Abistract I

Homework 4 (chapter 5)

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1. Find the order of each of the following permutations.

a. (14) : order $= 2$

b. (147) : order $= 3$

c. (14762) : order $= 5$

d. $(a_1 a_2 \dots a_k)$: order $= k$

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2. Write each of the following permutations as a product of disjoint cycles.

a. $(1235)(413) = (15)(234)$

b. $(13256)(23)(46512) = (124)(35)(6) = (124)(35)$.

c. $(12)(13)(23)(142) = (1423)$.

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3. What is the order of each of the following permutation.

a. $(124)(357)$ = L.c.m $(3,3) = 3$

b. $(124)(3567)$: L.c.m $(3,4) = 12$

c. $(124)(35)$: L.c.m $(3,2) = 6$

d. $(124)(357869)$: L.c.m $(3,6) = 6$

بنرتبها لأنه ممنوع يكون في اشي مشترك

e. $(1235)(24567)$: $(124)(3567) \rightarrow$ L.c.d $(3,4) = 12$

f. $(345)(245)$: $(1)(25)(34) = (25)(34) \rightarrow$ L.c.d $(2,2) = 2$

4. What is the order of each of the following permutations.

a. $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 5 & 4 & 6 & 3 \end{bmatrix} = (12)(356)(4) = (12)(356)$.

$\rightarrow$ order : $L.c.m(2,3) = 6$

b. $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 6 & 1 & 2 & 3 & 4 & 5 \end{bmatrix} = (1753)(264)$

$\rightarrow$ order : $L.c.m(4,3) = 12$.

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6. Show that A_8 contains an element of order 15.

let $\alpha = (123)(45678) \in A_8$

$|\alpha| = L.c.m(3,5) = 15$ ✓

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8. What is the maximum order of any element in A_{10}.

In S_{10}.

$(1\ 2\ 3\ 4\ 5)(6\ 7\ 8)(9\ 10)$ has order $= 30$ and maximum in S_{10}

But this it is an odd permutation (5 cycles)(3 cycels)(2 cycles) $\notin A_{10}$

So

$\alpha = (2)(8) \rightarrow |\alpha| = 2$.

$\alpha = (3)(7) \rightarrow |\alpha| = 21$ ✓

$\alpha = (4)(6) \rightarrow |\alpha| = 2$

$\alpha = (1)(9) \rightarrow |\alpha| = 9$

$\alpha = (5)(5) \rightarrow |\alpha| = 5$

$\alpha = (6)(4) \rightarrow |\alpha| = 2$

So the maximum order in $A_{10} = \underline{\underline{21}}$

9. Determine whether the following permutations are even or odd.

a. $\overset{2}{(1 3 5)}$: $(15)(13)$ → even

b. $\overset{3}{(1 3 5 6)}$: $(16)(15)(13)$ → odd

c. $\overset{4}{(1 3 5 6 7)}$: $(17)(16)(15)(13)$ → even

d. $\overset{1}{(1 2)}\overset{2}{(1 3 4)}\overset{2}{(1 5 2)}$: $(12)(14)(13)(12)(15)$ → odd.

e. $\overset{3}{(1 2 4 3)}\overset{3}{(3 5 2 1)}$: $(13)(14)(12)(31)(32)(35)$ → even

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12. If α is even, prove that α^{-1} is even. If α is odd, prove that α^{-1} is odd.

If α is even → $\alpha = \alpha_1 \alpha_2 \cdots \alpha_{2n}$, α 2-cycles.

$\alpha^{-1} = [\alpha_1 \alpha_2 \cdots \alpha_{2n}]^{-1}$

$= \alpha_{2n} \alpha_{2n-1} \cdots \alpha_2 \alpha_1$ → even.

If α is odd → $\alpha = \alpha_1 \alpha_2 \cdots \alpha_{2n+1}$

$\alpha^{-1} = [\alpha_1 \alpha_2 \cdots \alpha_{2n+1}]^{-1}$

$= \alpha_{2n+1} \alpha_{2n} \cdots \alpha_2 \alpha_1$ → odd.

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14. In S_n, let α be an r-cycle, β an s-cycle and γ at t-cycle. complete the following

- $\alpha\beta$ is even iff $r+s$ is even
- $\alpha\beta\gamma$ is even iff $r+s+t$ is odd

17. let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{bmatrix}$ and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{bmatrix}$ compute each of the following

a. α^{-1} :

$$\alpha^{-1} = \begin{pmatrix} 2 & 1 & 3 & 5 & 4 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix}.$$

b. $\beta\alpha$:

$$\beta \circ \alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 6 & 2 & 3 & 4 & 5 \end{pmatrix} = (1)(26543) = (26543)$$

c. $\alpha\beta$:

$$\alpha \circ \beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 2 & 1 & 5 & 3 & 4 \end{pmatrix} = (16453)(2) = (16453).$$

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26. prove that (1234) is not the product of 3-cycles.

$\alpha = (1\ 2\ 3\ 4) = (14)(13)(12)$ is odd permutation.

and any 3-cycles $(a\,b\,c) = (ac)(ab)$ is even. $\times$

So α can't be written as product of 3-cycles

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35. How many elements of order 5 are in A_6 ?

A_6 is a group of all the even permutation and a cycle of odd length (even permutation)

→ 5 length cycle is even permutation,

→ the total number of order 5 in $A_6 = \frac{6!}{5} = 144$.

41. prove that S_n is non-Abelian for all $n \geq 3$.

let $\alpha = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$ $\in S_3$

$\alpha \circ \beta = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$ $\beta \alpha = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$

$\alpha\beta \neq \beta\alpha$ so is non-Abelian.

So S_n is non Abelian $\forall\, n \geq 3$.