4.9: Expectations of functions of Random variables.

exp 2: $X_1, \ldots, X_n$ with mean $\mu_1, \ldots, \mu_n$ and variance $\sigma_1^2, \ldots, \sigma_n^2$

Define $Y = \sum_{i=1}^{n} k_i X_i$, $k_i \in \mathbb{R}$.

→ $E(Y) = E(k_1 X_1 + \cdots + k_n X_n)$

$= k_1 E(X_1) + \cdots + k_n E(X_n)$.

$= k_1 \mu_1 + \cdots + k_n \mu_n$.

$$E(Y) = \mu_y = \sum_{i=1}^{n} k_i \mu_i .$$

→ $Var(Y) = Var(k_1 X_1 + \cdots + k_n X_n)$

$= Var\left(\sum_{i=1}^{n} k_i X_i\right)$

$= E\left[\left(\sum_{i=1}^{n} k_i X_i - \sum_{i=1}^{n} k_i \mu_i\right)^2\right]$

$= E\left[\left(\sum_{i=1}^{n} k_i (X_i - \mu_i)\right)^2\right]$

$= E\left[\left(\sum_{i=1}^{n} k_i (X_i - \mu_i)\right)\left(\sum_{i=1}^{n} k_i (X_i - \mu_i)\right)\right]$

$= E\left[\left(\sum_{i=1}^{n} k_i (X_i - \mu_i)\right)\left(\sum_{j=1}^{n} k_j (X_j - \mu_j)\right)\right]$

$= E\left[\sum_{j=1}^{n} \sum_{i=1}^{n} k_i k_j (X_i - \mu_i)(X_j - \mu_j)\right]$

$= \sum_{j=1}^{n} \sum_{i=1}^{n} k_i k_j \left(E(X_i - \mu_i)(X_j - \mu_j)\right)$

→

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$$Var(Y) = \sum_{j=1}^{n} \sum_{i=1}^{n} k_i k_j \; cov(X_i, X_j).$$

$$Var(Y) = \sum_{j=1}^{n} \sum_{i=1}^{n} k_i k_j \; \rho_{ij} \, \sigma_i \sigma_j .$$

→ Assuming $X_1, \dots, X_n$ indep.

$$Var(Y) = \sum_{i=1}^{n} k_i^2 \sigma_i^2 .$$

since it indep.

$$\rho_{ij} = \begin{cases} 0 , & i \neq j \\ 1 , & i = j \end{cases} .$$

→ Assuming $X_1, \dots, X_n$ r.v's with finite means and variances.

$$Var(Y) = \sum_{i=1}^{n} k_i^2 \sigma_i^2 + 2 \sum_{j=1}^{n} \sum_{\substack{i=1 \\ i<j}}^{n} k_i k_j \, \sigma_i \sigma_j \, \rho_{ij} .$$

Thm 5: $X_1, \dots, X_n$ r.v's with means $\mu_1, \dots, \mu_n$ and variances $\sigma_1^2, \dots, \sigma_n^2$.

Define $Y = \sum_{i=1}^{n} k_i X_i$, $k_i \in \mathbb{R}$ then

1. $E(Y) = \mu_Y = \sum_{i=1}^{n} k_i \mu_i$

2. $Var(Y) = \sigma_Y^2 = \sum_{i=1}^{n} k_i^2 \sigma_i^2 + 2 \sum_{j=1}^{n} \sum_{i=1}^{n} k_i k_j \, \sigma_i \sigma_j \, \rho_{ij}$

where $\rho_{ij} = \dfrac{cov(X_i, X_j)}{\sigma_i \sigma_j}$.

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Corollary: $X_1, \ldots, X_n$ random sample with mean μ and variance σ^2

Define $Y = \sum_{i=1}^{n} k_i X_i$, $k_i \in \mathbb{R}$ then

1. $E(Y) = \mu \left(\sum_{i=1}^{n} k_i \right)$.

2. $Var(Y) = \sigma^2 \left(\sum_{i=1}^{n} k_i^2 \right)$.

exp 3: if $\bar{X} = \frac{\sum X_i}{n}$ and $X_1, \ldots, X_n$ Random sample from $N(\mu, \sigma^2)$

$\rightarrow E(\bar{X}) = \mu \left(\sum_{i=1}^{n} \frac{1}{n} \right) = \mu \left(\frac{1}{n} \cdot n \right) = \mu = \mu_{\bar{X}}$

$\rightarrow Var(\bar{X}) = \sigma^2 \left(\sum_{i=1}^{n} \frac{1}{n^2} \right) = \sigma^2 \left(\frac{1}{n^2} \cdot n \right) = \frac{\sigma^2}{n} = \sigma_{\bar{X}}^2$.

Done ... ♡

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