Chapter 3: Motion in a circle.

Uniform Circular Motion: the motion of an object around a circular path with a constant (uniform) speed.

* For circular motion to happen, 2 things have to be true:
1) The net force has to be constant, and always pointing toward the center.
2) There has to be a constant velocity at a 90° angle to the net force.

[1] Centripetal Force (ΣF_c):

→ The net force on the object. It always points toward the center of motion. Measured in N.

[2] Centripetal Acceleration (a_c):

→ The acceleration of the object (points in the same direction as the centripetal force). measured in m/sec^2

→ Why does acceleration of an object in circular motion point toward the center?

1) The net force points toward the center, and acceleration always points in the same direction as the net force.

2) Acceleration shows the direction of the change in velocity. Vector addition shows how this keeps the object moving in a circle.

3 Period (T):

→ The amount of time an object takes to complete one full cycle. Measured in seconds.

4 frequency (f):

→ The amount of repetitions a cycle completes in one second. Here, it means how many times the object goes around the circle in 1 second. Measured in "hertz" or Hz.

$$T = \frac{1}{f}$$

5 Angular velocity (ω):

→ The change in angle around a circle over the change in time. Measured in rad/sec.

6 Tangential velocity (V_t):

→ The distance the object moves around the circle over the change in time. Measured in m/sec.

$$V_t = \frac{d}{t} \quad \rightarrow d = 2\pi r$$

$$V_t = \frac{2\pi r}{T} = 2\pi r f \quad \text{—— } 1$$

$$\omega = \frac{\Delta\theta}{t} \quad \text{—— } 2$$

$$2\pi f = \omega = \frac{2\pi}{T} \quad \text{—— } 3$$

$$v_t = \omega r \quad \text{——— (4)}$$

$$a_c = \frac{v_t^2}{r} \quad \text{——— (5)}$$

$$\Sigma F_c = m a_c = m\frac{v_t^2}{r} \quad \text{——— (6)}$$

* **Example 1:** A 1000 kg car takes 4 seconds to turn from north to west around a corner. If the radius of curvature of the road is 30 m, what force do the car's wheels apply?

$$F_c = m\frac{v_t^2}{r}$$

$$= 1000 \times \frac{v_t^2}{30}$$

$$\rightarrow v_t = \omega r$$

$$\omega = \frac{\Delta\theta}{\Delta t} = 90 \times \frac{2\pi}{360} \times \frac{1}{4} = 0.393 \text{ rad/s}$$

$$v_t = 0.393 \times 30 = 11.8 \text{ m/sec}$$

$$\therefore F_c = 1000 \times \frac{(11.8)^2}{30}$$

$$F_c = 4641.3 \text{ N}$$

* **Example 2:** The distance from the earth to the sun is 149.6 million km. How fast is the earth moving around its orbit? What is the earth's acceleration toward the sun?

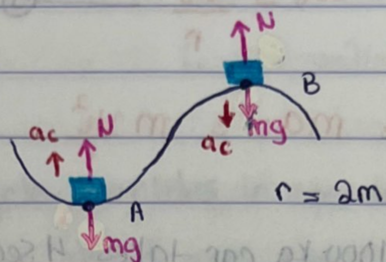
$$\rightarrow r = 149.6 \times 10^6 \text{ km} = 1.496 \times 10^{11} \text{ m}$$

$$T = 365 \text{ days} = 3.15 \times 10^7 \text{ sec.}$$

$$v_t = \frac{2\pi r}{T} = \frac{2\pi(1.496 \times 10^{11})}{3.15 \times 10^7} = 2.98 \times 10^5 \text{ m/sec.}$$

$$a_c = \frac{v_t^2}{r} = \frac{(2.98 \times 10^5)^2}{1.496 \times 10^{11}} = 5.9 \times 10^{-3} \text{ m/sec}^2$$

* Example 3: Calculate the normal force of a 5 kg box moving at a speed of 15 m/sec at points A and B shown below.



• At (A):

$\Sigma F = ma$

$N - mg = ma_c$

$N = \frac{mv_t^2}{r} + mg$

$N = \frac{5(15)^2}{2} + 5(9.8)$

$\therefore N_A = 611.5\ N$

• At (B):

$\Sigma F = ma$

$N - mg = m(-a_c)$

$N - mg = m\left(\frac{-v_t^2}{r}\right)$

$N = mg + -m\frac{(v_t)^2}{r}$

$N = 5(9.8) + -5\frac{(15)^2}{2}$

$\therefore N_B = -513.5\ N$

Chapter 3 problems' solutions :

3.1 :

(a) (i) $\frac{\pi}{6} \times \frac{180}{\pi} = 30^\circ$ (ii) $\frac{\pi}{4} \times \frac{180}{\pi} = 45^\circ$

(iii) $\frac{\pi}{2} \times \frac{180}{\pi} = 90^\circ$ (iv) $0.1 \times \frac{180}{\pi} = 5.73^\circ$

(v) $\frac{3\pi}{4} \times \frac{180}{\pi} = 135^\circ$

(b) (i) $1^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{180}$ (ii) $45 \times \frac{\pi}{180} = \frac{\pi}{4}$

(iii) $60 \times \frac{\pi}{180} = \frac{\pi}{3}$ (iv) $180^\circ \times \frac{\pi}{180} = \pi$

(v) $360 \times \frac{\pi}{180} = 2\pi$

3.2: $\theta = 45^\circ$ $t = 0.1$ sec

$$\omega = \frac{\Delta\theta}{\Delta t} = 45 \times \frac{\pi}{180} \times \frac{1}{0.1} = 7.85 \text{ rad/sec}.$$

3.3: $\theta = \pi$ $t = 0.2$ sec $m = 2$ kg $r = 0.45$ m

$$F_c = m \frac{V_t^2}{r}$$

$$V = \omega r$$

$$= \frac{\theta}{t}(r) = \frac{\pi}{0.2}(0.45) = 7.07 \text{ m/sec}$$

$$F_c = \frac{2(7.07)^2}{0.45} = 222.15 \text{ N}.$$

3.4: (a) $m = 3800\,kg$ $\quad V_t = 65\,\frac{km}{h} = 18.06\,m/sec$

$r = 80\,m.$

$$F_c = m\frac{V_t^2}{r} \cancel{= \frac{3800 \times (65)^2}{}}$$

$$F_c = \frac{3800(18.06)^2}{80} = 15.5 \times 10^3\,N$$

(b) $V_t = 100\,\frac{km}{h} = 27.8\,m/sec$

$$F_c = 3800 \times \frac{(27.8)^2}{80} = 36.7 \times 10^3\,N$$

3.5: $V_t = 180\,\frac{km}{h} = 50\,m/sec$ $\quad \cancel{C = 2.4\,km = 2400\,m}$

$$2\pi r = 2400$$

$$\cancel{\omega = \frac{V_t}{r} = \frac{50}{2400}}$$

$$r = 382\,m$$

$$\omega = \frac{V_t}{r} = \frac{50}{382} = 0.13\,rad/sec.$$