BIRZEIT UNIVERSITY

MATHEMATICS DEPARTMENT
MATH331, Quiz 3

• Name....Key.... • Number.................. • Section.......1..........

Q_1. Solve the following IVP

$$(9x^2 + y - 1) - (4y - x)y' = 0, \quad y(1) = 0.$$

$$(9x^2 + y - 1)dx - (4y - x)dy = 0$$

$$\frac{\partial M}{\partial y} = 1 = \frac{\partial N}{\partial x} \Rightarrow \text{(Exact Eq.)} \quad 1$$

$$f(x,y) = \int M(x,y)dx = \int (9x^2 + y - 1)dx$$

$$= 3x^3 + yx - x + h(y) \quad 2$$

$$f_y(x,y) = x + h'(y) = N(x,y) = x - 4y$$

$$\Rightarrow h'(y) = -4y \Rightarrow h(y) = -2y^2 \quad 2$$

$$\therefore f(x,y) = 3x^2 + xy - x - 2y^2 = C$$

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Using $y(1) = 0$: $\Rightarrow 3 - 1 = C \Rightarrow \boxed{C = 2}$ 1

$$\therefore f(x,y) = 3x^2 + xy - x - 2y^2 = 2. \quad 1$$

Q_2. Solve the following D.E: $y'' + y' = e^{-t}$. , Let $v = y' \Rightarrow v' = y''$ (1)

$$\Rightarrow v' + v = e^{-t}$$

(1) $$\mu(t) = e^{\int 1 dt} = e^{t} \Rightarrow v(t) = e^{-t}\left[\int e^{t} \cdot e^{-t} dt + c\right]$$ (1)

$$\Rightarrow v(t) = t e^{-t} + \frac{c}{e^{t}}$$ (2)

Now : $y' = te^{-t} + ce^{-t}$

$$\Rightarrow y(t) = -te^{-t} - e^{-t} - ce^{-t} + D$$ (2)

Q_3. Using Picard Iteration, determine $\phi_n(t)$ for an arbitrary value of n.

$$y' = t^2 y - t, \;\; y(0) = 0.$$, $\phi_0(t) = 0$

(1) $$\phi_1(t) = \int (t^2(0) - t) dt = -\frac{t^2}{2}$$

(2) $$\phi_2(t) = \int \left(t^2\left(-\frac{t^2}{2}\right) - t\right) dt = \int \left(\frac{-t^4}{2} - t\right) dt = -\frac{t^5}{10} - \frac{t^2}{2}$$

$$\phi_3(t) = \int t^2\left(\frac{-t^5}{10} - \frac{t^2}{2}\right) - t \; dt = \int -\frac{t^7}{10} - \frac{t^4}{2} - t \; dt$$

(2) $$= -\frac{t^8}{80} - \frac{t^5}{10} - \frac{t^2}{2}$$

$$\vdots$$

(1) $$\phi_n(t) = -t^2 \sum_{k=1}^{n} \frac{(t^3)^{k-1}}{2 \cdot 5 \cdot 8 \cdots [2 + 3(k-1)]}$$