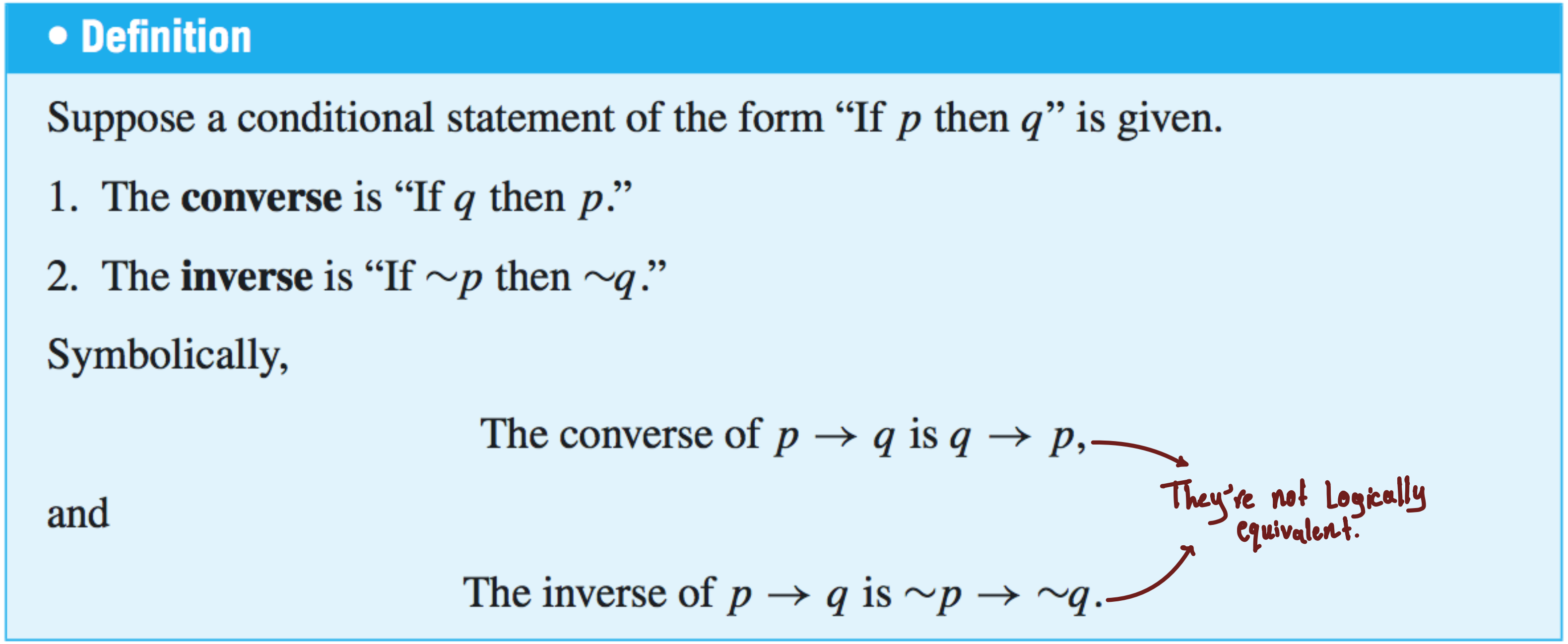
If my lecture is at Masri109, then I cannot buy coffee

If I can buy coffee then my lecture is not at Masri109

If Amjad loves Zatar, then Amjad is smart

If Amjad is not smart then Amjad does not love Zatar

Converse and Inverse



• Definition

Suppose a conditional statement of the form “If p then q” is given.

1. The **converse** is “If q then p.”
2. The **inverse** is “If $\sim p$ then $\sim q$.”

Symbolically,

$$\text{The converse of } p \to q \text{ is } q \to p,$$

and

$$\text{The inverse of } p \to q \text{ is } \sim p \to \sim q.$$

They're not Logically equivalent.

Converse مقلوب
Inverse معكوس

Converse and Inverse

Examples

If my lecture is at Masri109, then I cannot buy coffee

Converse: If I cannot buy coffee then my lecture is at Masri109

Inverse: If my lecture is not at Masri109 then I can buy coffee

If Amjad loves Zatar, then Amjad is smart

Converse: If Amjad is smart then Amjad loves Zatar

Inverse: If Amjad does not love Zatar, then Amjad is not smart

Converse and Inverse

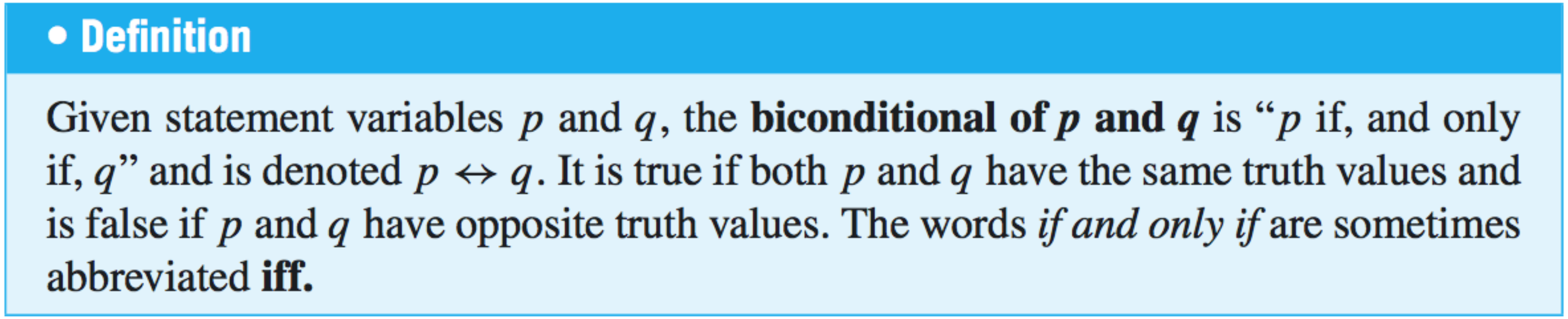
Caution! Many people believe that if a conditional statement is true, then its converse and inverse must also be true. This is not correct!

1. A conditional statement and its converse are *not* logically equivalent.
2. A conditional statement and its inverse are *not* logically equivalent.
3. The converse and the inverse of a conditional statement are logically equivalent to each other. $q \to p \equiv \sim p \to \sim q$

Biconditional

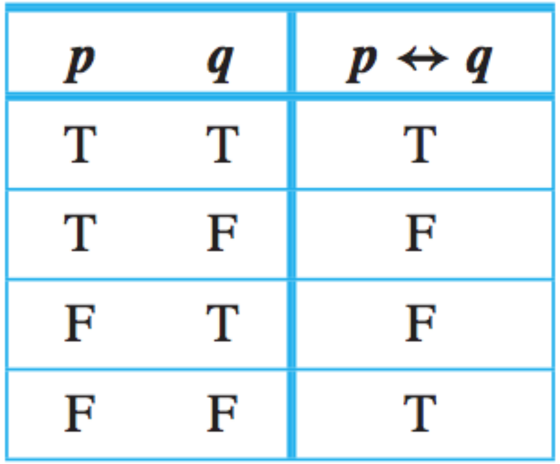
If and only if

iff



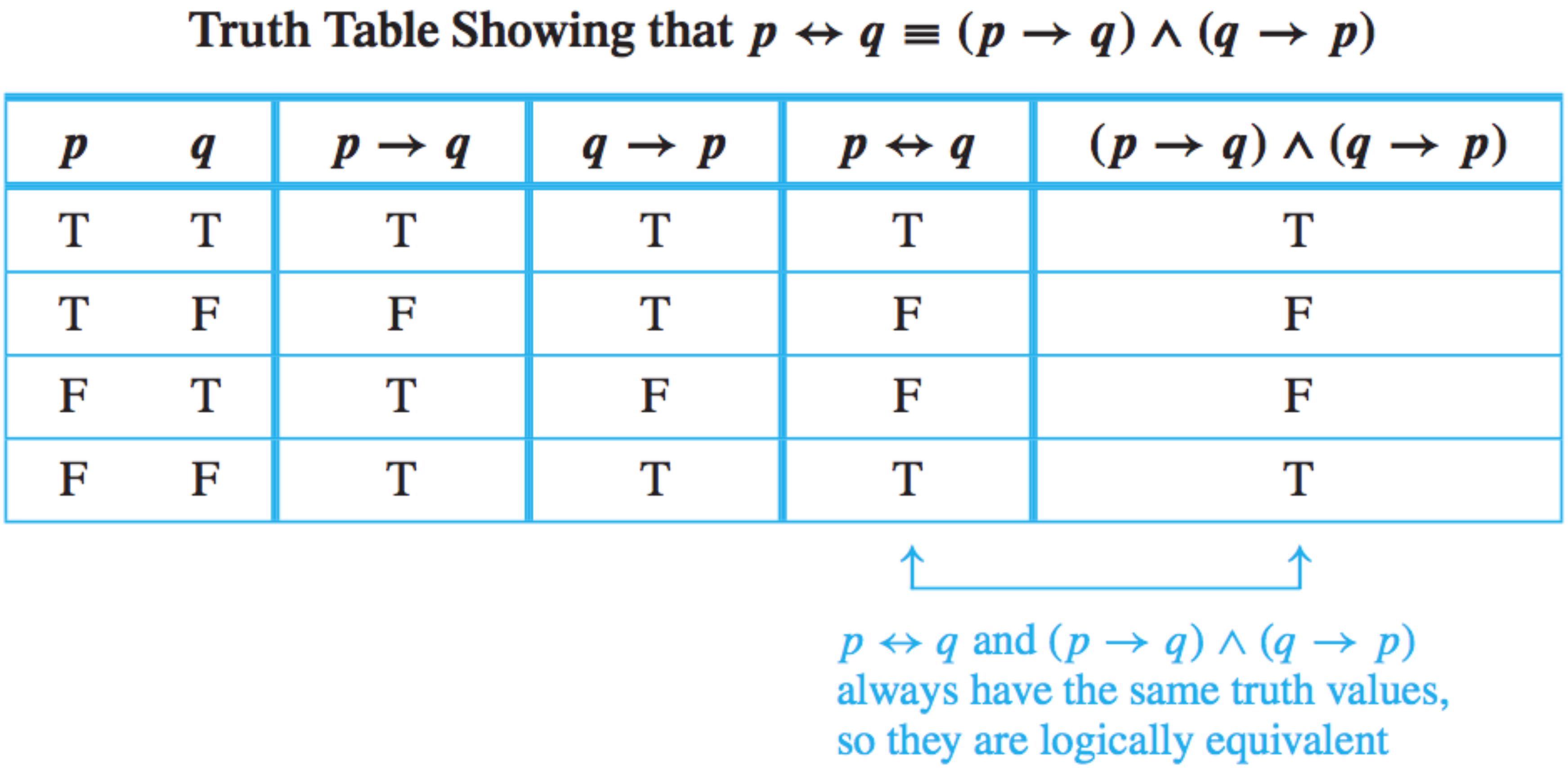
• Definition

Given statement variables p and q, the **biconditional of p and q** is "p if, and only if, q" and is denoted $p \leftrightarrow q$. It is true if both p and q have the same truth values and is false if p and q have opposite truth values. The words *if and only if* are sometimes abbreviated **iff.**



p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Biconditional



Truth Table Showing that $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

p	q	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$	$(p \rightarrow q) \wedge (q \rightarrow p)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

$p \leftrightarrow q$ and $(p \rightarrow q) \wedge (q \rightarrow p)$
always have the same truth values,
so they are logically equivalent

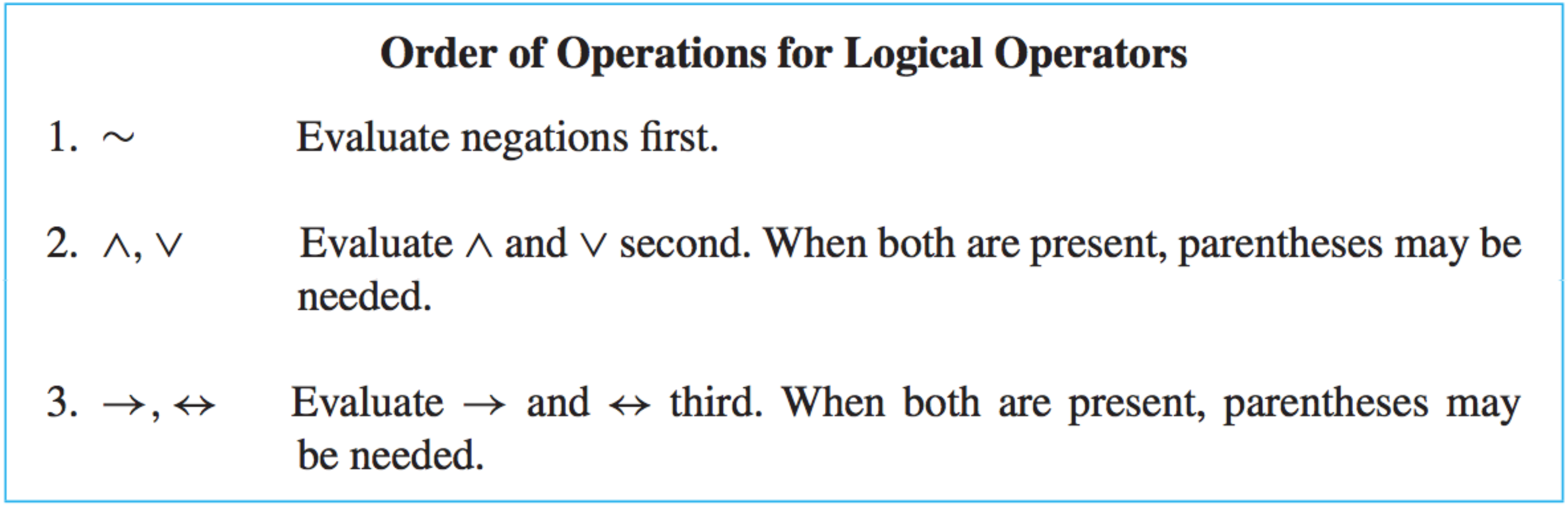
Biconditional

Examples

This computer program is correct if, and only if,
it produces correct answers for all possible sets of input data.

If this program produces the correct answers for all possible sets of input data, then it is correct.

If this program is correct, then it produces the correct answers for all possible sets of input data;



Order of Operations for Logical Operators

1.	~	Evaluate negations first.
2.	∧, ∨	Evaluate ∧ and ∨ second. When both are present, parentheses may be needed.
3.	→, ↔	Evaluate → and ↔ third. When both are present, parentheses may be needed.

Necessary and Sufficient Conditions

• Definition

If r and s are statements:

r is a **sufficient condition** for s means "if r then s."
r is a **necessary condition** for s means "if not r then not s."

r is a necessary condition for s also means "if s then r."

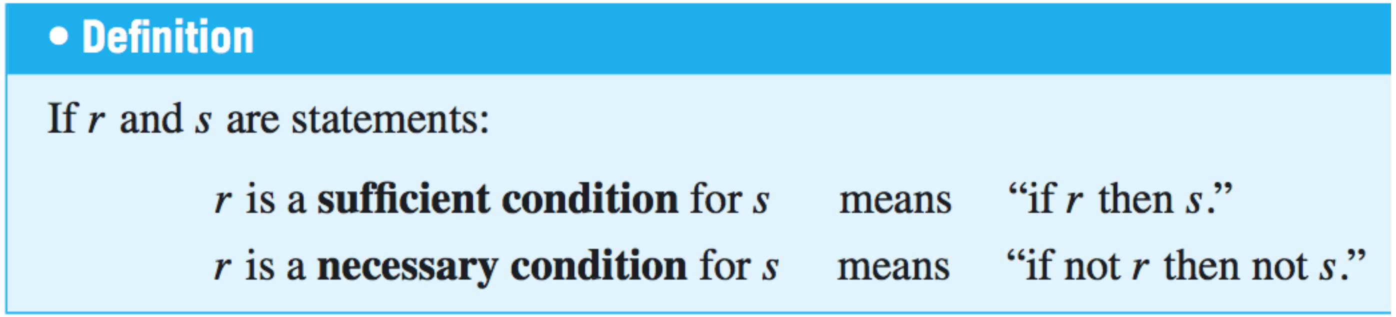
r is a necessary and sufficient condition for s means "r if, and only if, s."

Examples

	r
Studying is a sufficient condition for passing.	Study → Pass
In order to pass, it is sufficient to study.	Study → Pass
It is sufficient to study in order to pass.	Study → Pass
Studying is a necessary condition for passing.	~Study → ~Pass

,

Necessary and Sufficient Conditions

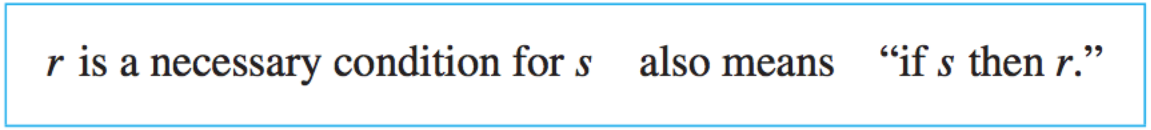


• Definition

If r and s are statements:

r is a **sufficient condition** for s means "if r then s."

r is a **necessary condition** for s means "if not r then not s."



r is a necessary condition for s also means "if s then r."

r is a necessary and sufficient condition for s means "r if, and only if, s."

Examples

Being above 16 is a sufficient condition for getting ID Card.

Above (16) → Get ID Card

Being above 16 is a necessary condition for getting ID Card

~Above (16) → ~Get ID Card

Necessary and Sufficient Conditions

Examples

A is a sufficient condition for B

→ if A then B

→ The occurrence of A guarantees the occurrence of B

A is a necessary condition for B

→ If ~A then ~B

→ If A did not occur, then B did not occur either

>> It also means If B then A (if B occurred then A had also occurred)

$\sim A \to \sim B \equiv B \to A$

Examples

Examples

- Passing all exams is a sufficient condition for passing the course
  - If a person passes all exams, then the person will pass the course
- Passing all exams is a necessary condition for passing the course
  - If a person does not pass all exams, then the person will not pass the course
  - OR if a person passes the course, then the person will have passed all exams

Propositional Logic

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Propositional Logic

2.3 Logical Inferencing

In this lecture:

- ☐ Part 1: **Numeration Method**
- ☐ Part 2: **Rules of Inference**
- ☐ Part 3: **Examples**

Numeration Method

Example 1

Premises

If today is Friday then today is holiday

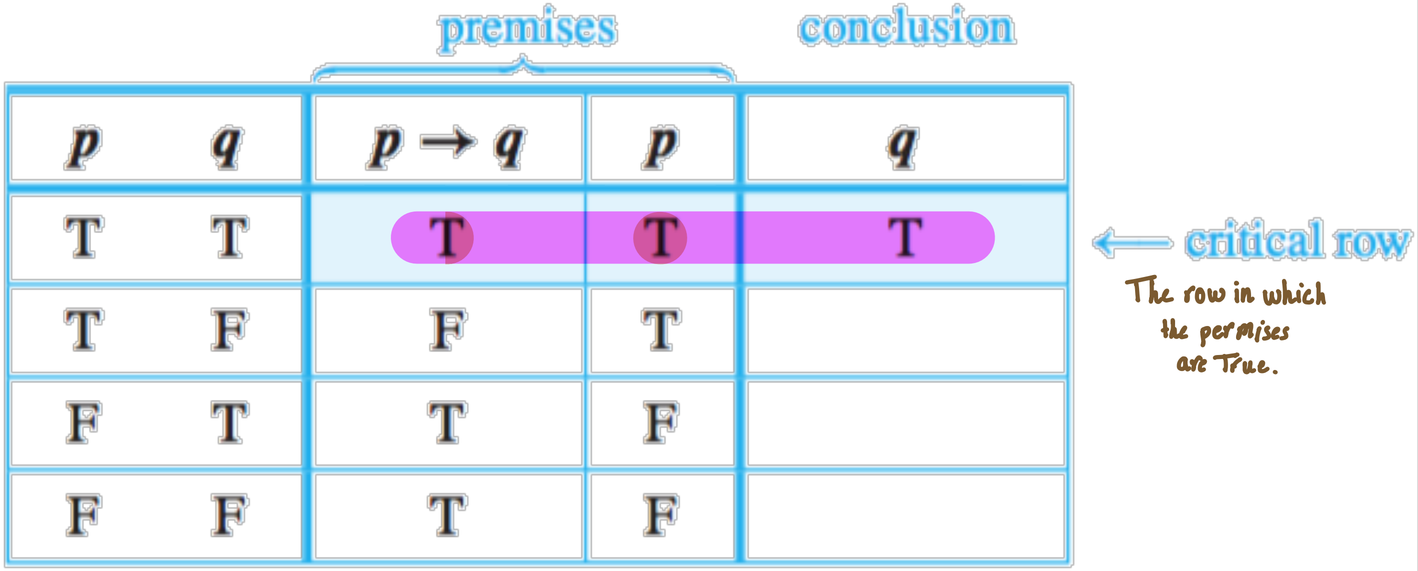
Today is Friday

conclusion → ∴today is holiday?

$p \rightarrow q$

p

$\therefore q?$



		premises		conclusion
p	q	$p \rightarrow q$	p	q
T	T	T	T	T
T	F	F	T	
F	T	T	F	
F	F	T	F	

← critical row

The row in which the permises are True.

Numeration Method

Example 1

If Socrates is a man, then Socrates is mortal.

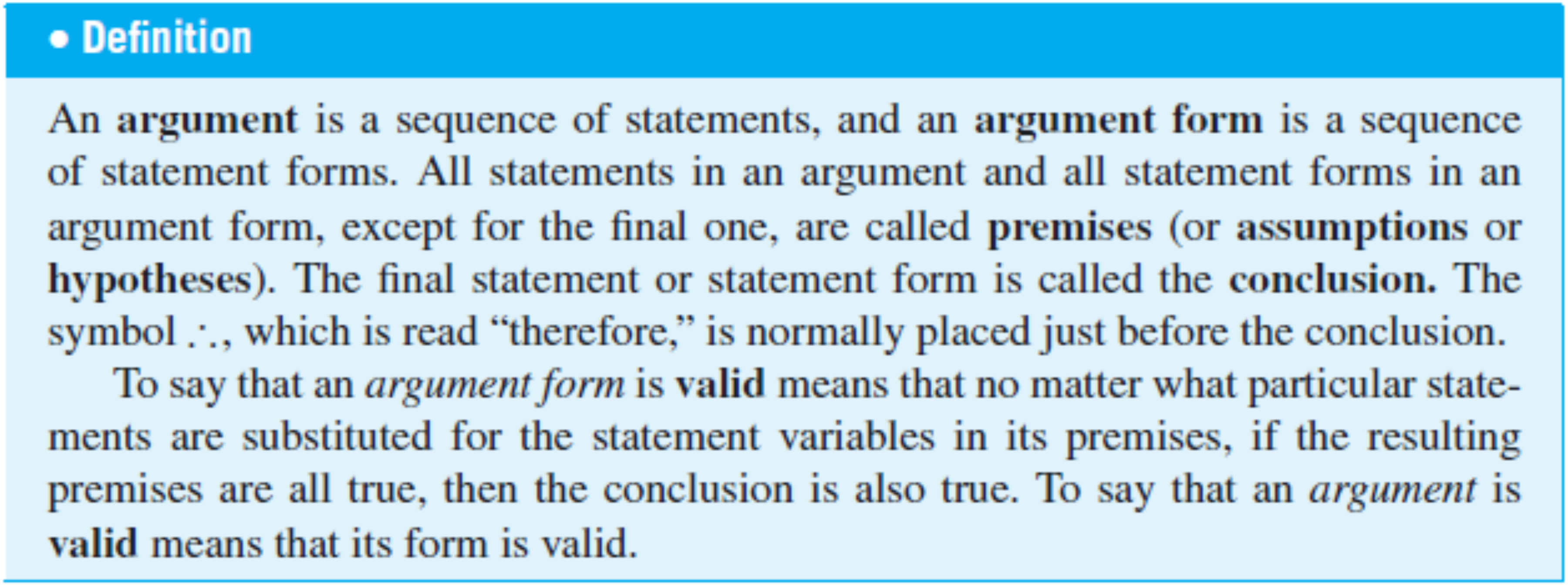
Socrates is a man.

∴ Socrates is mortal

$$p \rightarrow q$$

$$p$$

$$\therefore q?$$

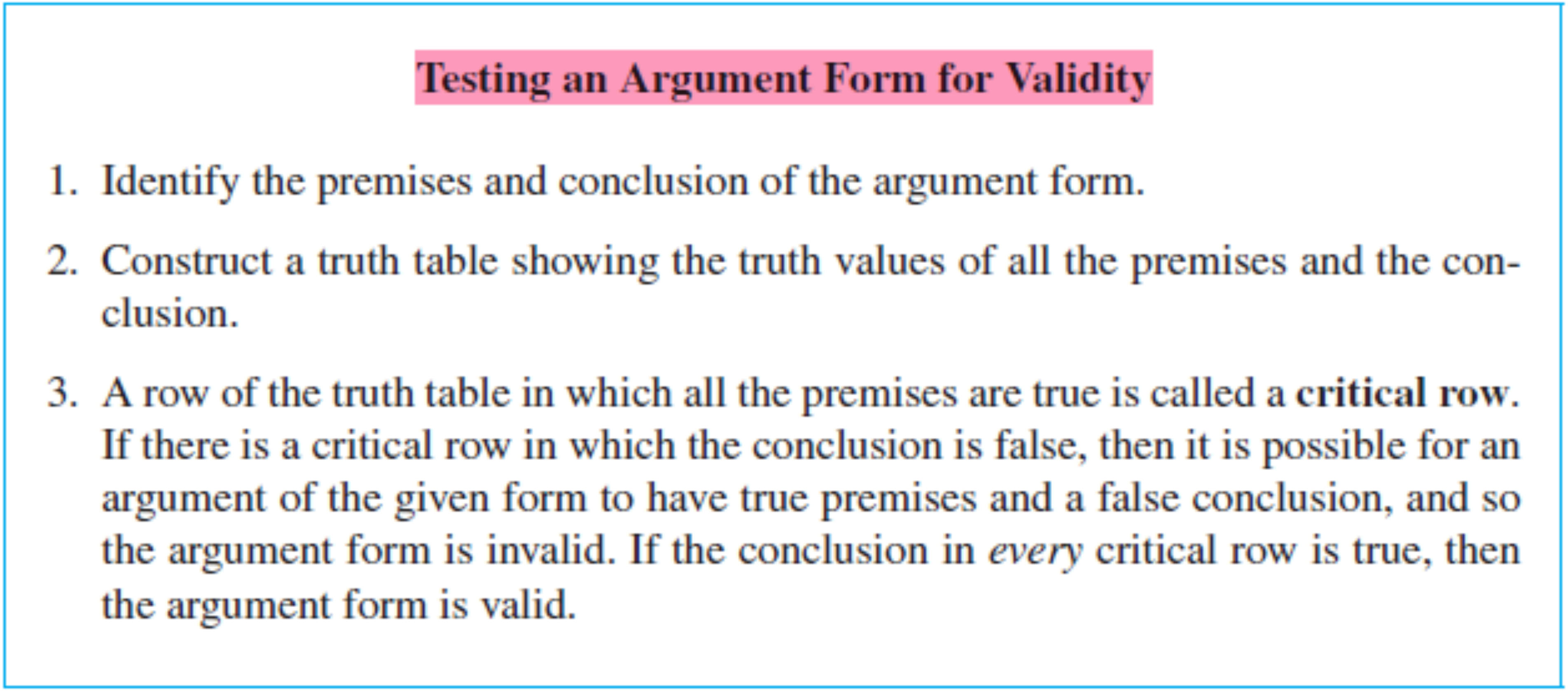


- **Definition**

An **argument** is a sequence of statements, and an **argument form** is a sequence of statement forms. All statements in an argument and all statement forms in an argument form, except for the final one, are called **premises** (or **assumptions** or **hypotheses**). The final statement or statement form is called the **conclusion.** The symbol ∴, which is read "therefore," is normally placed just before the conclusion.

To say that an *argument form* is **valid** means that no matter what particular statements are substituted for the statement variables in its premises, if the resulting premises are all true, then the conclusion is also true. To say that an *argument* is **valid** means that its form is valid.

Testing for validity



Testing an Argument Form for Validity

1. Identify the premises and conclusion of the argument form.
2. Construct a truth table showing the truth values of all the premises and the conclusion.
3. A row of the truth table in which all the premises are true is called a **critical row**. If there is a critical row in which the conclusion is false, then it is possible for an argument of the given form to have true premises and a false conclusion, and so the argument form is invalid. If the conclusion in *every* critical row is true, then the argument form is valid.

Numeration Method

Example 2

The argument form is invalid because the conclusion is false for one critical-row.

$p \to q \vee \sim r$

$q \to p \wedge r$

$\therefore p \to r$?

						premises		conclusion
p	q	r	$\sim r$	$q \vee \sim r$	$p \wedge r$	$p \to q \vee \sim r$	$q \to p \wedge r$	$p \to r$
T	T	T	F	T	T	T	T	T
T	T	F	T	T	F	T	F	
T	F	T	F	F	T	F	T	
T	F	F	T	T	F	T	T	F
F	T	T	F	T	F	T	F	
F	T	F	T	T	F	T	F	
F	F	T	F	F	F	T	T	T
F	F	F	T	T	F	T	T	T

Building a Valid Argument

- A valid argument is a sequence of statements where each statement is either a premise or follows from previous statements (called premises) by rules of inference. The last statement is called conclusion.

- A valid argument takes the following form:

Premise 1
Premise 2
.
.
.
Premise n
____
∴ Conclusion

Propositional Logic

2.3 Logical Inferencing

In this lecture:

- ☐ Part 1: **Numeration Method**
- ☐ Part 2: **Rules of Inference**
- ☐ Part 3: **Examples**

Rules of Inference

An argument form consisting of two premises and a conclusion is called a syllogism

- The first and second premises are called the major premise and minor premise, respectively

- A rule of inference is a form of argument that is valid

Rules of Inference

1. Modus Ponens

syllogism
valid argument

Corresponding Tautology:
$((p \rightarrow q) \wedge p) \rightarrow q$

$$p \longrightarrow q$$

$$p$$

$$\therefore \boldsymbol{q}$$

If today is Friday then today is holiday

Today is Friday

∴ Today is holiday

Modus Ponens = method of affirming (the conclusion is an affirmation)

Rules of Inference

1. Modus Ponens

Example:

Let p be “It is snowing.”
Let q be “I will study discrete math.”
“If it is snowing, then I will study discrete math.”
“It is snowing.”

“Therefore , I will study discrete math.”

Rules of Inference

2. Modus Tollens:

Corresponding Tautology:
$((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$

$p \longrightarrow q$

$\sim q$

$\therefore \sim p$

If today is Friday then today is holiday

Today is not holiday

∴ Today is not Friday

Modus Tollens = method of denying (the conclusion is a denial)

Rules of Inference

2. Modus Tollens

Example:

Let p be “it is snowing.”

Let q be “I will study discrete math.”

“If it is snowing, then I will study discrete math.” “I will not study discrete math.”

“Therefore , it is not snowing.”

Rules of Inference

3. Generalization:

$$\begin{array}{c} p \\ \hline \therefore p \vee q \end{array}$$

Today is Saturday

∴ Today is Saturday or today is Sunday

$$\begin{array}{c} q \\ \hline \therefore p \vee q \end{array}$$

According to the first, if p is true, then, more generally, “p or q” is true for any other statement q

Rules of Inference

4. Specialization: aka Conjunction Elimination

$p \land q$

$\therefore \boldsymbol{p}$

Today is Friday and today is holiday

∴ **Today is Friday**

$p \land q$

$\therefore \boldsymbol{q}$

∴ **Today is Holiday**

Rules of Inference

5. Conjunction:

$$p$$

$$q$$

$$\therefore p \wedge q$$

Today is Friday

Today is Holiday

∴ Today is Friday and today is holiday

Rules of Inference

6. Elimination: also known as Disjunctive Elimination

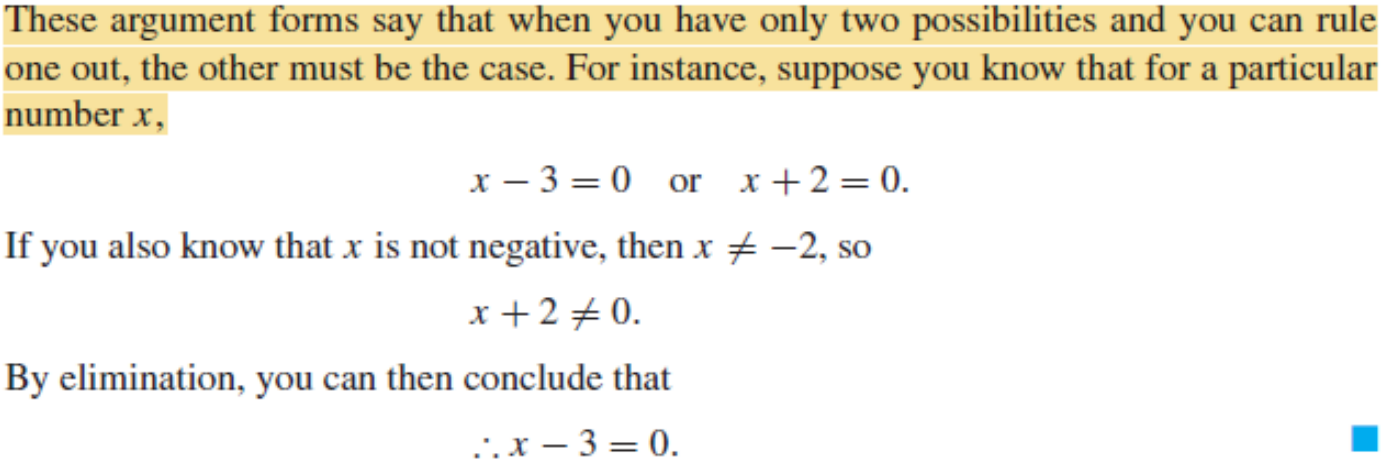
$$p \vee q$$
$$\sim q$$
$$\therefore \boldsymbol{p}$$

Today is Saturday or today is Sunday

Today is not Saturday

∴ **Today is Sunday**

$$p \vee q$$
$$\sim p$$
$$\therefore \boldsymbol{q}$$



These argument forms say that when you have only two possibilities and you can rule one out, the other must be the case. For instance, suppose you know that for a particular number x,

$$x - 3 = 0 \quad \text{or} \quad x + 2 = 0.$$

If you also know that x is not negative, then $x \neq -2$, so

$$x + 2 \neq 0.$$

By elimination, you can then conclude that

$$\therefore x - 3 = 0.$$

Rules of Inference

6. Elimination: also known as Disjunctive Elimination

$$p \vee q$$

$$\sim q$$

$$\therefore \boldsymbol{p}$$

$$p \vee q$$

$$\sim p$$

$$\therefore \boldsymbol{q}$$

Example:

Let p be “I will study discrete math.” Let q be “I will study English literature.”
“I will study discrete math or I will study English literature.” “I will not study discrete math.”
“Therefore , I will study English literature.”

Rules of Inference

7. Transitivity: also known as Chain Argument

$$p \rightarrow q$$

$$q \rightarrow r$$

$$\therefore\ p \rightarrow r$$

If today is Friday then Today is holiday

If today is holiday then I am happy

∴ If today is Friday then I am happy

Rules of Inference

7. Transitivity: also known as Chain Argument

$$p \rightarrow q$$

$$q \rightarrow r$$

$$\therefore p \rightarrow r$$

Example:

Let p be “it snows.”

Let q be “I will study discrete math.” Let r be “I will get an A.”

“If it snows, then I will study discrete math.”

“If I study discrete math, I will get an A.”

“Therefore , If it snows, I will get an A.”

Rules of Inference

8. Division into Cases:

$p \vee q$

$p \longrightarrow r$

$q \longrightarrow r$

$\therefore r$

Today is Friday or today is Sunday

If today is Friday then I am happy

If today is Sunday then I am happy

∴ **I am happy**

Rules of Inference

8. Division into Cases:

$$p \lor q$$
$$p \rightarrow r$$
$$q \rightarrow r$$
$$\therefore r$$

Example:

Let p be “I will study discrete math.”
Let q be “I will study Computer Science.”
Let r be “I will study databases.”

“If I will study discrete math, then I will study Computer Science.”
“If I will study databases, then I will study Computer Science.”
“I will study discrete math or I will study databases.”

“Therefore, I will study Computer Science.”

Rules of Inference

9. Contradiction Rule:

$$\sim p \rightarrow c$$

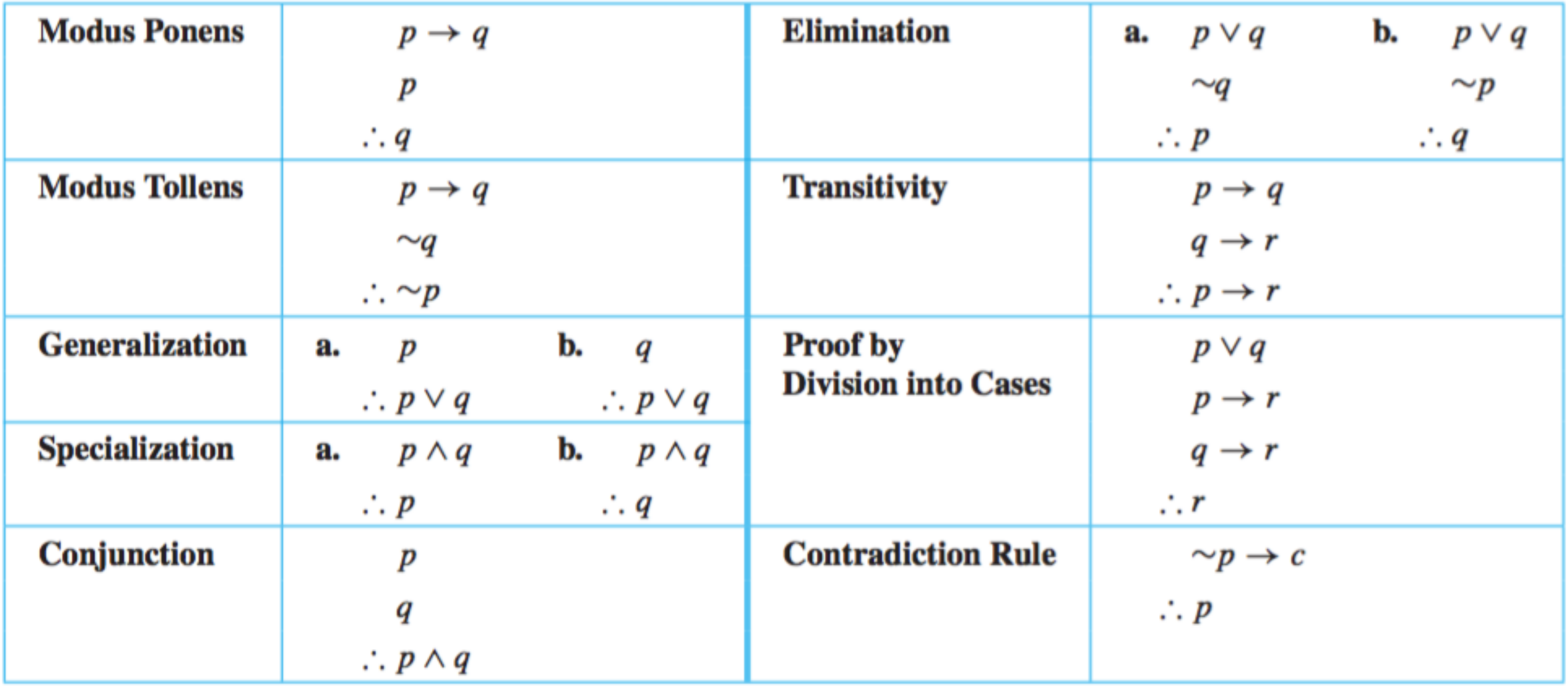
$$\therefore \boldsymbol{p}$$

If "Today is not Friday" is false

∴ **Today is Friday**

Rules of Inference

Summary



Modus Ponens	$p \to q$ p $\therefore q$	**Elimination**	**a.** $p \lor q$ $\sim q$ $\therefore p$ **b.** $p \lor q$ $\sim p$ $\therefore q$
Modus Tollens	$p \to q$ $\sim q$ $\therefore \sim p$	**Transitivity**	$p \to q$ $q \to r$ $\therefore p \to r$
Generalization	**a.** p $\therefore p \lor q$ **b.** q $\therefore p \lor q$	**Proof by Division into Cases**	$p \lor q$ $p \to r$ $q \to r$ $\therefore r$
Specialization	**a.** $p \land q$ $\therefore p$ **b.** $p \land q$ $\therefore q$		
Conjunction	p q $\therefore p \land q$	**Contradiction Rule**	$\sim p \to c$ $\therefore p$

Propositional Logic

2.3 Logical Inferencing

In this lecture:

- ☐ Part 1: **Numeration Method**
- ☐ Part 2: **Rules of Inference**
- ☐ Part 3: **Examples**

Inferencing Example

Formalize the following text in propositional logic and use the inference rules find the glasses.

If I was reading the newspaper in the kitchen, then my glasses are on the kitchen table. $RK \rightarrow GK$

If my glasses are on the kitchen table, then I saw them at breakfast. $GK \rightarrow SB$

I did not see my glasses at breakfast. $\sim SB$

I was reading the newspaper in the living room or I was reading the newspaper in the kitchen. $RL \lor RK$

If I was reading the newspaper in the living room then my glasses are on the coffee table. $RL \rightarrow GC$

Where are the glasses?

Inferencing Example

Let

RK = I was reading the newspaper in the kitchen.

GK = My glasses are on the kitchen table.

SB = I saw my glasses at breakfast.

RL = I was reading the newspaper in the living room.

GC = My glasses are on the coffee table.

$RK \rightarrow GK$
$GK \rightarrow SB$
$\sim SB$
$RL \vee RK$
$RL \rightarrow GC$

$RK \rightarrow GK$
$GK \rightarrow SB$
$\therefore RK \rightarrow SB$ by transitivity

$RL \vee RK$
$\sim RK$
$\therefore RL$ by elimination

$RK \rightarrow SB$
$\sim SB$
$\therefore \sim RK$ by modus tollens

$RL \rightarrow GC$
RL
$\therefore GC$ by modus ponenes

Thus the glasses are on the coffee table.