

1.2 Combining functions; shifting and scaling Graphs 10

* If f and g are functions, then for every

$x \in D(f) \cap D(g)$, we define:

$$(f+g)(x) = f(x) + g(x) \quad \begin{array}{l} \text{operation of addition of functions} \\ \text{addition of real numbers} \\ \text{"sum"} \end{array}$$

$$(f-g)(x) = f(x) - g(x) \quad \text{"Difference"}$$

$$(fg)(x) = f(x)g(x) \quad \text{"product"}$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0 \quad \text{"Quotient"}$$

$$(cf)(x) = cf(x), c \in \mathbb{R} \quad \text{"Multiply by constant"}$$

Example: If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{1-x}$, then

$$(2f)(x) = 2f(x) = 2\sqrt{x} \quad \text{with domain } [0, \infty)$$

$$(f+g)(x) = f(x) + g(x) = \sqrt{x} + \sqrt{1-x} \quad \text{with domain } [0, \infty) \cap (-\infty, 1] = [0, 1]$$

$$(f-g)(x) = \sqrt{x} - \sqrt{1-x} \quad \text{with domain } [0, 1]$$

$$(f \cdot g)(x) = \sqrt{x} \sqrt{1-x} = \sqrt{x(1-x)} \quad \text{with domain } [0, 1]$$

$$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}}{\sqrt{1-x}} = \sqrt{\frac{x}{1-x}} \quad \text{with domain } [0, 1)$$

$$\left(\frac{g}{f}\right)(x) = \frac{\sqrt{1-x}}{\sqrt{x}} = \sqrt{\frac{1-x}{x}} \quad \text{with domain } (0, 1]$$

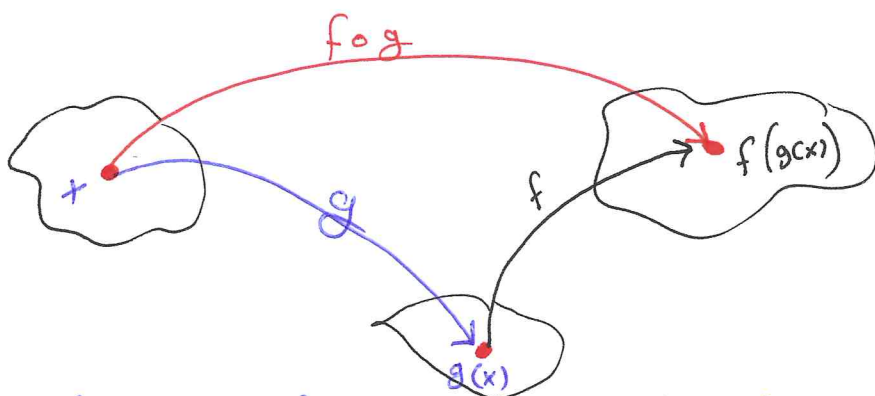
Composite functions

(11)

Def: Let f and g be two functions. The composite function $f \circ g$ "f composed with g" is defined by

$$(f \circ g)(x) = f(g(x))$$

$$R(g) \subset D(f)$$



The domain of $f \circ g$ consists of the numbers x in the domain of g for which $g(x)$ lies in the domain of f .

Example If $f(x) = \sqrt{x}$ and $g(x) = x+1$ find

(a) $(f \circ g)(x) = f(g(x)) = f(x+1) = \sqrt{x+1}$, $D = [-1, \infty)$

(b) $(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{x} + 1$, $D = [0, \infty)$

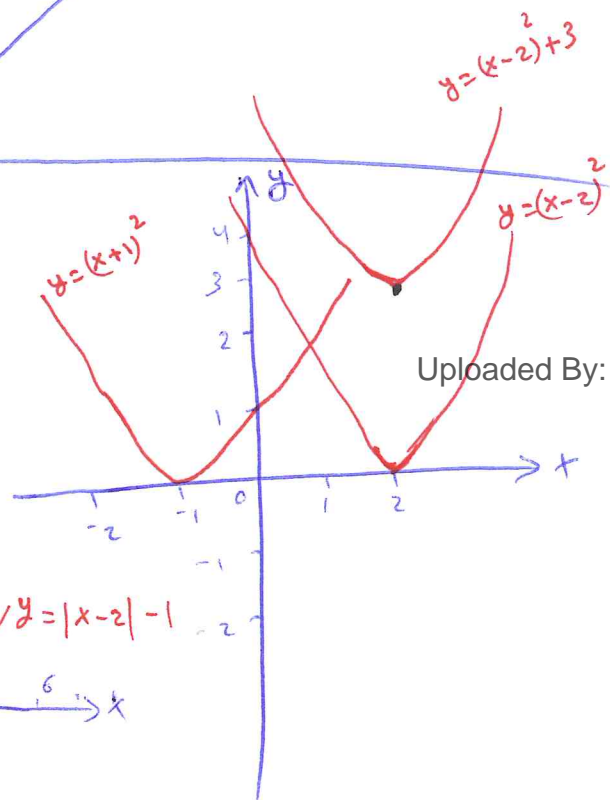
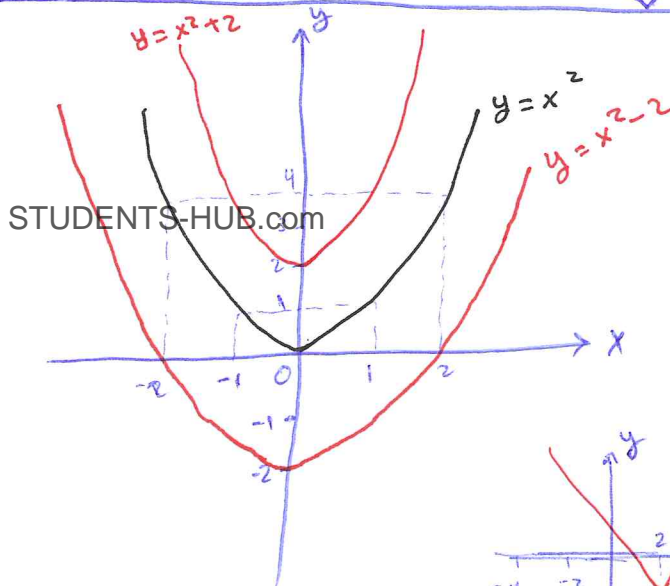
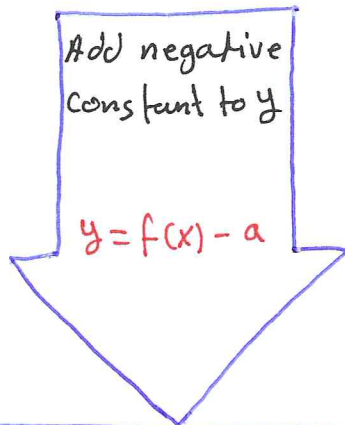
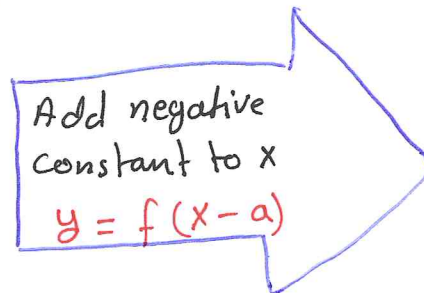
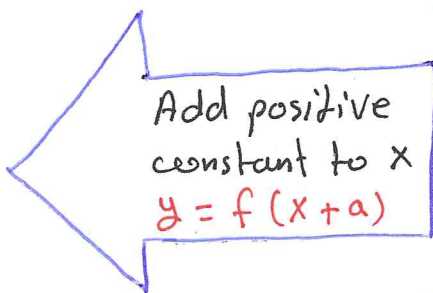
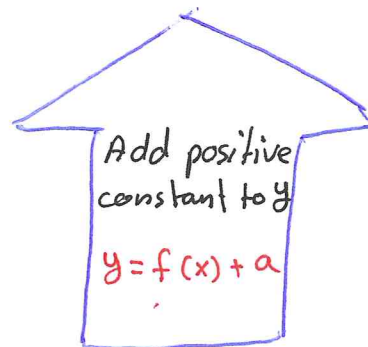
(c) $(f \circ f)(x) = f(f(x)) = f(\sqrt{x}) = \sqrt{\sqrt{x}} = x^{\frac{1}{4}}$, $D = [0, \infty)$

(d) $(g \circ g)(x) = g(g(x)) = g(x+1) = x+1+1 = x+2$, $D = \mathbb{R}$

Shifting Graphs

(12)

* Horizontal and Vertical shift : Let $a > 0$ and $y = f(x)$



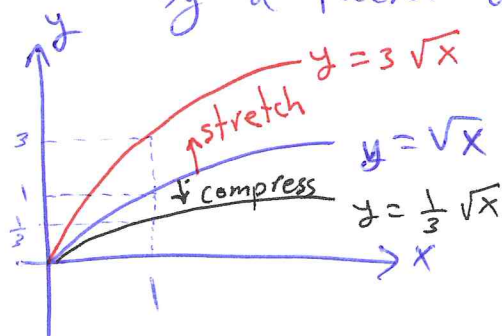
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Vertical scaling:

If $c > 1$, then

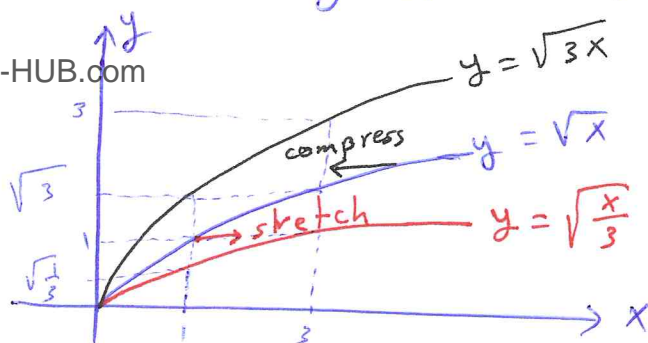
- $y = c f(x)$ stretches the graph of f vertically by a factor of c .

- $y = \frac{1}{c} f(x)$ compresses the graph of f vertically by a factor of c .

Horizontal scaling

If $c > 1$, then

- $y = f(cx)$ compresses the graph of f horizontally by a factor of c .
- $y = f\left(\frac{x}{c}\right)$ stretches the graph of f horizontally by a factor of c .

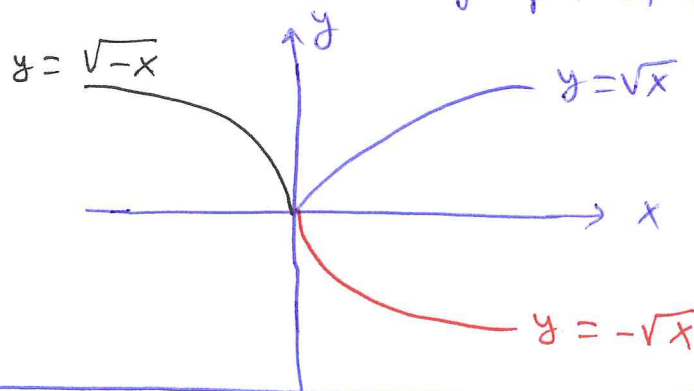


Reflections

(14)

* If $c = -1$, then

- $y = -f(x)$ reflects the graph of f across the x -axis
- $y = f(-x)$ reflects the graph of f across the y -axis



Example: Let $f(x) = x^4 - 2x^3 + 1$. Find formulas to

- (a) compress the graph horizontally by a factor of 2 followed by a reflection across the y -axis.

compress horizontally by 2 $\Rightarrow f(2x)$

reflection across y -axis $\Rightarrow f(-2x)$

$$\begin{aligned} f(-2x) &= (-2x)^4 - 2(-2x)^3 + 1 \\ &= 16x^4 + 16x^3 + 1 \end{aligned}$$

- (b) compress the graph vertically by a factor 2 followed by a reflection across the x -axis.

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compress the graph vertically by 2 $\Rightarrow \frac{1}{2} f(x)$

reflection across x -axis $\Rightarrow -\frac{1}{2} f(x)$

$$\begin{aligned} -\frac{1}{2} f(x) &= -\frac{1}{2} [x^4 - 2x^3 + 1] \\ &= -\frac{1}{2} x^4 + x^3 - \frac{1}{2} \end{aligned}$$

Ellipses مقطع ناقص

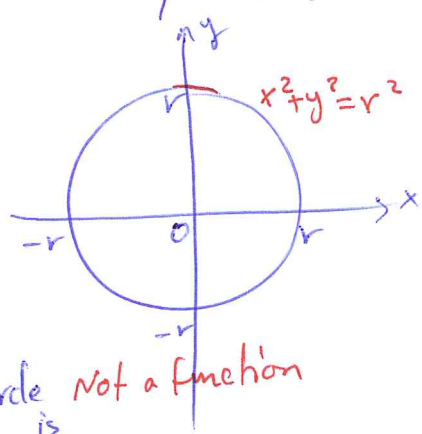
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* A standard equation for the circle of radius r and centered at point (h, k) is

$$(x-h)^2 + (y-k)^2 = r^2$$

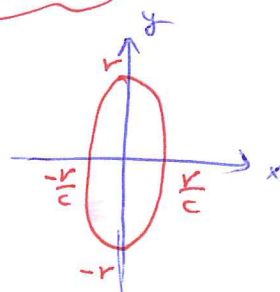
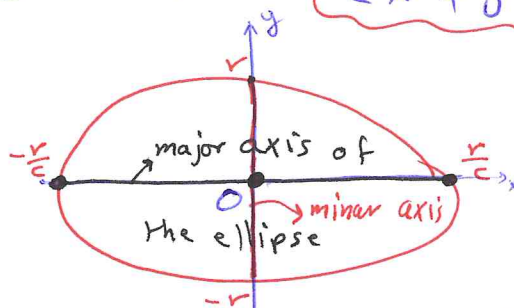
* Circle centered at origin has the following equation

$$x^2 + y^2 = r^2 \quad \text{--- (1)}$$



Substitute cx for x in equation

① we get $c^2x^2 + y^2 = r^2$ ~~X~~



y-intercept is the same $r, -r$ in the three figures

• Major axis is the longer line segment.

- ①. ellipse: $0 < c < 1$
- ②. not a function
- ③. stretches the circle

- ①. ellipses $c > 1$
- ②. not a function
- ③. compressed horizontally

④ Major axis is the line segment joining the points $(-r/c, 0)$ and $(r/c, 0)$

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④ Major axis is the line segment joining the points $(0, -r)$ and $(0, r)$

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⑤ Minor axis is the line segment joining the points $(0, -r)$ and $(0, r)$

⑤ Minor axis is the line segment joining the points $(-r/c, 0)$ and $(r/c, 0)$

Divide $*$ by $r^2 \Rightarrow \boxed{\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1} \rightarrow (2) \quad (16)$

Take $a = \frac{r}{c}$ and $b = r$

Equation (2) becomes:

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

→ Ellipse centered at origin

• If $a > b$, then the major axis is horizontal

• If $a < b$, then = = vertical

The standard equation of an ellipse centered at (h, k) is

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

