

3.7 + 3.6 The Chain Rule

(66)

Theorem If f is differentiable at x and g is differentiable at $f(x)$, then $g \circ f$ is differentiable at x

$$y = (g \circ f)(x) = g(f(x))$$

$$f = \frac{dy}{dx} = (g \circ f)'(x) = g'(f(x)) f'(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$y = g(u) \\ u = f(x)$$

Example Find the derivatives of

[a] $x(t) = \cos(t^2 + 1)$

$$\frac{dx}{dt} = -\sin(t^2 + 1) (2t) = -2t \sin(t^2 + 1)$$

[b] $y = \sin(x^2 + x)$

$$y' = \cos(x^2 + x) (2x + 1) = (2x + 1) \cos(x^2 + x)$$

[c] $g(t) = \tan(5 - \sin 2t)$

$$\frac{dg}{dt} = \sec^2(5 - \sin 2t) (-\cos 2t (2))$$

STUDENTS-HUB.COM - 2 $\cos 2t \sec^2(5 - \sin 2t)$ Uploaded By: Malak Obaid

[d] $f(x) = (5x^3 - x^4)^7$

$$f' = 7(5x^3 - x^4)^6 (15x^2 - 4x^3)$$

[e] $y = \sin^5 x \Rightarrow y = [\sin x]^5$

$$y' = 5 \sin^4 x \cos x$$

$$[f] \quad h(x) = |x| = \sqrt{x^2} = (x^2)^{\frac{1}{2}} \quad (67)$$

$$h'(x) = \frac{1}{2} (x^2)^{-\frac{1}{2}} 2x$$

$$= \frac{x}{\sqrt{x^2}}, \quad x \neq 0$$

$$= \frac{x}{|x|}, \quad x \neq 0$$

$$[g] \quad y = \frac{1}{(1-2x)^3} = (1-2x)^{-3}$$

$$y' = -3 (1-2x)^{-4} (-2)$$

$$= \frac{6}{(1-2x)^4}$$
