

(1.2) Exercises

Find domain and range for f , g , $f+g$ and $f \cdot g$

1) $f(x) = x$, $g(x) = \sqrt{x-1}$

$D: [1, \infty)$

$D: f(x) \Rightarrow (-\infty, \infty)$, $D: g(x) \Rightarrow [1, \infty)$, $D: (f+g)(x) = x + \sqrt{x-1} \Rightarrow D: [1, \infty)$, $(f \cdot g)(x) = \sqrt{x-1} \cdot x$

2) $f(x) = \sqrt{x+1}$, $g(x) = \sqrt{x-1}$

$D: (-\infty, -1) \cup (1, \infty)$

$D: f(x) \Rightarrow [-1, \infty)$, $D: g(x) \Rightarrow [1, \infty)$, $D: (f+g)(x) = \sqrt{x+1} + \sqrt{x-1} \Rightarrow [1, \infty)$, $D: (f \cdot g)(x) = \sqrt{(x-1)(x+1)}$

Find domains and ranges of f , g , f/g , and g/f !!

3) $f(x) = 2$, $g(x) = x^2 + 1$

$D: f(x) \Rightarrow \mathbb{R}$, $D: g(x) \Rightarrow \mathbb{R}$, $D: \frac{f}{g} \Rightarrow \mathbb{R}$, $D: \frac{g}{f} \Rightarrow \mathbb{R}$

4) $f(x) = 1$, $g(x) = 1 + \sqrt{x}$

$D: f(x) \Rightarrow \mathbb{R}$, $D: g(x) \Rightarrow [0, \infty)$, $D: (\frac{f}{g})(x) = \frac{1}{1+\sqrt{x}} \Rightarrow [0, \infty)$, $D: (\frac{g}{f})(x) \Rightarrow 1 + \sqrt{x} ; [0, \infty)$

5) if $f(x) = x+5$, $g(x) = x^2 - 3$ Find:

a. $f(g(0)) = f(0-3) = f(-3) = -3+5 = 2$ e. $f(f(-5)) = f(-5+5) = f(0) = 5$

b. $g(f(0)) = g(5) = 25 - 3 = 22$ f. $g(g(2)) = g(2^2 - 3) = g(1) = 1 - 3 = -2$

c. $f(g(x)) = (x^2 - 3) + 5 = x^2 + 2$ g. $f(f(x)) = x + 5 + 5 = x + 10$

d. $g(f(x)) = (x+5)^2 - 3 = x^2 + 10x + 22$ h. $g(g(x)) = (x^2 - 3)^2 - 3 = x^4 - 6x^2 + 6$

6) if $f(x) = x-1$, $g(x) = \frac{1}{x+1}$ Find:

a. $f(g(\frac{1}{2})) = f(\frac{2}{3}) = -\frac{1}{3}$ e. $f(f(2)) = f(1) = 0$

b. $g(f(\frac{1}{2})) = g(-\frac{1}{2}) = 2$ f. $g(g(2)) = g(\frac{1}{3}) = \frac{3}{4}$

c. $f(g(x)) = \frac{1}{x-1} + 1$ g. $f(f(x)) = x-1-1 = x-2$

d. $g(g(x)) = \frac{1}{\frac{1}{x-1} + 1} = \frac{x-1}{1+x-1} = \frac{x-1}{x}$ h. $g(g(x)) = \frac{1}{\frac{1}{x+1} + 1} = \frac{x+1}{1+x+1} = \frac{x+1}{x+2}$

in exercises (7-10) write formula $(f \circ g \circ h)(x)$

7) $f(x) = x+1, g(x) = 3x, h(x) = 4-x$

$f \circ g \circ h(x) = f(g(h(x))) = f(g(4-x)) = f(3(4-x)) = 12-3x+1 = 13-3x$

8) $f(x) = 3x+4, g(x) = 2x-1, h(x) = x^2$

$f \circ g \circ h(x) = f(g(h(x))) = f(g(x^2)) = f(2x^2-1) = 3(2x^2-1)+4 = 6x^2+1$

9) $f(x) = \sqrt{x+1}, g(x) = \frac{1}{x+4}, h(x) = \frac{1}{x}$

$f \circ g \circ h(x) = f(g(h(x))) = f(g(\frac{1}{x})) = f(\frac{1}{\frac{1}{x}+4}) = f(\frac{x}{4x+1}) = \sqrt{\frac{x}{4x+1}+1} = \sqrt{\frac{5x+1}{4x+1}} \quad x \neq -\frac{1}{4}$

10) $f(x) = \frac{x+2}{x-3}, g(x) = \frac{x^2}{x^2+1}, h(x) = \sqrt{2-x}$

$(f \circ g \circ h)(x) = f(g(h(x))) = f(g(\sqrt{2-x})) = f(\frac{2-x}{2-x+1}) = f(\frac{2-x}{3-x}) = \frac{\frac{2-x}{3-x}+2}{\frac{2-x}{3-x}-3} = \frac{2-x+2(3-x)}{2-x-3(3-x)} = \frac{2-x+2(3-x)}{2-x-9+3x} = \frac{8-3x}{2x-7} \quad x \neq 3$

press each functions in 11 $\Rightarrow$ 12 as a composite in volving one or more f, g, h, i, j

11) a. $y = \sqrt{x-3} \Rightarrow f(x) = x-3, g(x) = \sqrt{x} \quad (f \circ g)(x)$

b. $y = 2\sqrt{x} \Rightarrow f(x) = 2x, g(x) = \sqrt{x} \quad (f \circ g)(x)$

c. $y = x^4 \Rightarrow f(x) = x, g(x) = x^4 \quad (f \circ g)(x)$

d. $y = 4x \Rightarrow f(x) = 4x, g(x) = x \quad (f \circ g)(x)$

e. $y = \sqrt{x-3} \Rightarrow f(x) = \sqrt{x}, g(x) = x+3, h(x) = x-3 \quad (f \circ g \circ h)(x)$

f. $y = x^{\frac{3}{2}} \Rightarrow f(x) = \sqrt{x}, g(x) = x^3 \quad (f \circ g)(x)$

12) a. $y = 2x-3 \Rightarrow f(x) = x, g(x) = 2x-3$

13) a. $(f \circ g)(x) = \sqrt{x-7}, b. (f \circ g)(x) = 3x+6, c. g(x) = x^2+1$

d. $(f \circ g)(x) = \frac{x}{x-1} = \frac{x(x-1)}{x-(x-1)} = \frac{x^2-x}{1}, e. g(x) = \frac{1}{x-1}, f. f(x) = \frac{1}{x}$

14) a. $(f \circ g)(x) = \frac{1}{x-1}$

b. $(f \circ g)(x) = \frac{g(x)-1}{g(x)} = \frac{x}{x+1}$

$x \cdot g(x) - x + g(x) - 1 = x \cdot g(x)$

$g(x) = x+1$

c. $g(x) = x^2$

d. $f(x) = x^2$

15) a. $f(g(0)) = f(-1) = 3$

b. $g(f(3)) = g(-1) = 1$

c. $g(g(-1)) = g(1) = 0$

d. $f(f(3)) = f(0) = 2$

e. $g(f(0)) = g(2) = 1$

f. $f(g(\frac{1}{2})) = f(\frac{1}{2}) = \frac{5}{2}$

g. $\frac{1}{2} = (\frac{1}{2})^2 = (\frac{1}{2})^2$

h. $5-x = 4-1-x = 3-x$

i. $\frac{5-x}{1+x} = \frac{4-x}{1+x} = \frac{1}{1+x}$

j. $\frac{1}{x} = \frac{1}{1+x}$

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(1.2) exercises

17 → 18 / Write formulas for $(f \circ g)$, $(g \circ f)$ and find the domain and range?

17) $f(x) = \sqrt{x+1}$, $g(x) = \frac{1}{x}$

$(f \circ g)(x) = \sqrt{\frac{1}{x}+1} = \sqrt{\frac{1+x}{x}} \Rightarrow D: (-\infty, 0) \cup (0, \infty) \quad | \quad R: [0, \infty)$

$(g \circ f)(x) = \frac{1}{\sqrt{x+1}} \Rightarrow D: x > -1 \quad | \quad R: (-1, \infty)$

18) $f(x) = x^2$, $g(x) = 1 - \sqrt{x}$

$(f \circ g)(x) = (1 - \sqrt{x})^2$, $D: [0, \infty)$, $R: (-\infty, 1]$

$(g \circ f)(x) = 1 - |x|$, $D: x \geq 0$, $R: (-\infty, 1]$

19) Let $f(x) = \frac{x}{x-2}$, find $y = g(x)$ where $(f \circ g)(x) = x$

$(f \circ g)(x) = \frac{g(x)}{g(x)-2} = x \Rightarrow g(x) = x(g(x)-2) \Rightarrow g(x) - x(g(x)-2) = -2x \Rightarrow g(x) = \frac{2x}{x-1}$

20) Let $f(x) = 2x^3 + 4$ find a function $y = g(x)$ so $(f \circ g)(x) = x + 2$

$(f \circ g)(x) = 2g(x)^3 + 4 = x + 2$

$g(x) = \sqrt[3]{\frac{x-2}{2}}$

21) position (a): $(4x^2 + 7)^2 = (7+x)^2$, position (b): $(4-x^2)^2$

22) position (a): $y = x^2 + 3$, position (b): $y = x^2 - 5$

23) a. $y = (x-1)^2 - 4$ is position (4), b. $y = (x-2)^2 + 2$ is position (1), c. $y = (x+2)^2 + 2$ is position (2)

d. $y = (x+3)^2 - 2$ is position (3)

24) a: $f(x) = (1-x)^2 + 4$ b: $f(x) = (-2-x)^2 + 3$ c: $f(x) = (4-x)^2 - 1$ d: $f(x) = (2-x)^2$

25) → 34

25) $(x+2)^2 + (y+3)^2 = 49$

30) $y = -\sqrt{x-3}$

26) $(x+4)^2 + (y-3)^2 = 25$

31) $y = 2x$

27) $y = (x+1)^3 - 1$

32) $y = \frac{1}{2}(x) -$

28) $y = \sqrt{x+0.81}$

33) $y = \frac{1}{x-1} + 1$

29) $y = \sqrt[3]{(x-1)^2} - 1$

34) $y = \frac{1}{(x^2+2)^2} - 1$

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35) $y = \sqrt{x+4}$

36) $y = \sqrt{9-x}$

41) $y = (x+1)^{\frac{2}{3}}$

42) $y = (x-8)^{\frac{2}{3}}$

37) $y = 1-x-2$

38) $y = 1-x-1$

43) $y = 1-x^{\frac{2}{3}}$

44) $y = x^{\frac{2}{3}} - 1$

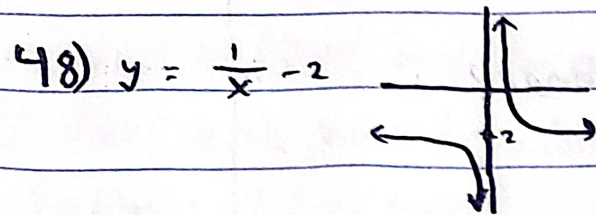
39) $y = 1+\sqrt{x-1}$

40) $y = 1-\sqrt{x}$

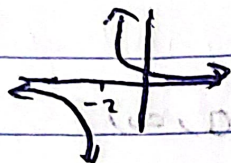
45) $y = (x-1)^{\frac{3}{2}} - 1$

46) $y = (x+2)^{\frac{3}{2}} + 1$

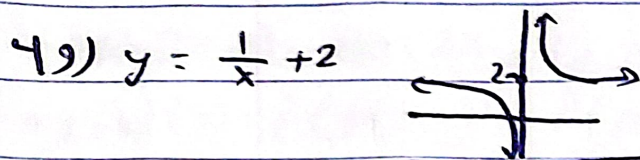
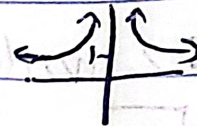
47) $y = \frac{1}{x-2}$



50) $y = \frac{1}{x+2}$



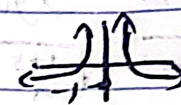
53) $y = \frac{1}{x^2} + 1$



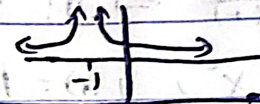
51) $y = \frac{1}{(x-1)^2}$



52) $y = \frac{1}{x^2} - 1$



54) $y = \frac{1}{(x+1)^2}$



- 55) a. $f(x) + 2 \Rightarrow D: [0, 2]$, $R: [2, 3]$, b. $f(x) - 1 \Rightarrow D: [0, 2]$, $R: [-1, 0]$
 c. $2f(x) \Rightarrow D: [0, 2]$, $R: [0, 2]$, d. $-f(x) \Rightarrow D: [0, 2]$, $R: [-1, 0]$
 e. $f(x+2) \Rightarrow D: [-2, 0]$, $R: [0, 1]$, f. $f(x-1) \Rightarrow D: [1, 3]$, $R: [0, 1]$
 g. $f(-x) \Rightarrow D: [-2, 0]$, $R: [0, 1]$, h. $-f(x+1) + 1 \Rightarrow D: [-1, 0]$, $R: [0, 1]$

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