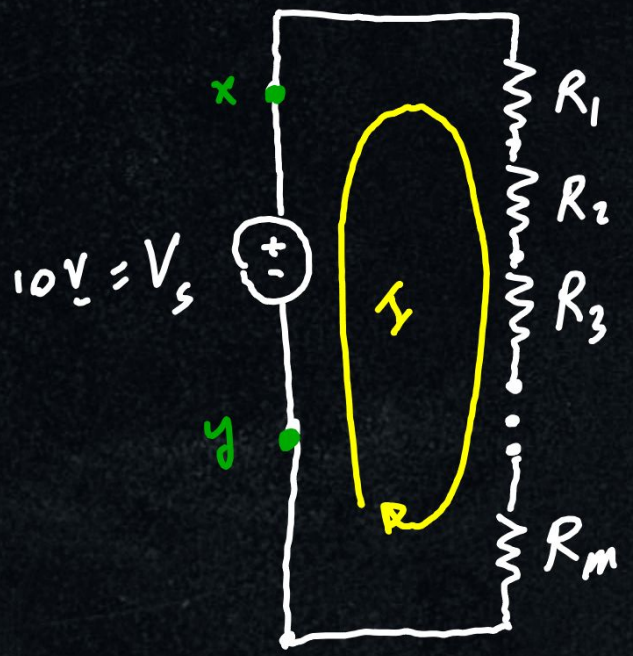
CH3: Simple Resistive Circuits

Resistors in series



$I = \frac{V_s}{R_{eq}}$.

series connected resistors carry the same current

Using KVL:

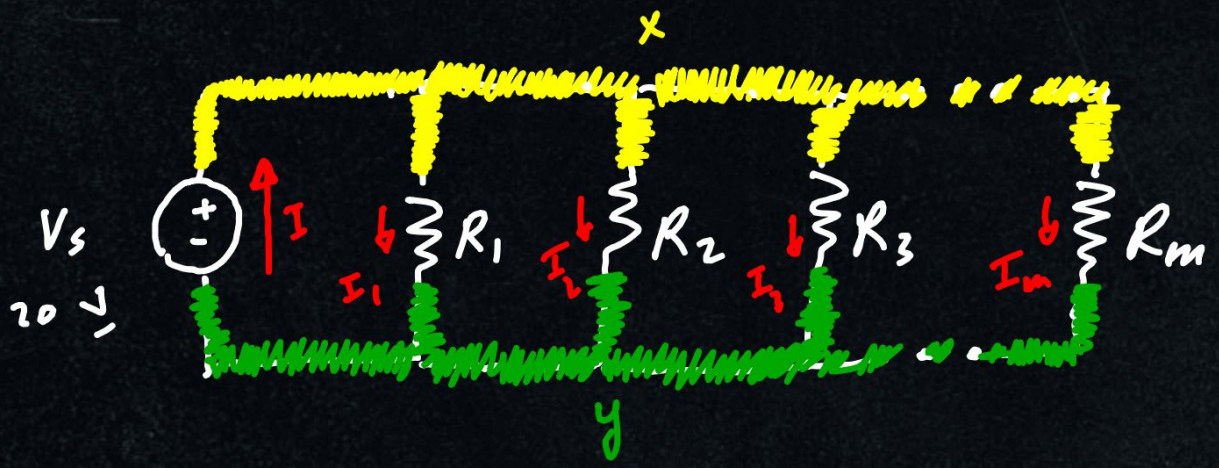
$$-V_s + R_1 I + R_2 I + R_3 I + \cdots + R_m I = 0$$

$$V_s = (R_1 + R_2 + \cdots + R_m) I$$

$$V_s = R_{eq} I$$

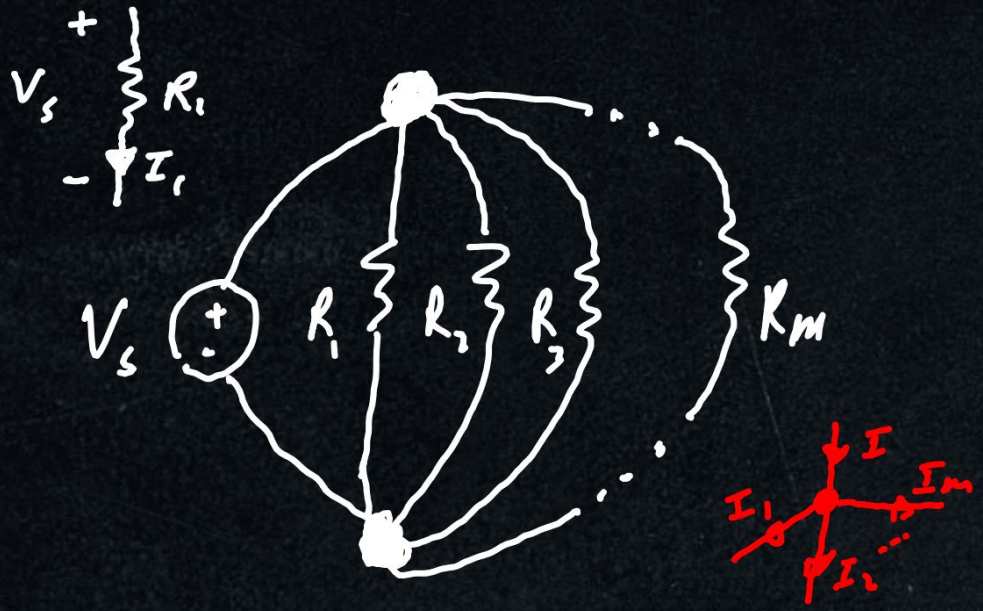
$$\boxed{R_{eq} = \sum_{i=1}^{m} R_i}$$

Resistors in parallel



parallel connected resistors have the same voltage across their terminal.

Number of nodes = 2

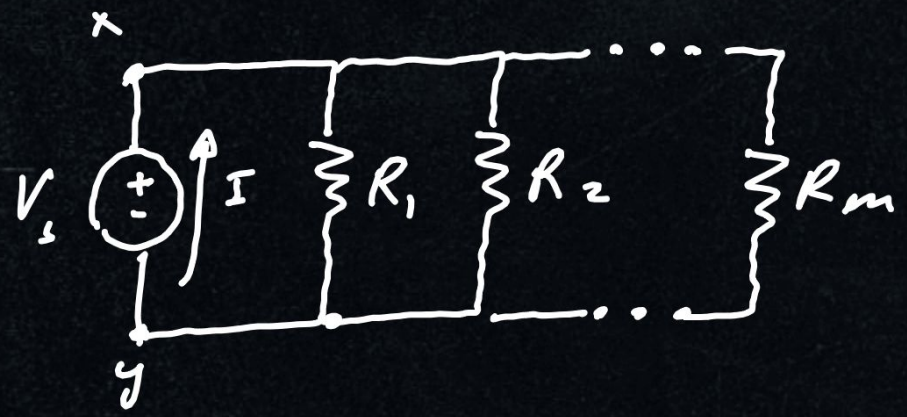


Based on using KCL:

$$\sum I\Big|_{\text{node x}} = 0$$

$$I_1 + I_2 + I_3 + \cdots + I_m - I = 0$$

$$\frac{V_s}{R_1} + \frac{V_s}{R_2} + \frac{V_s}{R_3} + \cdots + \frac{V_s}{R_m} = I$$

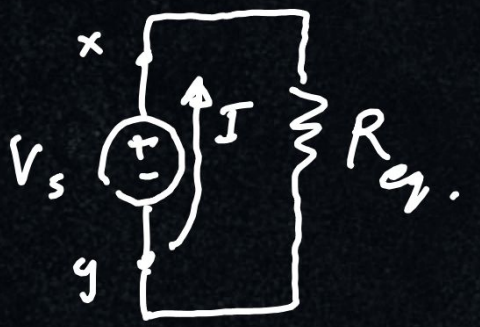


$$\frac{V_s}{R_1} + \frac{V_s}{R_2} + \frac{V_s}{R_3} + \cdots + \frac{V_s}{R_m} = I$$

$$V_s \left[\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_m} \right] = I$$

$$V_s = R_{eq} I$$

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_m}}$$



$$I = \frac{V_s}{R_{eq}}.$$

or

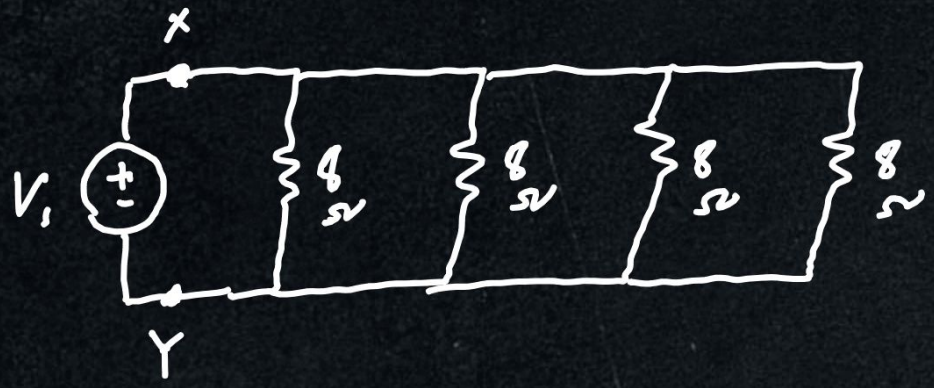
$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_m}}$$

1) If $m = 2$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$

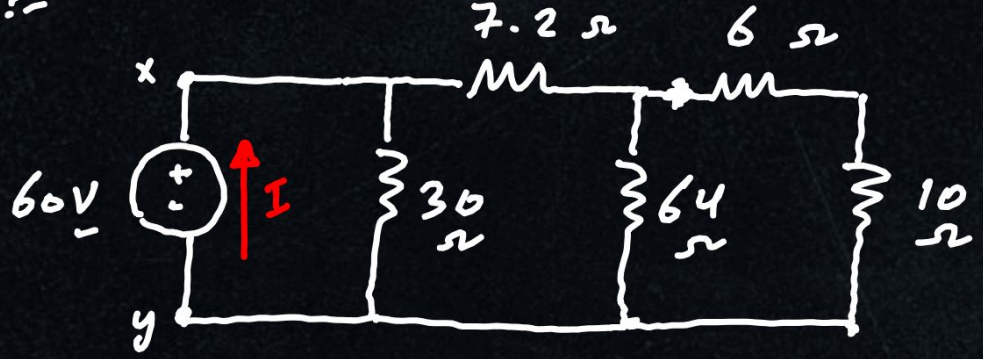
2) If $R_1 = R_2 = R_3 = \cdots = R_m = R$

$$\frac{1}{R_{eq}} = \underbrace{\frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \cdots + \frac{1}{R}}_{m \text{ terms}} = \frac{m}{R} \Rightarrow R_{eq} = \frac{R}{m}$$



$$R_{XY} = \frac{8}{4} = 2\ \Omega$$

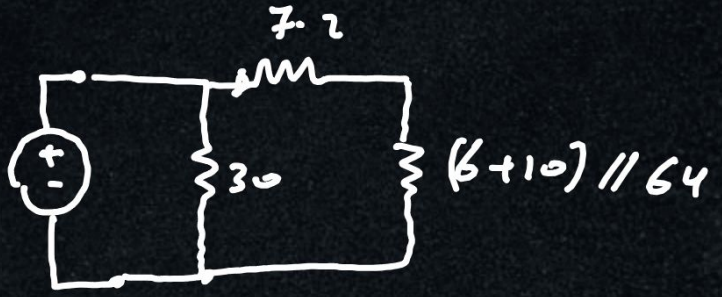
Ex :-



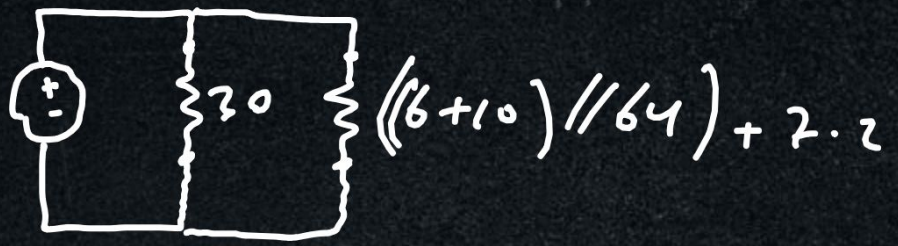
① Calculate R_{xy}

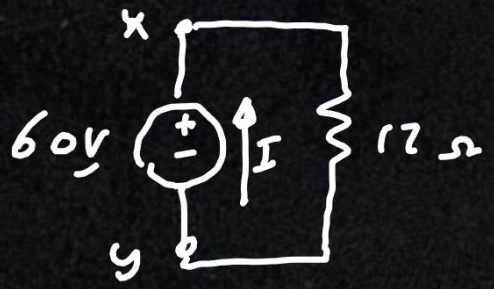
$$R_{xy} = \left[\left((6+10) \,//\, 64\right) + 7.2\right] \,//\, 30$$

12.8



$$R_{xy} = 12\ \Omega$$

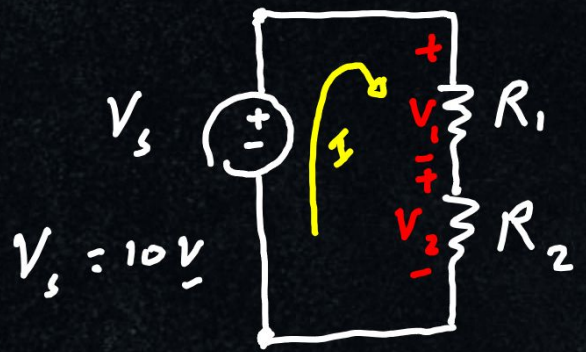




② Calculate I

$$I = \frac{60}{12} = 5A.$$

Voltage Divider



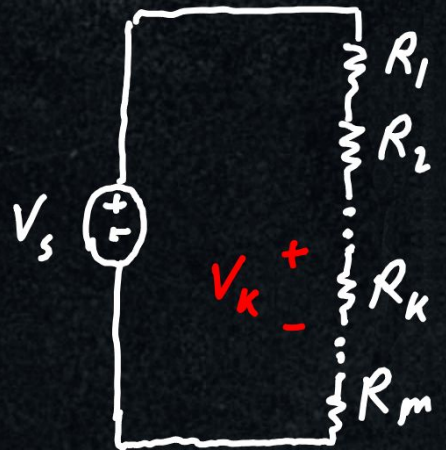
$$-V_s + R_1 I + R_2 I = 0$$

solve for I

$$I = \frac{V_s}{R_1 + R_2}$$

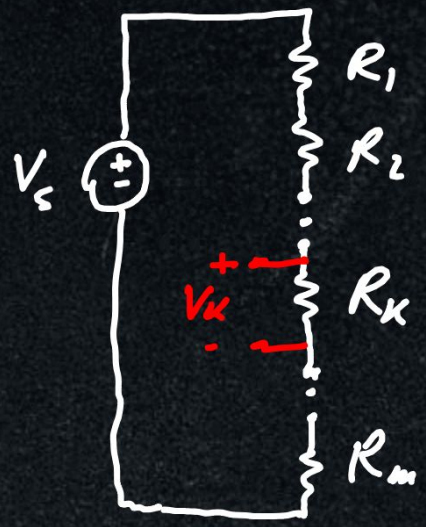
$$V_1 = R_1 I = \frac{R_1}{R_1 + R_2} V_s$$

$$V_2 = R_2 I = \frac{R_2}{R_1 + R_2} V_s$$

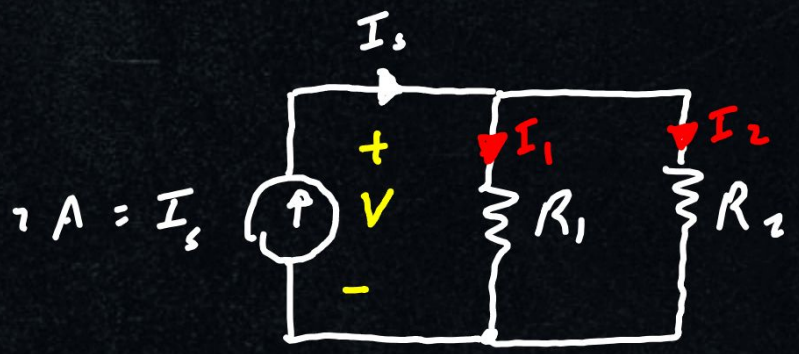


$k = 3, 4, \cdots, m-1$

$$V_k = \frac{R_k}{R_1 + R_2 + \cdots + R_m} \cdot V_s$$



Current Divider



$$V = R_1 I_1 = R_2 I_2 = \overbrace{(R_1 \| R_2)}^{R_{eq}} I_s$$

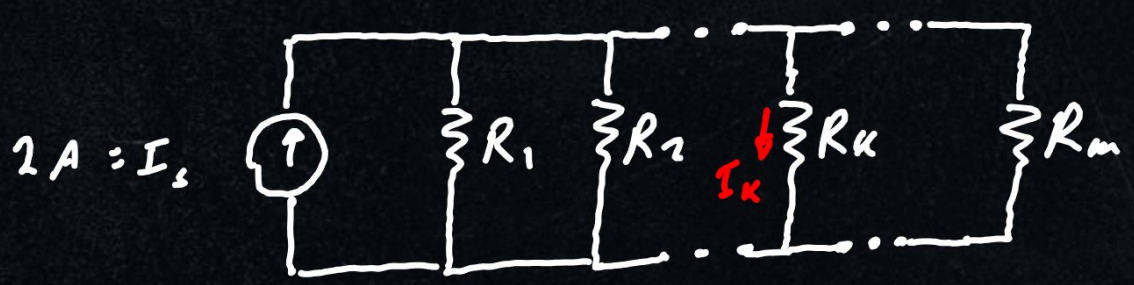
$$I_1 = \frac{R_{eq}}{R_1} \cdot I_s$$

$$I_2 = \frac{R_{eq}}{R_2} \cdot I_s$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$I_1 = \frac{R_2}{R_1 + R_2} \cdot I_s$$

$$I_2 = \frac{R_1}{R_1 + R_2} \cdot I_s$$

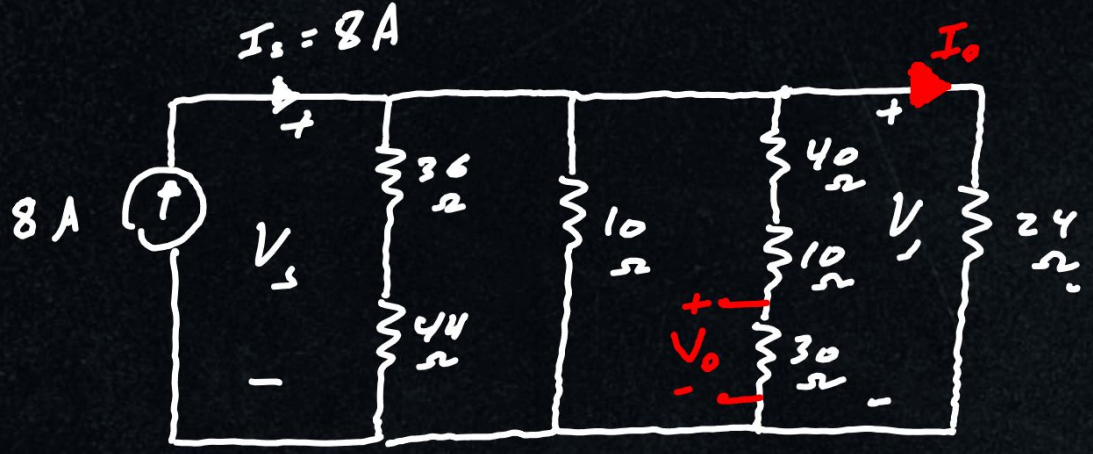


$k = 3, 4, \dots, m-1$

$$I_k = \frac{R_{eq}}{R_k} \cdot I_s \quad , \quad R_{eq} = R_1 // R_2 // \dots // R_m$$

$$= \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_m}}$$

EX:



Find I_o & V_o ?? using voltage & current dividers.

$$I_o = \frac{R_{eq}}{24} \cdot I_s \quad ; \quad R_{eq} = (36+44) \,//\, 10 \,//\, (40+10+30) \,//\, 24$$

$$R_{eq} = 80 \,//\, 10 \,//\, 80 \,//\, 24 = \frac{1}{\frac{1}{80}+\frac{1}{10}+\frac{1}{80}+\frac{1}{24}} = 6\ \Omega$$

$$I_o = \frac{6}{24}(8) = \boxed{2\ A}$$

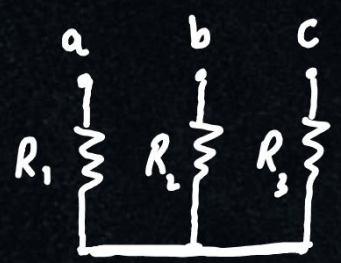
$$V_o = \frac{30}{30+10+40} \cdot V_s \quad ; \quad V_s = 8 R_{eq} = 2(24) = 48\ V$$

$$\Rightarrow V_o = \frac{30}{80} \cdot 48 = \boxed{18\ V}$$

Delta - Wye Equivalent Circuits

Δ Y

Y - connection



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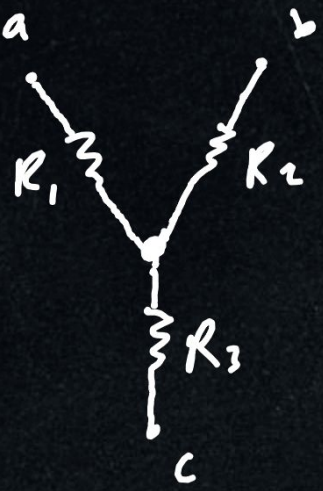
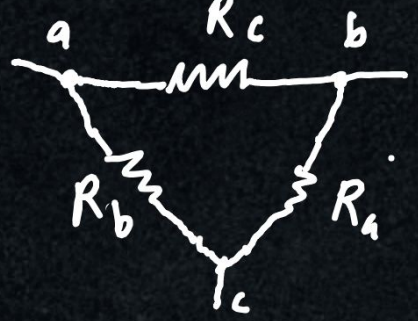


Y or star connection

Δ - connection



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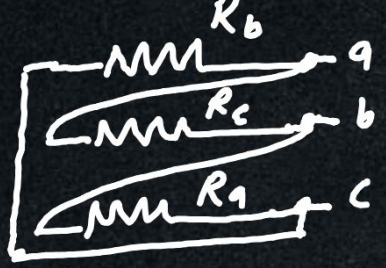
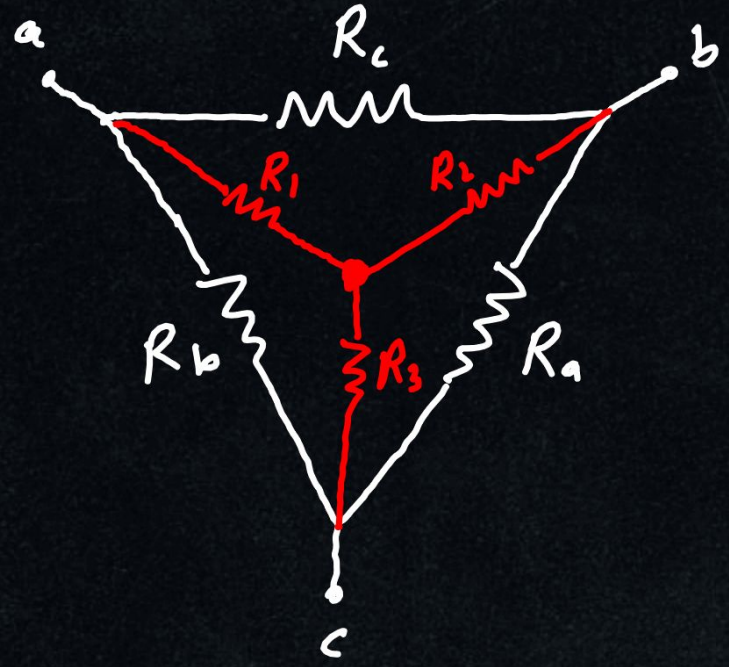
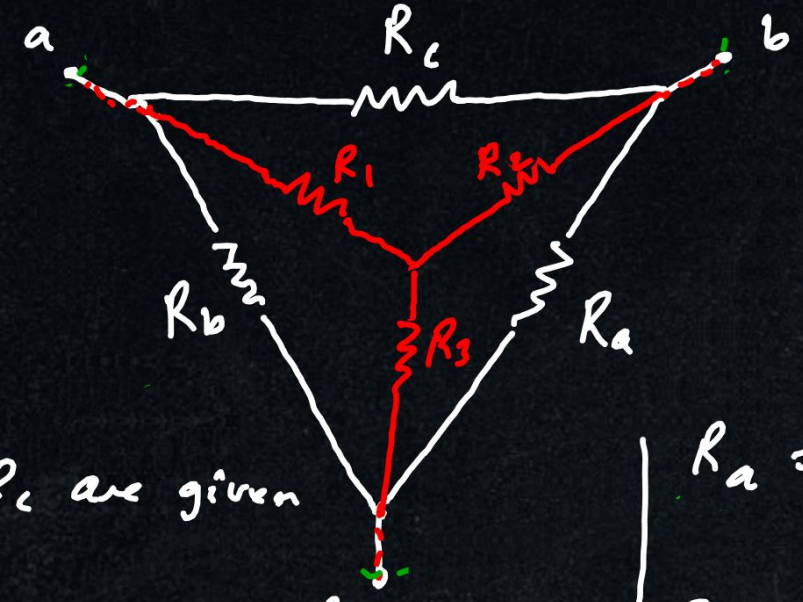




Y - Δ Transformation

Δ - Y Transformation



$$R_{ab} = R_1 + R_2 = (R_a + R_b) \parallel R_c$$

$$R_{bc} = R_2 + R_3 = (R_b + R_c) \parallel R_a$$

$$R_{ca} = R_1 + R_3 = (R_a + R_c) \parallel R_b$$

Δ → Y

$R_a, R_b + R_c$ are given

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c} \quad / \quad R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Y → Δ

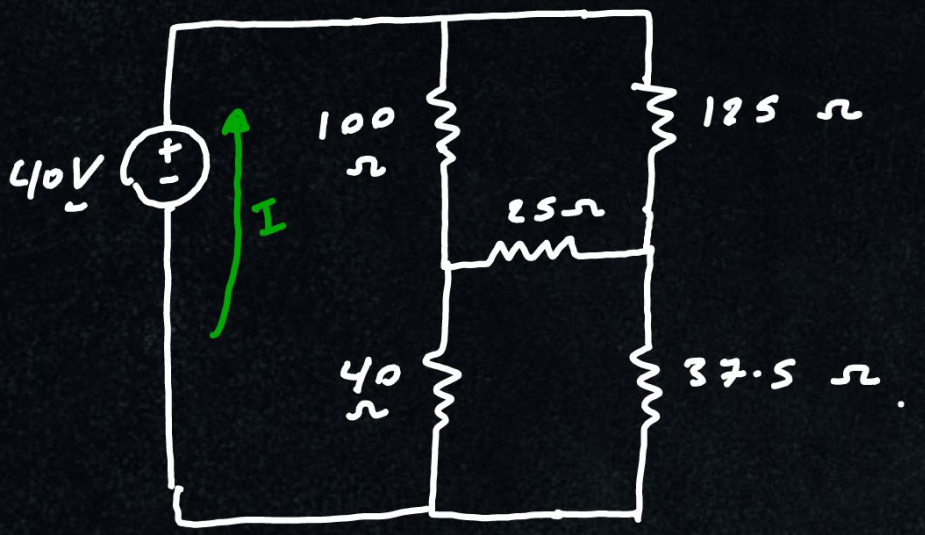
$R_1, R_2 + R_3$ are given

$$R_a = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1}$$

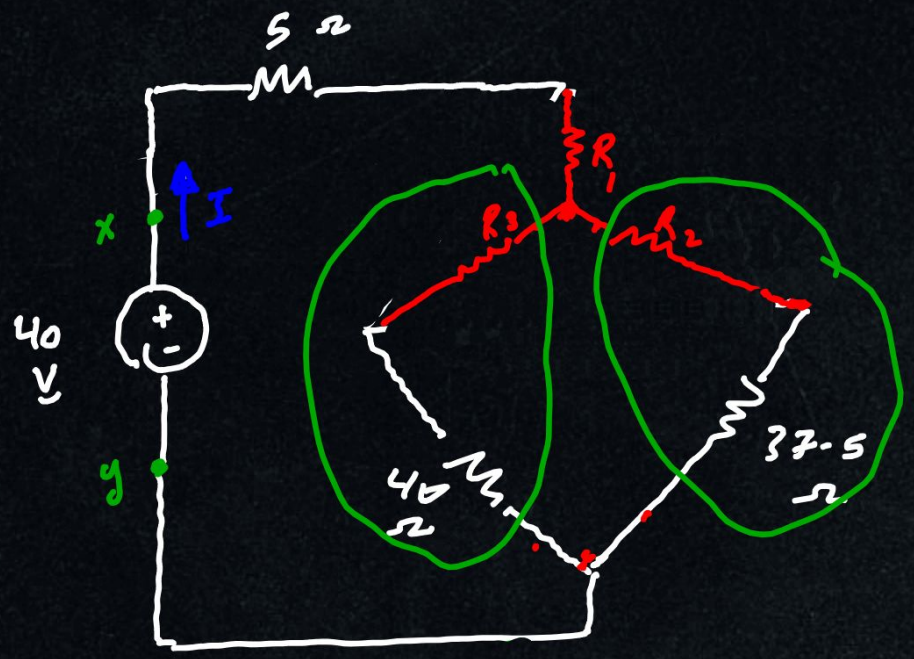
$$R_b = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3}$$

EX :- Calculate I



EX

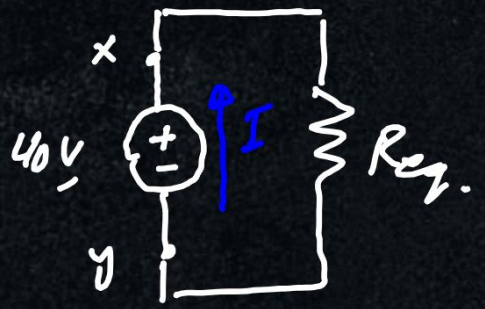


Calculate I ?

$$R_1 = \frac{100(125)}{250} = 50\ \Omega$$

$$R_2 = \frac{125(25)}{250} = 12.5\ \Omega$$

$$R_3 = \frac{100(25)}{250} = 10\ \Omega$$



$$I = \frac{40}{R_{eq}}$$

$$R_{eq} = [(12.5 + 37.5) \,//\, (10 + 40)] + 50 + 5$$

$$R_{eq} = 80\ \Omega$$

$$I = \frac{40}{80} = 0.5\ A$$