

• Inverse Fourier Transform

Example: Suppose that

$$X(e^{j\omega}) = \frac{1}{(1 - a e^{j\omega})(1 - b e^{-j\omega})}$$

Evaluate $X[n]$

Ans:

$$X(e^{j\omega}) = \frac{1}{(1 - a e^{j\omega})(1 - b e^{-j\omega})}$$

$$= \frac{A}{1 - a e^{j\omega}} + \frac{B}{1 - b e^{-j\omega}}$$

$$1 = A(1 - b e^{-j\omega}) + B(1 - a e^{j\omega})$$

when $e^{-j\omega} = \frac{1}{b}$

$\Rightarrow$

$$1 = B(1 - \frac{a}{b})$$

$$1 = B(\frac{b-a}{b}) \Rightarrow B = \frac{a}{b-a}$$

when $e^{j\omega} = \frac{1}{a}$

$$1 = A(1 - \frac{b}{a}) \Rightarrow 1 = A(\frac{a-b}{a}) \rightarrow A = \frac{a}{a-b}$$

$$X(e^{j\omega}) = \frac{a}{a-b} \frac{1}{1 - a e^{j\omega}} + \frac{a}{b-a} \frac{1}{1 - b e^{-j\omega}}$$

$$= \frac{a}{a-b} (a)^n u[n] - \frac{a}{a-b} (b)^n u[n]$$

• Z-transform

The Fourier Transform of a sequence $x[n]$ was defined as

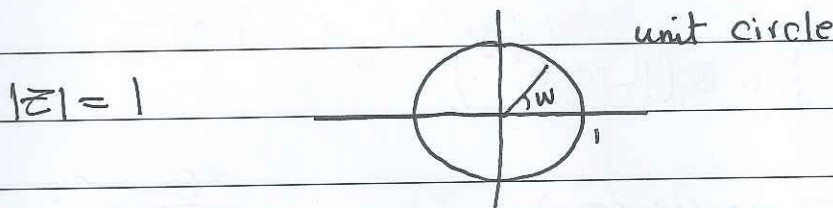
$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

The z-transform of sequence $x[n]$ is defined as

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

where z can be expressed in polar form as

$$z = r e^{j\omega}$$



Example: Consider the following signal

$$x[n] = \left(\frac{1}{2}\right)^n u[n] \quad \text{[called right-sided exponential seq]}$$

① Evaluate $X(z)$.

② Evaluate and Plot ROC

$$\text{Ans: } X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n} = \sum_{n=0}^{\infty} \left(\frac{1}{2} z^{-1}\right)^n$$

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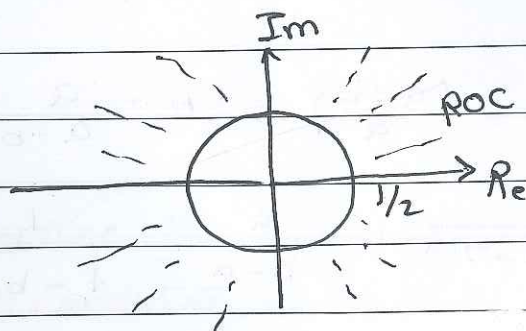
$n=-\infty$

$n=0$

$n=\infty$

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$$= \frac{1}{1 - \frac{1}{2} z^{-1}} = \frac{z}{z - \frac{1}{2}}$$



$$|z| > 1/2$$

FT exist

(66)

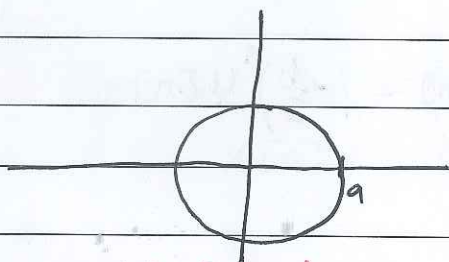
Note that: FT converges (exists) only if z -transform ($X(z)$) converges at $|z|=1 \Rightarrow$ i.e., ROC contains unit circle ($|z|=1$).

Example: Consider the signal $x[n] = a^n u[n]$

- (a) Evaluate $X(z)$
- (b) Plot the ROC
- (c) Does FT exist?

Ans:

$$\begin{aligned} X(z) &= \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=0}^{\infty} a^n z^{-n} = \sum_{n=0}^{\infty} (a z^{-1})^n \\ &= \frac{1}{1 - a z^{-1}} \\ &= \frac{z}{z - a} \end{aligned}$$



It can be noted that:

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$|a| < 1$

a. The Fourier Transform of $x[n]$ exists if

b. For $|a| = 1$, $x[n]$ is the unit step sequence with z -transform

$$X(z) = \frac{1}{1 - z^{-1}}, \quad |z| > 1$$

c. For $|a| > 1$, the ROC does not include the unit circle, consistent with the fact that, for these values of a , the Fourier transform of the exponentially growing sequence $a^n u[n]$ does not converge.

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Example: Consider the following signal $x[n] = -a^n u[-n-1]$
(called left-Sided Exponential Sequence).

- ① Evaluate $X(z)$
- ② Plot ROC
- ③ Does FT exists

Ans: $x[n] = -a^n u[-n-1]$

$$X(z) = - \sum_{n=-\infty}^{\infty} a^n u[-n-1] z^{-n} = - \sum_{n=-\infty}^{\infty} a^n z^{-n}$$

$$= - \sum_{n=1}^{\infty} a^{-n} z^n = 1 - \sum_{n=0}^{\infty} (a^{-1} z)^n$$

$$= 1 - \frac{1}{1 - a^{-1} z} \quad |z| < |a|$$

the FT does not exists in case $|a| < 1$.

Example: Consider a signal that is the sum of two real exponentials:

$$x[n] = \left(\frac{1}{2}\right)^n u[n] + \left(-\frac{1}{3}\right)^n u[n]$$

- ① Evaluate $X(z)$
- ② Plot ROC
- ③ Specify zeros and poles.

Ans: $x[n] = \left(\frac{1}{2}\right)^n u[n] + \left(-\frac{1}{3}\right)^n u[n]$

$$X(z) = \frac{1}{1 - \frac{1}{2} z^{-1}} + \frac{1}{1 + \frac{1}{3} z^{-1}}$$

$$= \frac{z}{z - \frac{1}{2}} + \frac{z}{z + \frac{1}{3}}$$

$$= \frac{(z + \frac{1}{3})z + z(z - \frac{1}{2})}{(z - \frac{1}{2})(z + \frac{1}{3})}$$

$$X(z) = \frac{z^2 + \frac{1}{3}z + z^2 - \frac{1}{2}z}{(z - 1/2)(z + 1/3)}$$

$$= \frac{2z^2 - \frac{1}{6}z}{(z - 1/2)(z + 1/3)}$$

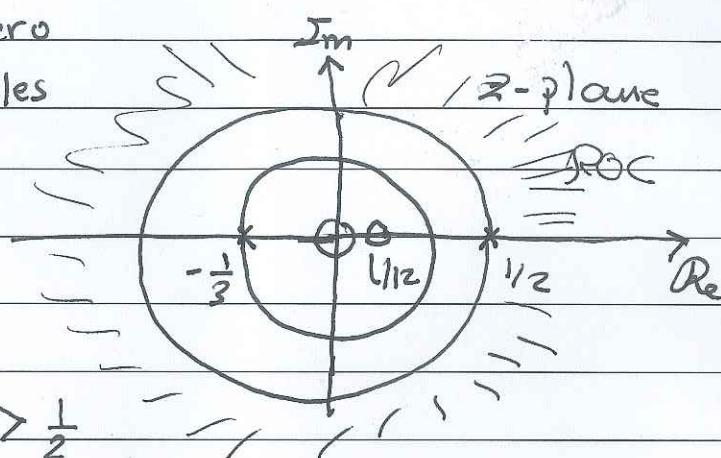
$$= \frac{2z(z - 1/12)}{(z - 1/2)(z + 1/3)} \quad \leftarrow \text{zeros}$$

$$(z - 1/2)(z + 1/3) \quad \leftarrow \text{poles}$$

let us defined :-

o zero

x poles



$$\text{ROC } |z| > \frac{1}{2}$$

Example: Consider the sequences

$$x[n] = \left(-\frac{1}{3}\right)^n u[n] - \left(\frac{1}{2}\right)^n u[-n-1]$$

(a) Evaluate $X(z)$

(b) Plot ROC

Ans: