Linear

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Chapter one :

1.1: system of linear equations:

A linear equation : $a_1x_1+a_2x_2+\dots\dots+a_nx_n=b$, where x are variables, a,b are real numbers.

$2x_1-x_2+x_3=5$
$X_1+3x_2+x_3=6$
Is 2x3 linear system.

m x n ⇒ number of unknowns (variables).
number of equations.

A non linear system : at least one of the equations in the system are nonlinear :

Exp>> $x+y=1$
$X^2+y=5$

Remark : in general there are three possibilities for 2x2 linear system:

1- the lines intersect at a point (unique solution).
2- they are parallel (no solution).
3- both equations represent the same line (infinite solution).

Remark : in general there are three possibilities for mxn linear system:

1- (unique solution).
2- (no solution).
3- (infinite solution).

لـ بس آجي أكتبها كيف بكتبها ⇐ (راح اعطي مثال):

Exp⇒ $+\begin{array}{r} 4x_1-2x_2=6 \\ -4x_1+2x_2=-6 \\ \hline 0 \quad = 0 \end{array}$

$\hookrightarrow x_2=2x_1-3$

free ← Let $x_1=t$ ⇒ then $x_2=2t-3$ → leading.

The solution set = $\{(x_1,x_2)=(t,2t-3)\}$ ⇒ $t\in\mathbb{R}$.

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m x n

- Consistent
  - Unique Solution
  - Infinite Solution
- In Consistent
  - No solution

Equivalent systems :

Two systems of equations are called equivalent systems if they have the same variables (unknowns) and the same solution set.

Square system: If m=n and it is called an nxn linear system.

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Strict triangular form :
1- nxn system .
2- if in the k equation the coefficient of the first k-1 variables are all zero and the coefficient of sky is nonzero (k=1,2,3,….,n)

Exp:

X1+x2+x3=8
 0 +2x2+x3=5
 0 + 0   + x3=9

This example is strict triangular form.

1- the system is strict triangular form is easy to solve and has a unique solution.
2- we have the system by backward substitution method .برجع رجوع بالحل

How to transform a system in strict triangular form?

1- interchange two rows
2- multiply a row by a nonzero constant
3- replace a row by its sum with a multiple of another row.

Augmented matrix = [A:b]

Coefficient matrix = [A] <<<< بكتب معاملات الاكسات كمصفوفة

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7. The two systems

$$\begin{aligned} 2x_1 + x_2 &= 3 \\ 4x_1 + 3x_2 &= 5 \end{aligned} \qquad \text{and} \qquad \begin{aligned} 2x_1 + x_2 &= -1 \\ 4x_1 + 3x_2 &= 1 \end{aligned}$$

have the same coefficient matrix but different right-hand sides. Solve both systems simultaneously by eliminating the first entry in the second row of the augmented matrix

$$\left(\begin{array}{cc|cc} 2 & 1 & 3 & -1 \\ 4 & 3 & 5 & 1 \end{array}\right)$$

and then performing back substitutions for each of the columns corresponding to the right-hand sides.

$$\left[\begin{array}{cc|cc} 2 & 1 & 3 & -1 \\ 4 & 3 & 5 & 1 \end{array}\right]$$

بيصير هاد بيصير هيك :

$$\left[\begin{array}{cc|cc} 2 & 1 & 3 & -1 \\ 0 & 1 & -1 & 3 \end{array}\right]$$

$\boxed{x_2 = -1}$ $\qquad$ $\boxed{x_2 = 3}$

$2x_1 - 1 = 3 \Rightarrow \boxed{x_1 = 2}$ $\qquad$ $2x_1 + 3 = -1 \Rightarrow \boxed{x_1 = -2}$

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1.2: Row Echelon Form(REF)

1- the first nonzero entry in each nonzero row is 1 called the leading one or the pivot 1.

2- the leading 1 in the k row is do the right of the leading 1 in the (k-1) row.

3- zero rows are below the nonzero rows.

Exp= $\begin{bmatrix} 1 & 4 & 2 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow$ REF.

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is not REF.

$\begin{bmatrix} 2 & 4 & 6 \\ 0 & 3 & 5 \\ 0 & 0 & 4 \end{bmatrix} \Rightarrow$ is not REF.

$\begin{bmatrix} 1 & 3 & 1 & 6 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow$ is REF.

Gaussian elimination method (REF)

Gaussian Jordan elimination method (RREF)

1- the matrix is in REF.

2- the first nonzero entry in each row 1's is the only nonzero entry in its column.

$A = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ is RREF.

$B = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is RREF.

$C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is RREF.

$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is Not RREF.

Under determined : m<n

- No solution
- Infinite

Over determined : m>n

- No solution
- Unique
- Infinite

Homogeneous system : Ax=0

Homogeneous system is always consistent since there is a solution called the zero solution or the trivial solution

Under determined homogeneous linear system has always infinite solution

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$$\left[\begin{array}{ccc|c} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \left.\begin{array}{l} x_1 = t \\ x_2 = 2 \\ x_3 = -1 \end{array}\right\} \Rightarrow (t, 2, -1)$$

. الجواب

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 2 & 5 & 3 & 0 \\ 1 & 1 & B & 0 \end{array}\right]$$

هل ممكن B تكون inconsistent ⇐ ?

⇓

نعملها REF

⇐ مستحيل تكون inconsistent لأنه هاي homogeneous.

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example ⇒ let $Ax=b$ be a linear system whose augmented matrix $(A|b)$ has reduced row echelon form:-

$$\left[\begin{array}{ccccc|c} 1 & 2 & 0 & 3 & 1 & -2 \\ 0 & 0 & 1 & 2 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right], \text{ if } a_1 = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 4 \end{bmatrix} \text{ and } a_3 = \begin{bmatrix} 2 \\ -1 \\ 1 \\ 3 \end{bmatrix} \text{ determine } b.$$

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*If A and B are singular then A+B is not also nonsingular.

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1.3 : Matrix arithmetic

$m \times n$ is the size (order) or the dimension of $A_{m \times n}$

For similarity we use the notation : $A = a_{ij}$: $i = 1, 2, \dots, m$
$J = 1, 2, \dots, n$

a_{45} → الصف ، العمود

Column vector is an $m \times 1$ matrix. $\Rightarrow A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

Row vector is an $1 \times n$ matrix. $B = [1.\ 3.\ 4.\ 7.]$

عدد الصفوف ، عدد الأعمدة

$R^{2 \times 3} : \begin{bmatrix} 10 & 5 & 7 \\ 2 & 13 & 60 \end{bmatrix}$

* column vectors $\Rightarrow a_j = \begin{pmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{nj} \end{pmatrix}$. * Row vector $\Rightarrow A = \begin{pmatrix} \vec{a_1} \\ \vec{a_2} \\ \vdots \\ \vec{a_m} \end{pmatrix}$.

The equality matrices : two matrices A and B are equal iff they have the same size and $a_{ij} = b_{ij}$.

Properties of addition and scalar multiplication:

1- $k(A+B) = kA + kB$.
2- $(k\,r)A = r(kA) = k(rA)$.
3- $A+B = B+A$.
4- $A+(B+C) = (A+B)+C$.
5- $A+0 = 0+A = A$.
6- $A-A = A+(-A) = 0$.
$-A$ is called the additive inverse of A.

When I want to multiply two matrices A and B they should have the same size.

ع explain
⇐ by the exp

↳ $A = \begin{bmatrix} 1 & 3 \\ 6 & -1 \end{bmatrix}_{2 \times 2}$ بضرب $B = \begin{bmatrix} 1 & 5 & 2 \\ 0 & 1 & -3 \end{bmatrix}_{2 \times 3}$

زي بعض لازم يكونوا.

↳ $AB = \begin{bmatrix} 1\times1+3\times0 & 1\times5+3\times1 & 1\times2+3\times-3 \\ 6\times1+-1\times0 & 6\times5+-1\times1 & 6\times2+-1\times-3 \end{bmatrix}$ $2 \times 3 \Rightarrow$ المصفوفة اللي رح تنتج.

$AB \neq BA$

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If it is told me to write the system in a matrix form :

$$4x_1 + 2x_2 + x_3 = 1.$$
$$5x_1 + 3x_2 + 7x_3 = 2.$$

$$\underbrace{\begin{bmatrix} 4 & 2 & 1 \\ 5 & 3 & 7 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 1 \\ 2 \end{bmatrix}}_{b}$$

طريقة ثانية

$$x_1 \begin{bmatrix} 4 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

Linear combination : c1a1+c2a2+….+cnan

↳ exp ⇒ Is $b = \begin{pmatrix} 2 \\ 24 \end{pmatrix}$ a linear combination of $a_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $a_2 \begin{pmatrix} 0 \\ 5 \end{pmatrix}$?

Answer ⇒ $b = c_1 a_1 + c_2 a_2$

$$\begin{pmatrix} 2 \\ 24 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 5 \end{pmatrix} \Rightarrow \quad 2 = c_1 + \cancel{c_2}^{\,0} \Rightarrow \boxed{c_1 = 2}$$

$$24 = 2c_1 + 5c_2$$

$$24 = 4 + 5c_2 \Rightarrow \boxed{c_2 = 4}$$

$b = 2a_1 + 4a_2 \Rightarrow$ b is a linear combination of a_1 and a_2.

فإذا ما طلعلي جواب لـ c_1 and c_2 تناقض ← not linear combination

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The transpose of a matrix : $\Rightarrow A^T = (a_{ij})^T = (a_{ji}) = B_{n \times m}$

↳ exp ⇒ $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}.$

An nxn matrix A is said to be symmetric if A^T=A

An nxn matrix A is said to be skew symmetric if A^T=-A

The zero matrix is always skew or symmetric .

$$\Rightarrow \left.\begin{aligned} A^T &= A \\ -A^T &= A \end{aligned}\right\} + \Rightarrow 0 = 2A \Rightarrow \boxed{A = 0}$$

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Properties of the transpose :

1- $(A^T)^T = A$.

2- $(A \mp B)^T = A^T \mp B^T$.

3- $(KA)^T = KA^T$.

4- $(AB)^T = B^T A^T$.

5- If A ,B are symmetric, then A+B is symmetric. $\Rightarrow (A+B)^T = A^T + B^T = A + B$.

6- If A is symmetric, then kA is symmetric. $\Rightarrow (\alpha A)^T = \alpha A^T = \alpha A$ is symmetric.

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1.4:Matrix algebra:

$$A^n = \begin{bmatrix} 2^{n-1} & 2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{bmatrix}$$

The identity matrix : $\Rightarrow I = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{if } i \neq j \end{cases}$ example $\Rightarrow I_{3\times3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

لازم تكون مصفوفة مربعة nxn.

IB=BI=B

Matrix inversion :

An nxn matrix A is said to be invertible if there exists a matrix B such that AB=BA=I, the matrix B is called the inverse or multiplicative inverse of A denoted by A^{-1}, If A^{-1} does not exist,then A has no inverse(A is called singular or invertible).

$$A^{-1} \neq \frac{1}{A}.$$

How to find the inverse of 2x2?

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow \alpha = ad - bc \neq 0$ then $A^{-1} = \frac{1}{\alpha} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

If α=0, the A is singular.

$\times A \cdot A^{-1} = I$

If A and B are nonsigular nxn matrices ,then AB is also nonsigular and : $(AB)^{-1} = B^{-1}A^{-1}$.

Algebraic rules for inverses:

1-The inverse of A if exists is unique.
2- $(A^{-1})^{-1}=A$.
3-$(kA)^{-1}=1/k\,(A^{-1})$
4- If A is invertible then A^T is invertible and $(A^T)^{-1}=(A^{-1})^T$
5-$[(AB)^T]^{-1}=(A^T)^{-1}(B^T)^{-1}$.

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1.5: elementary matrices :

A matrix E is called an elemantary matrix if it is obtained from the identity In by performing exactly one row operations.

There are three types :

1- interchanging any two rows of In.

2- multiplying any row of In by a nonzero constant.

3- adding a multiple of one row of In to another row of In.

If E is an elementary matrix, then E is nonsingular (invertible) and E^{-1} is an elementary of the same type .

A matrix B is row equivalent to a matrix A : $B = E_k E_{k-1} \cdots E_1 A$.

Example ⇒ if we have $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 3 \\ 1 & 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 3 \\ 2 & 2 & 6 \end{bmatrix}$

find an elementary matrix E such that $EA = B$ ⇒ B is row equivalent to A.

أول إشي في A and B زي بعض . هسة بدي أوجد الـ E اللي أضربها بـ A مشان

أوصل لـ B هسة بلاحظ إذا جمعت الصف الأول والثالث في A بطلع نفس الصف الأخير

اللي في B . ⇐ $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ⇐ نجمع الصف الأول والثالث ⇐ $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

هاي بتكون الجواب.

If A is row equivalent to B, then B is row equivalent to A.

If A is row equivalent to B and B is row equivalent to C, then A is row equivalent to C.

The system Ax=b, where Anxn, has a unique solution, iff A is nonsingular (The solution is $x=A^{-1}b$).

If A is nonsingular then A is row equivalent to I.

$Ax = 0$
- $x = 0$ if A nonsingular.
- inf if A is singular.

$Ax = b$
- unique if A is nonsingular.
- No solution. / infinite. → A singular.

To find the inverse for 3x3 matrix and above :

↳ I will take an example ⇒ $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{bmatrix}$, find A^{-1}.

$[A \mid I_3] = \left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

إذا واحنا نحل لقينا صف كله أصفار يعني ال A فش إلها inverse (A is singular) .

نحل هاي بوصلها ل I_3 عن طريقة ال RREF

والي بنتج هون بكون هو ال A^{-1} .

If he tolds me to solve the system Ax=b using the inverse of A :

Example ⇒
$$\left.\begin{aligned} x_1 + x_2 + 2x_3 &= -2 \\ x_2 + 2x_3 &= 3 \\ 2x_1 \quad + x_3 &= 0 \end{aligned}\right\} \Rightarrow \underbrace{\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}}_{b}.$$

$x = A^{-1} b$

نطلعها ونضربها في b عن طريقة الطريقة اللي فوقها لجيب ال A^{-1} .

Diagonal and Triangular matrices:

A is said to be triangular if it is either upper or lower triangular.

$A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 5 \end{bmatrix}$ upper triangular.

$B = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 0 & 0 \\ 0 & 4 & 3 \end{bmatrix}$ lower triangular.

$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ Diagonal.

$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ diagonal and triangular.

* Every Diagonal is triangular.

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LU factorization : (Triangular factorization)

↳ $A = LU$.

*not every matrix has an LU factorization.

① كيف بوجد ال L ⇐ نطبق ال A

$\begin{bmatrix} & 0 & \\ & 0 & 0 \end{bmatrix}$ ← ما بنفع هدول.

② في طريقتين لوجد ال L ← الطريقة السهلة المعادلات اللي استخدمناهم فانه الصفر في ال L بوخذ وبعكس إشاراتهم ونحطهم هونه.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix}$$

الطريقة الأصعب والأطول: ⇐ نجيب ال E_1 وال E_2 وال E_3

$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ⇒ نطبق أول معادلة استخدمناها لل L نطبقها هونه

ونفس الاشي ب E_2 و E_3.

بعدين $E_3 E_2 E_1 A = U$.

$A = (E_3 E_2 E_1)^{-1} U$.

$A = \underbrace{E_1^{-1} E_2^{-1} E_3^{-1}}_{L} U$.

$[E_1 \mid I]$ وهيكنا بوجدهم.

1- if A has an LU factorization , then A is nonsingular iff L is nonsingular. False because L is always nonsingular.

2- if A has an LU factorization , then A is nonsingular iff U is nonsingular. True.

3- if A has an LU factorization , then A is row equivalent to U. True.

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2.1:The determinant of a matrix

If A was 2x2 matrix then the det(A) or the lAl is = $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$

خاص في المحددة الـ 2x2

Then an nxn matrix A is nonsingular iff lAl ≠ 0.

$* \; |AB| = |BA| = |B|\,|A| = |A|\,|B|.$

Cofactor method :

$\hookrightarrow m_{ij}$ = the minor of $a_{ij} = |M_{ij}|$

$A_{ij} = (-1)^{i+j} \, |M_{ij}|.$

example ⇒ if $A = \begin{bmatrix} 2 & 5 & 4 \\ 3 & 1 & 2 \\ 5 & 4 & 6 \end{bmatrix}$ then $A_{32} = (-1)^{3+2} \, m_{32} \rightarrow$ يعني حذف الصف 3 العمود 2.

$= -1 \begin{vmatrix} 2 & 4 \\ 3 & 2 \end{vmatrix} = \boxed{8}$. الجواب

2x2 $\rightarrow 2\times2 - 3\times4 = 4 - 12 = -8 \times -1 = 8$

If A was nxn matrix then the det(A) or the lAl is = ⇒ أكثر من 2x2

$\hookrightarrow a_{11}A_{11} + a_{12}A_{12} + \dots + a_{1n}A_{1n}$, if $n \geq 2$.

example ⇒ if $A = \begin{bmatrix} 3 & 2 & 4 \\ 1 & -2 & 3 \\ 2 & 3 & 2 \end{bmatrix}$ find $|A|$.

$|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

$= 3\,(-1)^2 m_{11} + 2\,(-1)^3 m_{12} + 4\,(-1)^4 m_{13}$.

$= -3$

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if we have $\begin{bmatrix} 6 & 0 & 0 \\ 8 & 2 & 0 \\ 1 & 5 & 6 \end{bmatrix} = 6 \times 2 \times 6 = 6 \times 12 = 72.$

⇒ triangular matrix ⇒ Theorem ⑥

$* \; |A^T| = |A|$

1-If A has a row or column consisting entirely of zeros,then lAl=0.
2-If A has two identical rows or two identical columns,then lAl=0.

$\boxed{1}$ $\begin{vmatrix} \boxed{1 \;\; 2 \;\; 4 \;\; 6} \\ 5 \;\; -1 \;\; 0 \;\; 4 \\ 6 \;\; 7 \;\; -9 \;\; 10 \\ \boxed{1 \;\; 2 \;\; 4 \;\; 6} \end{vmatrix}$ ⇒ تكرار

$\therefore |A| = 0.$

$\boxed{2}$ $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 6 & 9 & 10 & 20 \\ 0 & 0 & 0 & 0 \end{vmatrix}$ ⇒ صف أو عمود كله أصفار

$\Downarrow$

$|A| = 0.$

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2.2: Prosperities of determinants:

Row operations :

1- |B|=-|A|. $\Rightarrow A = \begin{bmatrix} 2 & 4 \\ 6 & 5 \end{bmatrix} \Rightarrow B = \begin{bmatrix} 6 & 5 \\ 2 & 4 \end{bmatrix} \Rightarrow$ بس أبدل صفين أو عمودين بضرب كل مرة بسالب.

2- |B|=K|A| $\Rightarrow A = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} \Rightarrow B = \begin{bmatrix} 2 & 4 \\ 4 & 5 \end{bmatrix} = B = 2|A|.$

3- |B|=|A| $\Rightarrow A = \begin{bmatrix} 1 & 4 \\ 5 & -5 \end{bmatrix} \xrightarrow{-5R_1+R_2} B = \begin{bmatrix} 1 & 4 \\ 0 & -25 \end{bmatrix} \Rightarrow |B| = -25 = |A|.$

Let E be an elementary matrix then :

|E| = 1- (-1) if E is of type 1.

= 2- k≠ 0 if E is type of 2. $\Rightarrow |E_2|$ بنوخد الثابت

= 3- (1) if E is type of 3.

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* $|kA| = k^n |A|.$ * $|I| = 1$
* $|EA| = |E|\,|A|.$
* If A and B matrices then $\Rightarrow |AB| = |A|\,|B|.$
* If A is singular, then $|A|=0$ and so AB is singular, therefore $|AB| = 0 = |A|\,|B|.$

$|A+B| \neq |A| + |B|$ ⇐ المحددة لبتفصلش في الجمع *

* $|A^{-1}| = \dfrac{1}{|A|} \Rightarrow$ A is nonsingular.
* If $A^TA = I$, then $|A| = \mp 1 \Rightarrow |A^TA| = |I|$

$|A^T|\,|A| = 1$

$|A|\,|A| = 1 \Rightarrow |A|^2 = 1 \Rightarrow |A| = \mp 1.$

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* If he told me to use elemination Method to evallate det(A).

REF

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2.3 : Additional topics and applications :

The adjoins of a matrix:

$$\operatorname{adj}(A) = \begin{bmatrix} A_{11} & A_{12} \cdots & A_{1n} \\ A_{21} & A_{22} \cdots & A_{2n} \\ \vdots & & \\ A_{n1} & A_{n2} \cdots & A_{nn} \end{bmatrix}^T$$

* $A_{ij} = (-1)^{i+j} |M_{ij}|$.

* $A \operatorname{adj}(A) = |A| I_n$.
* $A^{-1} = \frac{1}{|A|} \operatorname{adj}(A)$.
* $|\operatorname{adj} A| = |A|^{n-1}$.
* $(\operatorname{adj} A)^{-1} = |A^{-1}| A = \operatorname{adj} A^{-1}$.
* $\operatorname{adj}(\operatorname{adj} A) = |A|^{n-2} A$.

* cremmer's Rule $\Rightarrow$ $x_i = \frac{|A_i|}{|A|}$, $i = 1, 2, \dots, n$.

example $\Rightarrow$
$$\left.\begin{aligned} x_1 + 2x_2 + x_3 &= 5. \\ 2x_1 + 2x_2 + x_3 &= 6. \\ x_1 + 2x_2 + 3x_3 &= 9. \end{aligned}\right\} \Rightarrow A = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

$$|A_1| = \begin{vmatrix} 5 & 2 & 1 \\ 6 & 2 & 1 \\ 9 & 2 & 3 \end{vmatrix} \Rightarrow$$ حوطت الناتج في العمود الأول

$\to x_1 = \frac{|A_1|}{|A|}$. وهكذا.

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* Evaluate the following determinant. write your answer as a polynomial in x:- $\begin{vmatrix} a-x & b & c \\ 1 & -x & 0 \\ 0 & 1 & -x \end{vmatrix}$.

$$\to |A| = (a-x) \begin{vmatrix} -x & 0 \\ 1 & -x \end{vmatrix} - b \begin{vmatrix} 1 & 0 \\ 0 & -x \end{vmatrix} + c \begin{vmatrix} 1 & -x \\ 0 & 1 \end{vmatrix}.$$
$$= (a-x)x^2 - b(-x) + c.$$
$$= -x^3 + ax^2 + bx + c.$$

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* If A is singular then $A \operatorname{adj} A = ??$

$A \operatorname{adj} A = |A| I$ if A is singular $|A| = 0$ so $\operatorname{adj}(A) = 0$.

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3.1: Definitions and examples:

* $V \Rightarrow$ is a vector space is a set of elements together with the operations of addition and scalar multiplication :-

C_1 :- if $x \in V$ and α is scalar, then $\alpha x \in V$ "closed under scalar multiplication".

C_2 :- if $x, y \in V$, then $x + y \in V$ "closed under addition".

A_1: $x + y = y + x$.

A_2: $(x+y)+z = x+(y+z)$.

A_3: $\exists$ an element $0 \in V$ such that $x + 0 = 0 + x = x$, $\forall x \in V$.

A_4: $\forall x \in V$, $\exists -x \in V$ such that $x + -x = 0$.

A_5: $\alpha(x+y) = \alpha x + \alpha y$.

A_6: $(\alpha + \beta)x = \alpha x + \beta x$.

A_7: $(\alpha\beta)x = \alpha(\beta x)$.

A_8: $1x = x$.

$\Rightarrow$ Notation $(V, +, \cdot)$

$\Rightarrow$ أمثلة $\Rightarrow$ $\boxed{1}$ $(\mathbb{R}, +, \cdot)$ $\Rightarrow$ "$\mathbb{R}$ with usual addition and multiplication is a vector space".

$\boxed{2}$ $V = \mathbb{R}^2$ with usual $+$ and $\cdot$ is a vector space where $(a,b) + (c,d) = (a+c, b+d)$

$\alpha(a,b) = (\alpha a, \alpha b)$.

$\boxed{3}$ $M_{m\times n} = \mathbb{R}^{m\times n}$ is the set of all $m\times n$ matrices with real entries under addition and scalar multiplication is a vector space.

$\boxed{4}$ The set of all real valued functions under $+$ and $\cdot$:-

$$\begin{cases} (f+g)(x) = f(x) + g(x). \\ (\alpha f)(x) = \alpha f(x). \end{cases}$$

$\boxed{5}$ $C[a,b] := \{ f : [a,b] \to \mathbb{R} : f$ is continuous on $[a,b]$

$\hookrightarrow (f+g)(x) = f(x) + g(x)$.

$(\alpha f)(x) = \alpha f(x)$

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مشتقة n

$\boxed{6}$ $C^n[a,b] = \{ f:[a,b] \to \mathbb{R} : f^{(n)}$ is continuous on $[a,b] \}$.

$(f+g)(x) = f(x) + g(x)$.

$(\alpha f)(x) = \alpha f(x)$.

$\boxed{7}$ $P_n = \{ f(x) = a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \}$.

$(f+g)(x) = f(x) + g(x)$

$(\alpha f)(x) = \alpha f(x)$.

* irrationales $\Rightarrow (\sqrt{2}, \sqrt{3}, e, \pi, \ldots)$. is not a vector space.
* $Z = \{0, \mp 1, \mp 2, \ldots\}$ is not a vector space.
* $V = \{ f(x) = \deg(f) = 3 \}$ is not a vector space.
  ↳ $1 - x^3$, $1 + x^3 \in V$ but $1 - x^3 + 1 + x^3 = \boxed{2} \in \deg = 3$.

---

*Let V be a vector space then:-

$\boxed{1}$ $0 v = \vec{0}$, $\forall v \in V$.

$\boxed{2}$ if $x + y = \vec{0}$, then $y = -x$.

$\boxed{3}$ $-1 . v = -v$, $\forall v \in V$.

3.2:Subspace and spanning sets:

* Subspace $\Rightarrow$ 1. $x+y \in S$.
  2. $\alpha x \in S$.
  3. $0 \in S$, $S \neq \emptyset$.

---

* Let S and T be subspace of a vector space V then:-

$\boxed{1}$ $S \cap T$ is subspace

$\boxed{2}$ $S \cup T$ is not always is a subspace of V.

$\boxed{3}$ $S+T$ is subspace of V.

---

* Null space $\Rightarrow$ $Ax=0$. (RREF). $\Rightarrow$ is a subspace of $\mathbb{R}^n$.

م بطلع ال free variable عامل مشترك   و ا- المعادلات او المصفوفات ⇐ طريقة إيجاده.
لي بساويهم بالصفر.

---

* Linear combination:- $c_1U_1 + c_2U_2 + \cdots + c_nU_n$ $\Rightarrow$ is a subspace of V.
  ↳ span $(u_1, u_2, \ldots, u_k)$.

example $\Rightarrow$ $U = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $U_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $U_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\Rightarrow$ $V = c_1V + c_2U_2$.

$\begin{pmatrix} 2 \\ 3 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\Rightarrow$ $\begin{cases} c_1 + c_2 = 2. \\ c_2 = 3. \end{cases}$ $\begin{cases} c_1 = -1. \\ c_2 = 3. \end{cases}$

$\therefore V = -U_1 + 3U_2$ / V is a linear combination or span (v_1, v_2).

---

* spanning set:- if $V = \text{span}(v_1, \ldots, v_k)$. $\Rightarrow$. consistent لازم تكون دايماً
  ↳ $e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$. $e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

* $\{e_1, e_2, \ldots, e_n\}$ is the standard spanning set for $\mathbb{R}^n$.

$$X = x_1 \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} + x_n \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$$

* $\{1, x, x^2, x^3\}$ is a spanning set for P_4.

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* standard spanning for $R^{2\times2}$ are:-

$$\begin{bmatrix}1&0\\0&0\end{bmatrix}\begin{bmatrix}0&1\\0&0\end{bmatrix}\begin{bmatrix}0&0\\1&0\end{bmatrix}\begin{bmatrix}0&0\\0&1\end{bmatrix}.$$

---

* Let A be a 4×3 matrix and let $b\in R^4$. How many possible solutions could the system $Ax=b$ have if $N(A)=\{0\}$? Answer the same question in the case $N(A)\neq\{0\}$. Explain your answer.

↳ $\boxed{a}$ If $N(A)=0$ then $y=x_0+\boxed{z}\to 0$ so $y=x_0$ Therefore, $Ax=b$ has no solution or a unique solution.

$\boxed{b}$ If $N(A)\neq 0$ then $y=x_0+\boxed{z}\to\neq 0$ so $y\neq x_0$ Therefore $Ax=b$. has no solution or infinite solutions.

---

* Let $\{x_1,x_2,\dots\dots,x_k\}$ be a spanning set for a vector space V.

↳ if we add another vector x_{k+1}, to the set, will we still have a spanning set? ⇒ Yes.

* If we delete one of the vector say x_k from the set, will we still have a spanning set ⇒ No.

$\{(1,0,0),(0,1,0),(0,0,1)\}$ ⇒ spans to R^3 but

$\{(1,0,0),(0,1,0)\}$ does not.

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3.3:linear independence:

⇒ We called about V is linearly independent:-

↳ $c_1v_1 + c_2v_2 + \cdots + c_nv_n = 0 \Rightarrow \boxed{c_1 = c_2 = \cdots = c_n = 0}$

otherwise, they are linearly dependent.

⇒ Is $\begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}1\\2\end{pmatrix}$ Linearly independent??

$c_1\begin{pmatrix}1\\1\end{pmatrix} + c_2\begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix} \Rightarrow 0 = c_2, c_1$ قيم بطلع ، طلعوا

↳ REF.

*If the system was underdetermined homogeneous ⇒ it has infinite solution.

↳ so it is always Linearly dependent.

Theorem $\boxed{1}$:- A set of vectors in R^n are linearly independent iff the matrix $A = [v_1\ v_2 \cdots v_n]$ is nonsingular.

↳ Remark ⇒ $|A| \neq 0 \Rightarrow$ linearly independent.

هاد ال theorem فقط إذا كانت $n \times n$ مربعة.

سواء كانوا أعداد مصفوفة أو P_n ⇐

Exp ⇒ $P_1(x) = 2x^2 + x + 8$, $P_2(x) = x^2 + 8x + 7$, $P_3 = x^2 - 2x + 3$.

↳ $c_1P_1(x) + c_2P_2(x) + c_3P_3(x) = 0$

$c_1(2x^2 + x + 8) + c_2(x^2 + 8x + 7) + c_3(x^2 - 2x + 3) = 0$.

↳ x^2 terms , x terms, constant terms ⇒ بطلعهم

$$\left.\begin{array}{l} 2c_1 + c_2 + c_3 = 0 \\ c_1 + 8c_2 - 2c_3 = 0 \\ 8c_1 + 7c_2 + 3c_3 = 0 \end{array}\right\} \Rightarrow$$ بحلها عادي و بطلعها

Theorem 1 ←

Theorem $\boxed{2}$:- A set of vectors are linearly dependent iff one of them is a linear combination of the remaining set of vectors.

↳ example ⇒ $\left\{ \underset{v_1}{\begin{pmatrix}1\\0\end{pmatrix}}, \underset{v_2}{\begin{pmatrix}0\\1\end{pmatrix}}, \underset{v_3}{\begin{pmatrix}1\\1\end{pmatrix}} \right\}$ are linearly dependent

since $v_3 = v_1 + v_2 \Rightarrow$ (v_3 is a linear combination of v_1 and v_2).

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Theorem [3]:- A set of vectors are linearly independent iff every vector is uniquely written as a linear combination of $v_1, v_2, \dots, v_k$.

---

* The vector space $C^{n-1}[a,b]$.

$W(f_1, f_2, \dots, f_n)$ on $[a,b]$ by:-

$$W(f_1, \dots, f_n) = \begin{vmatrix} f_1 & \cdots & f_n \\ f_1' & \cdots & f_n' \\ \vdots & & \vdots \\ f_1^{(n-1)} & \cdots & f_n^{(n-1)} \end{vmatrix}$$

Wronskian of $f_1, \dots, f_n$

→ if $W(f_1, f_2, \dots, f_n)(x_0) \neq 0$ then $\{f_1, \dots, f_n\}$ are linearly independent.

* If $f_1, f_2, \dots$ are linear dependent then $W(f_1, \dots, f_n) = 0$.

if $W(f_1, f_2, \dots, f_n) = 0$ we can't say about anything. ⇒ نرجع للتعريف $c_1v_1 + c_2v_2 + \dots + c_nv_n = 0$

---

* If we add a vector X_{k+1} to set $\{x_1, x_2, \dots, x_k\}$, the set $\{x_1, x_2, \dots, x_k, x_{k+1}\}$ may or may not be a linear independent set.

* If we delete X_k from $\{x_1, x_2, \dots, x_k\}$, will still be linear independent.

* any nonempty subset of a linearly independent set of vectors $\{v_1, \dots, v_n\}$ is also linearly independent.

* Let A be an $m \times n$, show that if A has linearly independent column vector, then $N(A) = \{0\}$.

⇒ linearly dependent ⇒ $c_1 x + c_2 y = 0$, where c_1, c_2 are not $\neq 0$.

→ $x = -\frac{c_2}{c_1} y$.

⇒ both vectors (x and y) placed at the origin, they will lie along the same line, but if they are linearly independent will not lie at the same line.

If the matrix was not square we can not use the det:

1- we use homogeneous
2-REF if x involve free variables, if they are not trivial solution, then the vectors are linearly dependent , if there are no free variables then c=0 is the only solution and hence the vectors must be linearly independent.

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3.4:basis and dimension:

$\Downarrow$

[1] span V

[2] Linearly independent.

*The zero vector space $\{0\}$ has dimension zero basis $\emptyset$.

[1] $\dim R^n = n$. [2] $\dim P_n = n$. [3] $\dim R^{m\times n} = m.n$. [4] $\dim\{0\} = 0$.

[5] $\dim R = 1$. [6] $\dim C^n[a,b] = \infty$.

Theorem 1:- Let $\{v_1, v_2, \ldots, v_n\}$ be a spaning set for V. If $w_1, w_2, \ldots, w_k \in V$, $k > n$. then $w_1, w_2, \ldots, w_k$ are linearly dependent

↳ example ⇒ $\left\{\begin{pmatrix}2\\1\end{pmatrix}, \begin{pmatrix}4\\3\end{pmatrix}\right\}$ is a spanning set for R^2

$\left\{\begin{pmatrix}2\\1\end{pmatrix}, \begin{pmatrix}4\\3\end{pmatrix}, \begin{pmatrix}-7\\3\end{pmatrix}\right\}$ are linearly dependent.

Theorem 2:- Let $\{v_1, \ldots, v_n\}$ and $\{w_1, \ldots, w_k\}$ be two bases for a vector space V. then

Theorem 3:- Let V be a vector space with $\dim V = n$ then the follwing are equivalent.

↳ [i] $\{v_1, \ldots, v_n\}$ is a basis.

[ii] $\{v_1, \ldots, v_n\}$ span V.

[iii] $\{v_1, \ldots, v_n\}$ linearly independent.

Summery ⇒ $\dim V = n$ then:-

[1] A set $v_1, \ldots, v_k$, $k > n$ linearly dependent.

[2] A set $v_1, \ldots, v_k$, $k < n$ can not span V.

[3] If $k = n$ and $v_1, \ldots, v_k$ are linearly independent or span V, then $\{v_1, v_2, \ldots, v_k\}$ is a basis for V.

[4] A spanning set of $v_1, v_2, \ldots, v_k$, $k > n$ can be reduced (Pured down) to a basis for V.

[5] A linearly independent set $v_1, \ldots, v_k$, $k < n$ can be extended to a basis for V.

↳ Examples:-

$\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}2\\3\\0\end{bmatrix}\right\}$ Linearly independ but not span for R^3

$\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}2\\3\\0\end{bmatrix}, \begin{bmatrix}4\\5\\6\end{bmatrix}\right\}$ $\Downarrow$ A basis for R^3

$\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}2\\3\\0\end{bmatrix}, \begin{bmatrix}4\\5\\6\end{bmatrix}, \begin{bmatrix}7\\8\\9\end{bmatrix}\right\}$ $\Downarrow$ spans for R^3, but it is linearly dependent.

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* find a basis and dimension f:-

$S = \{ (a+3b+c \, , \, 2a+6b+c)^T : a, b, c \in \mathbb{R} \}$.

$a \overset{v_1}{(1,2,0)} + b \overset{v_2}{(3,6,0)} + c \overset{v_3}{(1,0,1)}$.

Notice that $v_2 = 3v_1 \Rightarrow$ the $\{v_1, v_2, v_3\}$ linearly dependent.

$\therefore S = \text{span} \{ \underbrace{\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}_{\text{linearly independent}} \} \rightarrow$ basis for S.

dimension = 2.

$\Rightarrow$ The standard basis for $\mathbb{R}^3$ is $\{e_1, e_2, e_3\}$.

$\Rightarrow$ The standard basis for $P_n = \{1, x, x^2, \ldots, x^{n-1}\}$.

$\Rightarrow$ since they are linearly independent, they form a basis for:-

$N(A) =$ [1] RREF.

[2] بساويهم بالصفر.

[3] بطلع ال free variables عامل مشترك هاد هو ال basis لل $N(A)$.

$\Rightarrow$ If V is a vector space of dimension $n > 0$, then:-

[1] any set of n linearly independent vector space V.

[2] any n vectors that spans V are linearly independent.

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3.5:change of basis:

* $E = \{u_1, u_2, \dots, u_n\}$ a basis of V.

then any $v \in V$ can be written uniquely as:-

$V = \alpha u_1 + \alpha u_2 + \alpha u_3 + \dots + \alpha v_n$.

$\vec{\alpha} = (\alpha_1, \alpha_2, \dots, \alpha_n)$ coordinate of v with respect to a basis E.

$\rightarrow [v]_E$ or $v_E \Rightarrow [v]_E = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix}$.

* If E is a standard basis for a vector space $\mathbb{R}$ then $[v]_E = v$, $\forall v \in \mathbb{R}^n$.

---

* U is called transition matrix from the basis:-

$E = \{u_1, u_2\}$.

$$\begin{tikzcd}
{[u_1, u_2]} \arrow[r, "U_1"] \arrow[dr, dashed] & {[e_1, e_2]} \arrow[d, "U_2^{-1}"] \\
 & {[u_1, u_2]}
\end{tikzcd}$$

$S = U_2^{-1} U_1$

* $v_B = P_{B' \to B} [v]_{B'}$, $v_{B'} = P_{B \to B'} [v]_B$.

$\Rightarrow$ Transition matrices $\Rightarrow P_{B' \to B}$

from B' to B.

$P_{B \to B'} \Rightarrow [B' | B]$ RREF $\rightarrow [I | P_{B \to B'}]$. , $P_{B' \to B} \Rightarrow [B | B']$ RREF. $\rightarrow [I | P_{B' \to B}]$

---

* إذا أعطاني معادلات وحكالي transition $\Leftarrow$

$\begin{bmatrix} x \\ 1 \end{bmatrix} \Rightarrow [2x-1, 2x+1]$

$x = c_1(2x-1) + c_2(2x+1) \Rightarrow c_1, c_2$. طلّع

$1 = c_1(2x-1) + c_2(2x+1) \Rightarrow c_1, c_2$ طلّع

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* Let V be a finite dimensional vector space with $\dim V = n$, if $E[v_1, v_2, \dots, v_n]$, $F = [w_1, w_2, \dots, w_n]$ be two basis.
Then the transition matrix from the basis E into the basis F is the $n \times n$ nonsingular matrix.

$\boxed{1}$ $[v_1 + v_2 + \dots + v_k]_E = [v_1]_E + [v_2]_E + \dots + [v_k]_E$.

$\boxed{2}$ $v_1, v_2, \dots, v_k$ are linearly independent iff $[v_1]_E, \dots [v_k]_E$ are linearly independent.

* $[x]_F = S_{E \to F} [x]_E = (U_2^{-1} U_1) [x]_E$.

$\hookrightarrow$ $x = \underbrace{\alpha_1 x_1 + \alpha_2 x_2}$.

بوصل ال α تكونوا ال $[x]_F$.

3.6:row space and column space :

basis for the null space ⇒ 1. REF , 2- نساويها بالصفر , 3- $x_1, x_2, \dots$ بطلع قيم

1 Row space ⇒ 1. REF ⇒ U بصير عنا

↳ $R(U) = R(A)$ = the nonzero rows is a basis for $R(A)$ قيم بتكون U = قيم بتكون A

بـ هاد بـ الـ Row space .

2 Column space ⇒ 1. REF ⇒ U بصير عنا ⇒ بـ نفس الترتيب بـ A بـ القيم $\neq$ U بـ القيم اللي بـ

is a column space. لـ نفترض أول عمودين في الـ U is column space بتكون أول عمودين في A

3 The nullity ⇒ $Null(A) = \dim N(A)$.

4 rank of A ⇒ $rank(A) = \dim(A)$.

* $Rank(A) + Null(A) = n$, ⇒ $rank + \dim(N(A)) = n$.

* dependency relation ⇒ إذا أكتب العمود أو أي عمود ما دخل الـ Row space كمل معادلة وهاد بحسب من الـ U

وبس أروح للـ A.

* If $m \times n$ matrix, the dimension of the row space of A = the dimension of the columns space of A. ↳ $A_{3\times 2}$, rank = 2.

1 $Ax = b$ is consistent iff b is in the column space of A.

2 $Ax = b$ is consistent iff the column vectors of A span R^m.

↳ The $Ax = b$ has at most one solution iff A is linearly independent

3 $n \times n$, A is nonsingular iff the column vector of A form a basis for R^n.

↳ $X = A^{-1} b$.

* إذا طلب الـ dimension يعني بطلع عدد الـ row space أو الـ column space.

↳ 1 REF.

كل الأعمدة اللي عندهم واحد هم الـ row space. dimension بتساوي عددهم

* إذا بتكون معطيني A و b وبدو أشوف إذا هي consistent وهل b هي من حل لـ الهم ⇐ $b = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$, $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ $a_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $a_2 = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

$x_1 a_1 + x_2 a_2 = b$.

$x_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + x_2 \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$ ⇒ وكلهم إذا طلع عندي أجوبة بتكون b ما طلع بتكون لا

* How many solutions will the linear system $Ax=b$ have if b is in the column space of A and the column vectors of A are linearly dependent.

↳ If $b \in C(A)$, then the system is consistent. If the column vector of A are linearly dependent. then the system will have infinitely many solutions.

---

* Let A be an $m \times n$ matrix with $m > n$, Let $b \in R^m$ and suppose that $N(A) = \{0\}$.

ⓐ What can you conclude about the column vector of A? Are they linearly independent? Do they span R^m? Explain

↳ $N(A) = \{0\} \Rightarrow$ Nulity $\Rightarrow \text{rank}(A) = n$. A has n columns and the dimension of its column space $= n \Rightarrow$ The columns are linearly independent. Dimension of $R^m = m$ and $m > n$. Therefore, columns of A can't span R^m.

ⓑ How many solutions will the system $Ax=b$ have if b is not in the column space of A? How many solutions will there be if b is in the column space of A?

↳ If b doesn't belong in the column space of A, system $Ax=b$ is inconsistent, and has no solutions. If b belongs in the column spaces, $Ax=b$ is consistent and has infinetly many solutions.

---

* Let A be 4×4 matrix with RREF given by $\Rightarrow$

$U = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ if $a_1 = \begin{bmatrix} -3 \\ 5 \\ 2 \\ 1 \end{bmatrix}$ and $a_2 = \begin{bmatrix} 4 \\ -3 \\ 7 \\ -1 \end{bmatrix}$ find a_3 and a_4.

$u_3 = 2u_1 + u_2$. | $u_4 = u_1 + 4u_2$.

$a_3 = 2a_1 + a_2$ | $a_4 = a_1 + 4a_2$

عوضها. | عوضها.

---

* A and B are $n \times n$ matrices $N(A-B) = R^n$ then $A = B$.

* $AB = 0$ iff the column space of B is a subspace of null space of A.

* $AB = 0$ then the sum of the ranks of A and B cannot exceed n.

$\boxed{4.1}$ Linear Transformation iff $L : V \Rightarrow W$

$\hookrightarrow$ $\boxed{1}$ $L(v_1 + v_2) = L(v_1) + L(v_2)$, $\forall v_1, v_2 \in V$.

$\boxed{2}$ $L(\alpha v) = \alpha L(v)$, $\forall v \in V$, $\forall \alpha \in R$.

$\Rightarrow$ If $V = W$ then $L : V \to V$ is said to be linear operator.

$\Rightarrow$ $L : V \to W$ is a linear Transformation iff :-

$L(\alpha v_1 + B v_2) = \alpha L(v_1) + B L(v_2)$.

$\Rightarrow$ Kernal and Images (Range) :-

$\Downarrow$

$L(v) = 0_w$

$\Downarrow$

إذا طلع الجواب صفر

one-one

$\Downarrow$

dim Kernal :-

بعوض الصورة الي معطيني اياها

بالسؤال بساوي الصفر وبوجد

القيم بعدها برجع للأصل نعوضها

بـ $L(v)$ والي طلع جزء خاص

مشترك.

$\Downarrow$

Example :- $L\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

لـ بنعوض صورة بعطيني

اياها بالسؤال

$\begin{pmatrix} x_1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Leftarrow$ إذا $x_1 = 0$

صفر $L\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ x_2 \end{pmatrix} = x_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

هاد هي kernal.

$\hookrightarrow$ $w = L(v)$

$R^3 \to R^2$

إذا كانت

$\textcircled{2}$ image وطلع عنا

تكون الجواب onto.

$\hookrightarrow$ $L\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ 0 \end{pmatrix} = x_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

هاد هي الـ Image.

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$\boxed{6.1}$ ⇒ Eigen values and Eigen vectors

λ $\qquad\qquad$ $A v = \lambda v$

⇒ λ قيمة ⇒ $|A - \lambda I_n| = 0$

⇒ $N(A - \lambda I) \neq \{0\}$, $A - \lambda I$ is singular.

$$\begin{vmatrix} 3-\lambda & 2 \\ 3 & -2-\lambda \end{vmatrix}$$ ⇐ نجيب الـ A ودائماً نحط λ في الـ main diagonal ⇐

ونوجد المحدد ونحل القيمة ونوجد قيمة الـ λ .

⇐ هسة الـ eigen values ⇐ [I] إذا طلعوا مختلفين القيم يعني $\lambda_1 = 4$ ، $\lambda_2 = -3$.

⇐ بعوض قيمة الـ λ في المحددة الأصلية مثلاً:

$$\begin{vmatrix} 3-4 & 2 \\ 3 & -2-4 \end{vmatrix} = \begin{vmatrix} -1 & 2 \\ 3 & -6 \end{vmatrix}$$

$$\left[\begin{array}{cc|c} -1 & 2 & 0 \\ 3 & -6 & 0 \end{array}\right]$$ ⇐ نوجد الـ $N(A - \lambda I)$

(RREF). ⇐ بوجد الـ null الطبيعي بيكون هادا الـ V_1 ونوجد لـ $\lambda_2 = -3$

[II] هسة إذا طلعوا الـ λ مختلفين ونفس القيمة بوجد الـ Null.

⇒ complex eigen value ⇒ $z = \alpha + \beta i$ ⇒ $\bar{z} = \alpha - \beta i$

⇐ conjugate (المرافق).

⇐ كل هاد لما إذا الـ طلع $\lambda^2 + 4 = 0$ ⇐ $\lambda = \mp 2i$.

⇐ لنفترض عندي ⇐ $\lambda_1 = 2 + i$ ، $\lambda_2 = 2 - i$.

⇐ ابدي أوجد الـ V بوجد لوحدة والثانية ~~بسط~~ مثلاً:

$v_1 = \begin{pmatrix} i \\ 5 \end{pmatrix}$ ⇐ لطالما

$v_2 = \begin{pmatrix} -i \\ 5 \end{pmatrix}$

$A\vec{v} = \lambda \vec{v}$, $i^2 = -1 \Rightarrow -i^2 = 1$

Trace denoted tr(A) : is the sum of all entries on the main diagonal.

$\Rightarrow |A| = \lambda_1 \lambda_2 \cdots \lambda_n$

$\Rightarrow tr(A) = \lambda_1 + \lambda_2 + \cdots + \lambda_n.$

يعني بوجد ال λ بالأول

$|A - \lambda I| = 0$

Let A be an nxn matrix, then A is singular if is an eigenvalue of A.

Let A be an nxn matrix, then A and A^T have the same eigenvalue.

$\Rightarrow A^n v = \lambda^n v \Rightarrow Av = \lambda v$

Eigenvalue for triangular : will be the main diagonal. $= \lambda$.

Let A be an nxn nonsingular matrix. If λ is an eigenvalue of A, then $\frac{1}{\lambda}$ is an eigenvalue for A with the same eigenvectors. $\Rightarrow A^{-1} v = \frac{1}{\lambda} v.$

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