– Sec 7.7: Hyperbolic functions

الاقترانات الزائدة.

*** Definitions:-** تعاريف الاقترانات الزائدة

1. $\sinh x = \dfrac{e^x - e^{-x}}{2}$; $x \in \mathbb{R}$.

2. $\cosh x = \dfrac{e^x + e^{-x}}{2}$; $x \in \mathbb{R}$.

3. $\tanh x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$; $x \in \mathbb{R}$.

4. $\coth x = \dfrac{1}{\tanh x} = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$; $x \in \mathbb{R} \setminus \{0\}$.

5. $\operatorname{csch} x = \dfrac{1}{\sinh x} = \dfrac{2}{e^x - e^{-x}}$; $x \in \mathbb{R} \setminus \{0\}$.

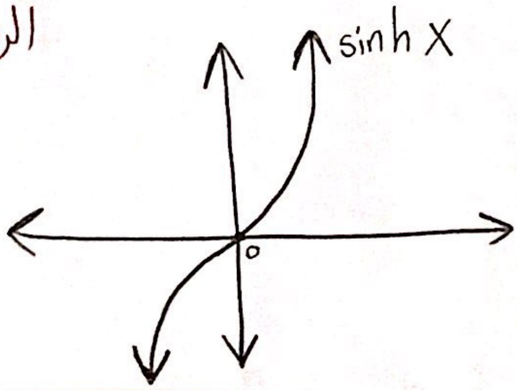
6. $\operatorname{sech} x = \dfrac{1}{\cosh x} = \dfrac{2}{e^x + e^{-x}}$; $x \in \mathbb{R}$.

– Notes: ملاحظات

1. $\sinh x$, $\tanh x$, $\coth x$ and $\operatorname{csch} x$ are odd functions.
2. $\cosh x$, and $\operatorname{sech} x$ are even functions.

– Graphs: الرسومات

1. $f(x) = \sinh x$



$\sinh 0 = 0$

$D: (-\infty, \infty) = \mathbb{R}$

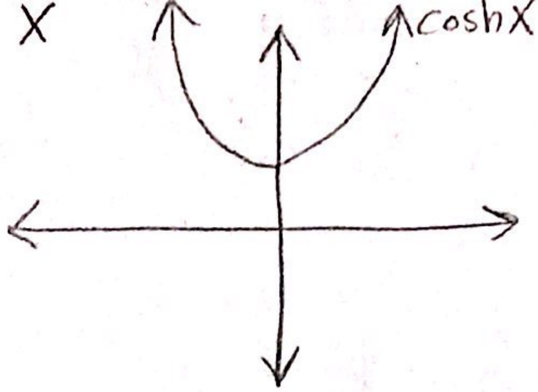
$R: (-\infty, \infty)$

$\lim_{x \to \infty} \sinh x = \infty$

$\lim_{x \to -\infty} \sinh x = -\infty$

odd $\sinh x = \sinh - x$

② $f(x) = \cosh x$



$\cosh 0 = 1$

D: $(-\infty, \infty)$

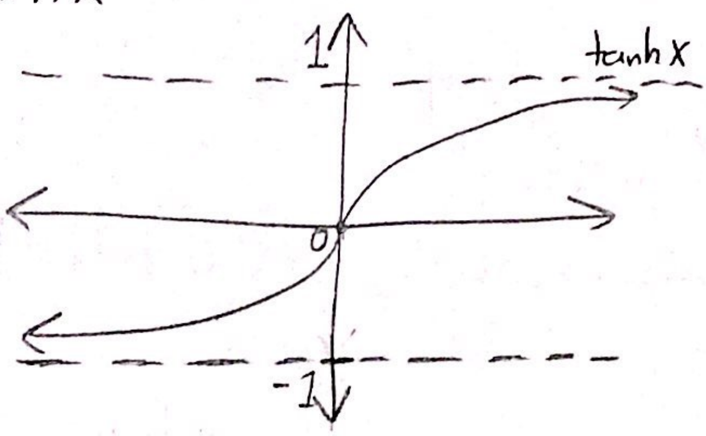
R: $[1, \infty)$

even: $\cosh(x) = \cosh(-x)$

↳ symmetric about the y-axis.

$\lim_{x \to \mp\infty} \cosh x = \infty$

③ $f(x) = \tanh x$



$\tanh 0 = 0$

D: $(-\infty, \infty)$

R: $(-1, 1)$

odd: $\tanh x = -\tanh(-x)$

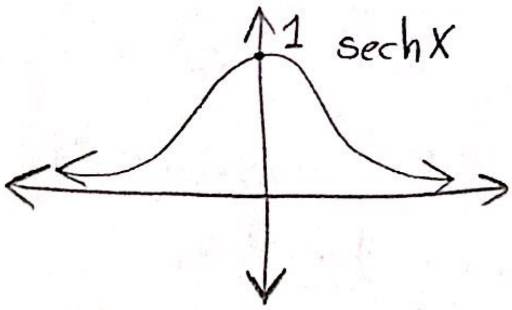
↳ symmetric about the origin.

$\lim_{x \to \infty} \tanh x = 1$

and $\lim_{x \to -\infty} \tanh x = -1$

$\therefore y = 1, -1$ are H. A.

④ $y = \operatorname{sech} x$



$\operatorname{sech} 0 = 1$

D: $(-\infty, \infty)$

R: $(0, 1]$

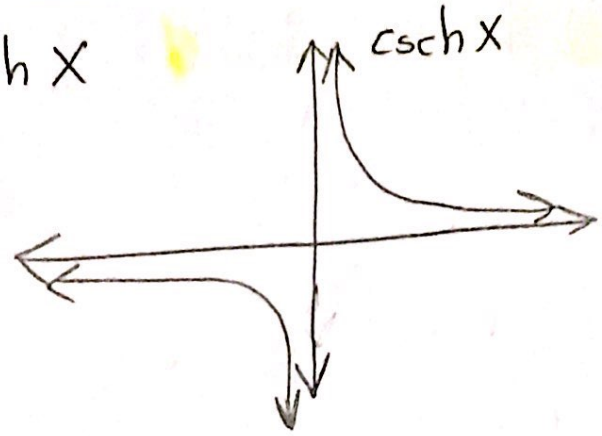
even: $\operatorname{sech} x = \operatorname{sech}(-x)$

↳ symmetric about the y-axis.

$\lim_{x \to \mp\infty} \operatorname{sech} x = 0$

$\therefore y = 0$ is a H. A.

⑤ $y = \operatorname{csch} x$



$\lim_{x\to 0^+} \operatorname{csch} x = \infty$

$\lim_{x\to 0^-} \operatorname{csch} x = -\infty$

$\therefore\ x = 0$ is a V.A

D: $\mathbb{R} \setminus \{0\}$.

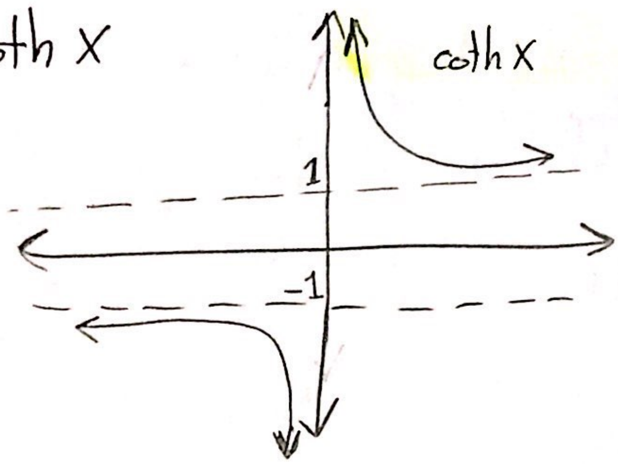
R: $\mathbb{R} \setminus \{0\}$

$\lim_{x\to \mp\infty} \operatorname{csch} x = 0$

$\therefore\ y = 0$ is a H.A

odd: $\operatorname{csch} x = \operatorname{csch}(-x)$
↳ symmetric about the origin.

⑥ $y = \coth x$



$\lim_{x\to 0^+} \coth x = \infty$

$\lim_{x\to 0^-} \coth x = -\infty$

$\therefore\ x = 0$ is a V.A

D: $(-\infty, \infty) \setminus \{0\}$

R: $(-\infty, -1) \cup (1, \infty) = \mathbb{R} \setminus [-1, 1]$

$\lim_{x\to \infty} \coth x = 1$ and

$\lim_{x\to -\infty} \coth x = -1$

$\therefore\ y = 1, -1$ are H.A

odd → $\coth x = \coth -x$
↳ symmetric about the origin.

* Identities for hyperbolic functions:

متطابقات

1. $\cosh^2 x - \sinh^2 x = 1$
2. $\sinh(2x) = 2 \sinh x \cosh x$
3. $\cosh(2x) = \cosh^2 x + \sinh^2 x$.
4. $\cosh^2 x = \frac{\cosh(2x) + 1}{2}$.
5. $\sinh^2 x = \frac{\cosh(2x) - 1}{2}$.
6. $\coth^2 x = 1 + \operatorname{csch}^2 x$
7. $1 - \tanh^2 x = \operatorname{sech}^2 x$.

* Derivatives of hyperbolic functions:

المشتقات

1. $\frac{d}{dx}(\sinh U) = \cosh U \cdot U'$
2. $\frac{d}{dx}(\cosh U) = \sinh U \cdot U'$
3. $\frac{d}{dx}(\tanh U) = \operatorname{sech}^2 U \cdot U'$
4. $\frac{d}{dx}(\coth U) = -\operatorname{csch}^2 U \cdot U'$
5. $\frac{d}{dx}(\operatorname{sech} U) = -\operatorname{sech} U \tanh U \cdot U'$
6. $\frac{d}{dx}(\operatorname{csch} U) = -\operatorname{csch} U \coth U \cdot U'$