

Chapter I

Q1

Find the Domain and Range

(a) $f(x) = \frac{1}{\sqrt{x}}$

$D(f) = (0, \infty)$

$R(f) = (0, \infty)$

$x > 0$



(b) $f(x) = \tan(\pi x)$

$D(f) = \mathbb{R} \setminus \left\{ \frac{1}{2} + n, n = 0, \pm 1, \pm 2, \dots \right\}$

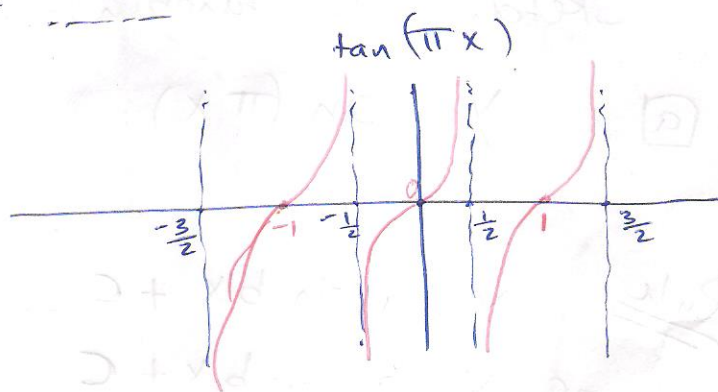
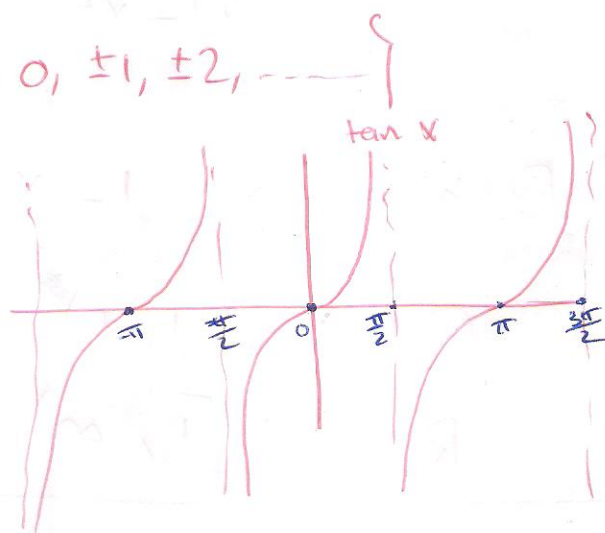
$R(f) = \mathbb{R}$

$\tan(\pi x) = \frac{\sin \pi x}{\cos \pi x}$

$\cos \pi x \neq 0$

$\pi x \neq \frac{\pi}{2}, -\frac{\pi}{2}, \dots$

$x \neq \frac{1}{2} + n$



(c) $f(x) = 1 + |x|$

$D(f) = \mathbb{R}$

$R(f) = [1, \infty)$



(d) $f(x) = \sec^2 x = \frac{1}{\cos^2 x}$

$\cos^2 x \neq 0$

$\cos x \neq 0$

$\rightarrow x \neq \frac{\pi}{2} + n\pi$

$D(f) = \mathbb{R} \setminus \left\{ \frac{\pi}{2} + n\pi, n = 0, \pm 1, \pm 2, \dots \right\}$

$R(f) = [1, \infty)$

$$(e) \quad g(x) = \frac{1}{x^2}$$

$$D(g) = \boxed{x \neq 0}$$

$$D(g) = \mathbb{R} \setminus \{0\}$$

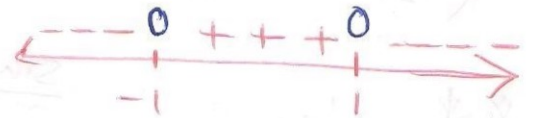
$$R(g) = (0, \infty)$$

$$(f) \quad h(x) = \frac{1}{\sqrt{1-x^2}}$$

$$D(h) \longrightarrow 1 - x^2 > 0$$

$$D(h) = (-1, 1)$$

$$R(h) = [1, \infty)$$



Q2

Sketch the following functions.

a) $y = \sin(\pi x)$

Rule

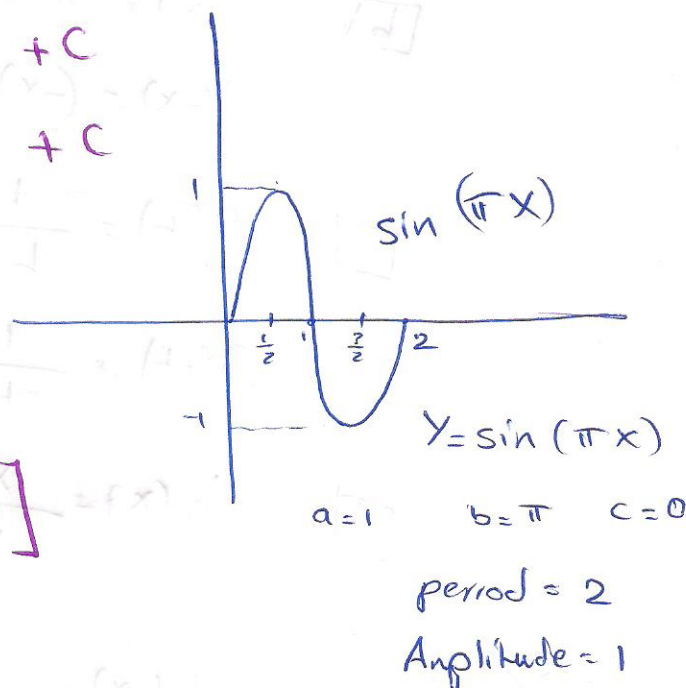
$$y = a \sin(bx) + c$$

or $y = a \cos(bx) + c$

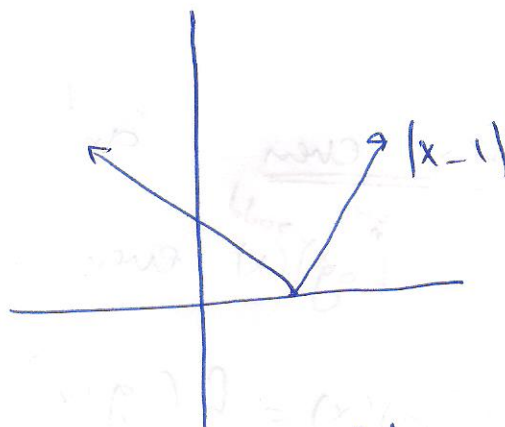
$$\text{period} = \frac{2\pi}{|b|}$$

$$\text{Amplitude} = |a|$$

$$\text{Range} = \left[\underbrace{-|a|+c}_{\text{Min}}, \underbrace{|a|+c}_{\text{Max}} \right]$$



b) $y = |x-1|$



c)

$$y = \cos(x) + 1$$

$$a=1$$

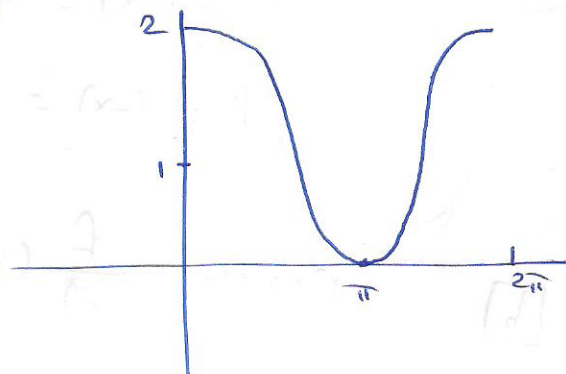
$$b=1$$

$$c=1$$

$$\text{Range} = [0, 2]$$

$$\text{period} = 2\pi$$

$$\text{Amplitude} = 1$$



Q3

a) $f(x) = x^2 + 1$
 $f(-x) = (-x)^2 + 1 = x^2 + 1 = f(x)$ even

b) $f(x) = x^3 + x$
 $f(-x) = (-x)^3 + (-x) = -x^3 - x = -(f(x))$ odd

c) $g(t) = \frac{1}{t-1}$
 $g(-t) = \frac{1}{-t-1}$ neither odd nor even.

d) $h(x) = \frac{x}{x^2-1}$
 $h(-x) = \frac{-x}{(-x)^2-1} = \frac{-x}{x^2-1} = -h(x)$ odd

Q4 If $f(x)$ even and $g(x)$ odd

a) prove $(f \circ g)(x)$ even

$$(f \circ g)(x) = f(g(x))$$

$$(f \circ g)(-x) = f(g(-x)) = \frac{f(-g(x))}{\text{even}} = f(g(x)) = f \circ g(x)$$
 even

b) prove $\frac{f}{g}(x)$ odd

$$\frac{f}{g}(x) = \frac{f(x)}{g(x)}$$

$$\frac{f}{g}(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -\left(\frac{f(x)}{g(x)}\right) = -\frac{f}{g}(x)$$
 odd

Chapter 2

$$\boxed{Q_1} (a) \lim_{t \rightarrow -1} \frac{t^2 + 3t + 2}{t^2 - t - 2} = \frac{(-1)^2 + 3(-1) + 2}{(-1)^2 + 1 - 2} = \frac{0}{0}$$

$$\lim_{t \rightarrow -1} \frac{(t+1)(t+2)}{(t-2)(t+1)} = \boxed{\frac{1}{-3}}$$

$$(b) \lim_{x \rightarrow 1} \frac{1 - \sqrt{x}}{1 - (x)^2} = \frac{1 - \sqrt{1}}{1 - 1} = \frac{0}{0}$$

$$\lim_{x \rightarrow 1} \frac{1 - \sqrt{x}}{(1 - \sqrt{x})(1 + \sqrt{x})} = \frac{1}{1 + \sqrt{1}} = \boxed{\frac{1}{2}}$$

$$(c) \lim_{\theta \rightarrow 1} \frac{\theta^4 - 1}{\theta^3 - 1}$$

$$\begin{aligned} \lim_{\theta \rightarrow 1} \frac{(\theta^2 - 1)(\theta^2 + 1)}{(\theta - 1)(\theta^2 + \theta + 1)} &= \lim_{\theta \rightarrow 1} \frac{(\theta - 1)(\theta + 1)(\theta^2 + 1)}{(\theta - 1)(\theta^2 + \theta + 1)} \\ &= \frac{2 \cdot 2}{3} = \boxed{\frac{4}{3}} \end{aligned}$$

$$(d) \lim_{\theta \rightarrow 0} \frac{\sin 2\theta}{3\theta} = \boxed{\frac{2}{3}}$$

Rule

$$\lim_{\theta \rightarrow 0} \frac{\sin a\theta}{b\theta} = \frac{a}{b}$$

OR

$$\lim_{\theta \rightarrow 0} \frac{\tan a\theta}{b\theta} = \frac{a}{b}$$

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$$\lim_{\theta \rightarrow 0} \frac{\frac{\sin 2\theta}{\theta}}{\frac{3\theta}{\theta}} =$$

$$\frac{\lim_{\theta \rightarrow 0} \frac{\sin 2\theta}{\theta}}{\lim_{\theta \rightarrow 0} 3} = \frac{2}{3}$$