Chapter 7.1, Problem 50E

Problem

Given a set S and a subset A, the **characteristic function of *A*,** denoted χa, is the function defined from S to **Z** with the property that for all $u \in S$,

$$\chi_A(u) = \begin{cases} 1 & \text{if } u \in A \\ 0 & \text{if } u \notin A. \end{cases}$$

Show that each of the following holds for all subsets A and B of S and all $u \in S$.

a. $\chi A \cap B(u) = \chi A(u) \cdot \chi B(u)$

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b. $\chi A \cap B(u) = \chi A(u) + \chi B(u) - \chi A(u) \cdot \chi B(u)$

Step-by-step solution

Step 1 of 3

Consider that the characteristic function χ_A from S to Z is defined as follows:

$$\chi_A(u) = \begin{cases} 1 & \text{if } u \in A \\ 0 & \text{if } u \notin A \end{cases}$$

(a)

If A and B are subsets of S, then,`

$$\chi_{A\cap B}(u) = \begin{cases} 1 & \text{if } u \in A \cap B \\ 0 & \text{if } u \notin A \cap B \end{cases}$$

Now,

$$u \in A \cap B$$
$$u \in A \text{ and } u \in B$$
$$\chi_A(u) = 1 \text{ and } \chi_B(u) = 1$$
$$\begin{aligned} \chi_A(u) \cdot \chi_B(u) &= 1 \cdot 1 \\ &= 1 \end{aligned}$$
$$\begin{aligned} \chi_{A\cap B}(u) &= 1 \\ &= 1 \cdot 1 \\ &= \chi_A(u) \cdot \chi_B(u) \qquad \text{............}(1) \end{aligned}$$

Step 2 of 3

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If $u \notin A \cap B$, then $u \notin A$ or $u \notin B$.

$\chi_A(u) = 0$ or $\chi_B(u) = 0$

$\chi_A(u) \cdot \chi_B(u) = 0$

When, $u \notin A \cap B$,

$$\begin{aligned}\chi_{A\cap B}(u) &= 0 \\ &= 0 \cdot 1 \\ &= 1 \cdot 0 \\ &= 0.0\end{aligned}$$

$= \chi_A(u) \cdot \chi_B(u)$ ....(2)

From equations (1) and (2), obtained as,

$\boxed{\chi_{A\cap B}(u) = \chi_A(u) \cdot \chi_B(u)}$.

Step 3 of 3

(b)

Consider that the characteristic function, $\chi_{A\cup B}$ from *S* to *Z* is defined as follows:

$$\chi_{A\cup B}(u) = \begin{cases} 1 & \text{if } u \in A \cup B \\ 0 & \text{if } u \notin A \cup B \end{cases}$$

Now, $\chi_{A\cup B}(u) = 1$

$u \in A \cup B$

$u \in A$ or $u \in B$

$\chi_A(u) = 1$ or $\chi_B(u) = 1$

$\chi_A(u) + \chi_B(u) - \chi_A(u)\chi_B(u) = 1$

Therefore,

$\boxed{\chi_{A\cup B}(u) = \chi_A(u) + \chi_B(u) - \chi_A(u)\chi_B(u)}$.

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