STAT2361

ملاحظات المحاضرات

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BZU-HUB

Chapter 4

ملاحظة: المحاضرات من شرح الدكتور محمد مضية

Chapter 4 - Introduction To Probability

1. Probability : Numarical measure of the likelihood (Chance) that an (uncertain) event will occur (happen)

   - Probability = measure of uncertainty

2. Experiment : Any process generates a well-defined outcomes

   - Random ← المطلوبة
   - Non-Random ← نتائجها معروفة وثابتة

3. Random :

   - Toss a Coin
   - Roll a die (حجر النرد)

4. Sample space : S

   The set of all possible outcomes in a randome experiment

   $$S = \{s_1, s_2, \dots s_n\} \qquad S_i : \text{Sample point}$$

* Examples :

One - Step

$\boxed{1}$ Toss a Coin

$S = \{\text{Head}, \text{Coin}\} = \{H, T\}$

$\boxed{2}$ Roll a die

$S = \{1, 2, 3, 4, 5, 6\}$

$\boxed{3}$ Play a football game

$S = \{\text{loss}, \text{win}, \text{tie}\}$

4. Select a student at random of your class (name)

$S = \{\text{All names}\}$

B. Event (المجموعة الجزئية): any subset of the sample space

*Ex: Roll a die $\quad S = \{1, 2, 3, 4, 5, 6\}$

- define the event $A \rightarrow$ an even number is observed

$A = \{2, 4, 6\}$

- define the event $B \rightarrow$ a number greater than 6 is observed

$B = \{\ \}$ $\emptyset \rightarrow$ impossible event

empty

* Multi-Steps Experiments ≥ 2 Steps

$\rightarrow \#S = ?$

Counting Rules

1. **Multiplication Rule:** An experiment consists of a sequence of k-steps $(k \geq 2)$ and:

Step 1 has n_1 outcomes

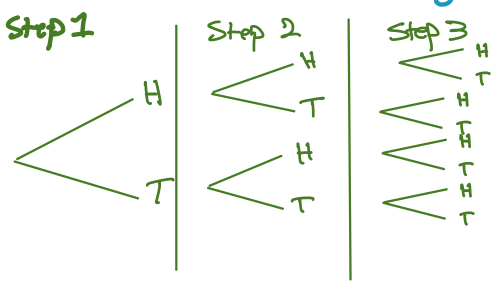
⋮

Step k has n_k outcomes

↳ Then the total number of all outcomes is: $(n_1) * (n_2) \dots (n_k)$

*Examples:

Tree Diagram



1. Toss 2 coins

$(2) \cdot (2) = 4$

$S = \{HH, HT, TH, TH\}$

2. Toss a coin 3 times

$S = \{HHH, HHT, HTH \dots TTT\}$

3. Toss a coin 6 times

$\#S = 2^6 = \underline{64}$

4 Roll 2 dice (a die 2 times)

$6 \cdot 6 = 36$ (ordered pairs)

5 Toss a coin and roll a die

$\# S = (2)(6) = 12$

* Select $\underline{r}$ objects from $\underline{n}$ objects $r \leq n$

order → Permutation

no order → Combination

$n!$ = The Factorial of n

$0! = 1$

* Combination (No order):

The number of $\underline{n}$ objects taken $\underline{r}$ at a time, nCr, $\binom{n}{r}$

$$nCr = \frac{n!}{(n-r)!\,r!}$$

*Ex: Select $\underline{3}$ students at random from your class of 30 students.

$30C3$ (nCr بالآلة الحاسبة)

$= 4060$

* Permutations (Selecting with order):

The number of permutations of $\underline{n}$ objects taken $\underline{n}$ at a time

*Ex: نفس الي فوق بس مرتب

$_{30}P_3$ (برضه من الحاسبة)

$= 24360$

*Ex: Consider the following names (class of 5 times)

A, B, C, D, E

1. Select $\underline{3}$ students at random from this class $\binom{n=5}{r=3}$

\# of combinations $= 5C3 = 10$

- list of all possible combination:

ABC	ADE
ABD	BCD
ABE	BDE
ACD	BCE
ACE	CDE

② Select 3 Students for : President , Vice presetent, member

5 = كم طالب؟ ; 4 ; member → 3

order

$_5P_3 = 60$ $\qquad$ $5 * 4 * 3 = 60$

4.1 : Probability Methods

* $P(A)$ = The Probability that event (A) will occur (happen)

→ 2 Rules:-

① $0 \leq P(A) \leq 1$

$P(\emptyset) = 0$ (impossible)

$P(S) = 1$ (Certain)

② $\sum P(A_i) = 1$

A_i = Disjoint events

* 3 Methods to assign probability:

① The classical Method

Let $S = \{S_1, S_2 \ldots S_i\}$ sample space for a random expirement

Assum that all sample points are equally likely (the same probability)

$P(S_i) = \frac{1}{n}$

↳ If A is an event of S, then:

$$P(A) = \frac{\text{\# of Sample Points in } A}{n}$$

* Ex :

① Toss (a fair) Coin

$P(H) = P(T) = \frac{1}{2}$ $\qquad$ $S = \{H, T\}$

② Roll (fair) die

$S = \{1, 2, 3, 4, 5, 6\}$

$P(1) = P(2) = \ldots = P(6) = \frac{1}{6}$

③ Roll 2 dice

$(6).(6) = 36$ pairs

Define event A = Sum of the 2 faces is 8, find (A)

$A = \{(4,4), (5,3), (3,5), (2,6), (6,2)\}$

$p(A) = \frac{5}{36}$

② The Relative Frequency Method

Suppose that an exprinment repeted n-times, event A repeated f times, the probability of $A = p(A) =$ Relative frequency of $A = \frac{f}{n}$.

*Ex:

In your class, # of Students is $= 30$, the number of female Students $= 18$, select a student at random

$p(F) = \frac{18}{30}$, $P(M) = \frac{12}{30}$

③ The subjective Method

4.2: Events and their Probabilities

* Event $\subseteq$ Sample space

مجموعة جزئية

Q14, P(154): 4 equally likely outcomes

① $P(E_2) = \frac{1}{4}$

② any $2 = \frac{2}{4} = \frac{1}{2}$

③ any $3 = 3/4$

4.3 : Probability Rules

① $P(A') \rightarrow$ The probability that A will not occur

$$P(A') = 1 - P(A)$$

مش عكس يصيروا مع بعض

② $P(A \cap B) = P(A \text{ and } B)$

= the probability that both event will occure

- If A, B are disjoint $\Rightarrow P(A \cap B) = P(\emptyset) = 0$
- If $P(A \cap B) = 0$, then A, B are called mutually exclusive

حادثيه منفصليه ← exclusive — مش ممكن يصيرو مع بعض

③ $P(A \cup B) = P(A \text{ or } B)$

↳ The probability that A or B will occur

A will occure or B will occur or both will occur.
(at least one event will occur)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

→ اصلاً صفر

إذا مش موجودة فبكون $P(A) + P(B)$

* $(A \cup B)' \Rightarrow A' \cap B'$
* $(A \cap B)' \Rightarrow A' \cup B'$

المراجعة :

- Venn Diagram

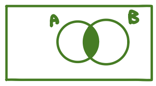
- operations

* Complement A', A^c

تقاطع

* Intersection

$A \cap B$



$A \cap B = \emptyset \Rightarrow$ Disjoint

اتحاد

* Union : $A \cup B$

Ex :

if $P(A) = 0.6$, $P(B) = 0.8$, $P(A \cap B) = 0.5$

مجموعة أكبر من واحد ← الاتحاد

① $P(A \cap B) \neq 0 \Rightarrow A, B$ are not mutually exclusive (الهم علاقة ببعض)

② $P(B') = 1 - P(B) = 0.2$

③ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.6 + 0.8 - 0.5$

$= 0.9$

④ $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B) = 1 - 0.9 = 0.1$

Ex :-

A = The 2 faces are the same (identical)

$= \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

① $P(A) = \frac{6}{36}$

② B = the sum of the 2 faces is 10

$= \{(5,5), (6,4), (4,6)\}$

$P(A) = \frac{3}{36}$

③ C = The sum of the 2 faces is 9

$= \{(5,4), (4,5), (3,6), (6,3)\}$

$P(C) = \frac{4}{36}$

④ $P(A') = 1 - P(A) = 1 - \frac{6}{36} = \frac{30}{36}$

شو الاشي المشترك ← $A \cap B = \{(5,5)\} \Rightarrow P(A \cap B) = 1/36$

⑤ $P(A \cup B) = P(A) + P(B) + P(A \cap B)$

$= \frac{6 + 3 - 1}{36} = \frac{8}{36}$

⑥ Give an example of 2 mutually exclusive events

$A \cap C = \emptyset$, $B \cap C = \emptyset$

⑦ $P(A \cup C) = P(A) + P(C) + 0$

$= \frac{6 + 4}{36} = \frac{10}{36}$

واجب من الدكتور

Ex: Roll a die 3 times

① Find $\#(S)$

$= 6^3 = 216$

② Find the probability of each sample point

$= \frac{1}{n} = \frac{1}{216} = 0.005$

③ Define A = Sum of the 3 faces is at least 4, find P(A)

$A = \{(1,1,2), (1,2,1), (2,1,1),\}$

$P(A) = \frac{3}{216} = 1.4\%$

4.4: Conditional Probabilities

Let A and B be two events:

- The Conditional Probability of A given B is: احتمالية حدوث A إذا B حدث

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

شرط ، تقاطع

- The Conditional Probability of B given A is:

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

- The events A and B are independent if:

$$P(A \cap B) = P(A) . P(B)$$

otherwise, they are dependent

- If A and B are independent, then

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)\, P(B)}{P(B)} = P(A)$$

$$P(B \mid A) = P(B)$$

- The events A and B are mutually exclusive if:

$P(A \cap B) = 0$ ← واحد منهم بكون صفر

***Example:** Suppose that we have two events A and B, with $P(A) = 0.2$, $P(B) = 0.4$, $P(A \cap B) = 0.08$

$\boxed{a}$ Find $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.08}{0.4} = 0.2$

$\boxed{b}$ Find $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{0.08}{0.2} = 0.4$

$\boxed{c}$ Are A and B independent? Why or why not?

بناخذ أي وحدة
من الحدود والأفهم

Yes, because $P(A \mid B) = P(A)$
$P(B \mid A) = P(B)$

* Independent events:

- A, B are independent if:
  $P(A|B) = P(A)$
  $P(B|A) = P(B)$

- A, B are independent if:
  $P(A \cap B) = P(A) \cdot P(B)$

*** Ex:** $P(A) = 0.6$, $P(B) = 0.4$, $P(A \cap B) = 0.3$
are A, B indepent? and why?

$$P(A|B) \overset{?}{=} P(A) \Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{0.3}{0.4}$$

$$= 0.75 \neq P(A) = 0.6$$

$\therefore$ are not independent.

*** Ex:** $P(A) = 0.6$, $P(B) = 0.4$, $P(A \cup B) = 0.76$, are A, B independent?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.76 = 0.6 + 0.4 - \underbrace{P(A \cap B)}$$

0.24 = المطلوب

- $P(A \cap B) = 0.24 \neq 0 \Rightarrow$ not mutually exclusive
  $0.24 = (0.4) \cdot (0.6)$
  $\Rightarrow$ A, B are independent

*** Ex:** $P(A) = 0.45$, $P(B) = 0.37$, find $P(A \cup B)$ if:

$\boxed{1}$ A, B are mutually exclusive $\rightarrow P(A \cap B) = 0$
$P(A \cap B) \overset{?}{=} 0$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.45 + 0.37 - 0 = \boxed{0.82}$$

$\boxed{2}$ A, B are independent

$$P(A \cap B) = P(A) \cdot P(B)$$

$$= 0.1665$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.45 + 0.37 - 0.1665$$

$$= 0.6535$$

* Example: (Q 32 page 168) sample data representative of the national health insurance coverage are shown here :

		Health Insurance Yes	Health Insurance No	Total
Age	18-34	750	170	920
	35 and older	950	130	1080
	Total	1700	300	2000

a) Develope a joint probability table for these data

Health Insurance

Joint probabilities

		Yes	No	Total
Age	18-34	$\frac{750}{2000} = 0.375$	0.085	0.46
	35 and older	0.476	0.065	0.54
	Total	0.850	0.150	1.00

Marginal probabilities

b) What do the marginal probabilities tell you about the age of the U.S population ?

46% of the population are within age 18-34
54% of the population are 35 and older

c) What is the probability that a randomly selected individual does not have health insurance ?

$$0.15 = \frac{300}{2000}$$

d) If the invidual is between the ages 18-34, what is the probability that the invidual does not have health insurance?

$$P(No \mid 18\text{-}34) = \frac{0.085}{0.46} = 0.1848$$

e) If the invidual age is 35 or older, what is the probability that the invidual does not have health Insurance?

$$P(No \mid 35 \text{ or older}) = \frac{0.065}{0.54} = 0.1204$$

f) If the invidual does not have health Insurance, What is the probability that the invidual is in 18-34 age?

$$P(18\text{-}34 \mid No) = \frac{0.085}{0.150} = 0.567$$

g) What does the Probability information tell you about the health Insurance in U.S?

From d and e, we see higher probability of (No) for (18-34) ages