

Center of Mass

- The center of mass of a system of particles is the point that moves as though :-
 - all of the system's mass were concentrated there
 - all external forces were applied there
- for a discrete system

1 Dimen $\rightarrow$ the location of com :- $X_{com} = \frac{m_1 X_1 + m_2 X_2}{m_1 + m_2}$

3 Dimen $\rightarrow$ the location of com :- $\vec{r}_{com} = x_{com} \hat{i} + y_{com} \hat{j} + z_{com} \hat{k}$

- for a continuous system (solid body) uniform objects

3 Dimen $\rightarrow \vec{r}_{com} = \frac{\int \vec{r} dm}{m_{tot}}$

- A uniform object has a uniform density or mass per unit volume which is :- $\rho = \frac{M}{V}$

$\nearrow$ mass
 $\nwarrow$ volume

PS :- the center of mass does not necessarily lie within the object : exp: a doughnut



- If an external force acts on a com

$\vec{F}_{net} = M \vec{a}_{com}$

$\rightarrow$ the total mass of the system (constant)

$\rightarrow$ acceleration of com (not the particles)

$\rightarrow$ net force of all external forces that acts on system

$\rightarrow$ internal forces are not included

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Linear Momentum

of a Particle

→ • $\vec{P} = m\vec{v}$ of a Particle
 Mass velocity

• $F_{\text{net}} = \frac{d\vec{P}}{dt}$
 extend ←

P changes when there is a net external force only

Newton's 2nd law in terms of momentum
 ← The time rate of change of the momentum of a particle is equal to the net force acting on the particle and is in the direction of that force

of a system of particles

• $\vec{P} = \vec{P}_1 + \vec{P}_2$ or of particle 2
 • $\vec{P} = M\vec{V}_{\text{cm}}$
 • $\vec{F}_{\text{net}} = \frac{d\vec{P}}{dt}$

Collision

brief collision :- in a small duration

$\vec{P}$ is conserved and impulse
 $\vec{P}_i = \vec{P}_f$ $J = \Delta P$

• in a single collision :- $\Delta P = J$

• in series of collisions :- n is the number of collisions

$J = -n \Delta P$

$F_{\text{avg}} = \frac{n}{\Delta t} m \Delta v$

$= -\frac{\Delta m}{\Delta t} \Delta v$ ($\Delta m = nm$)

Ps: the minus sign indicates that J and ΔP has opposite Directions

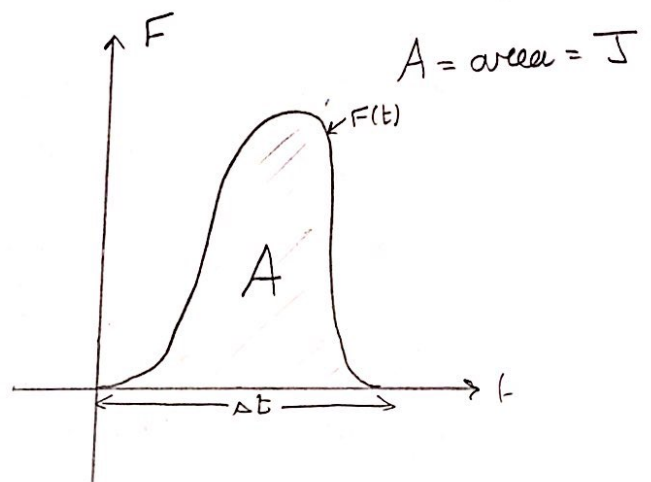
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impulse

$$J = \int_{t_i}^{t_f} \vec{F}(t) dt$$

$$\Delta P = J$$

$$J = F_{avg} \Delta t$$



Kinetic Energy in collisions

elastic collision

• K.E conserved

$$K.E_i = K.E_f$$

→ we use this equation

$$(v_{1i} - v_{2i}) = -(v_{1f} - v_{2f})$$

Completely inelastic collision

- greatest loss of K.E
- the bodies stick together

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

inelastic collision

• K.E is not conserved

$$K.E_f \neq K.E_i$$

T.S :- $\vec{V}_{com}$ is constant before and after a collision because $\sum \vec{F}_{net ext} = 0$
 Law of Conservation of Momentum

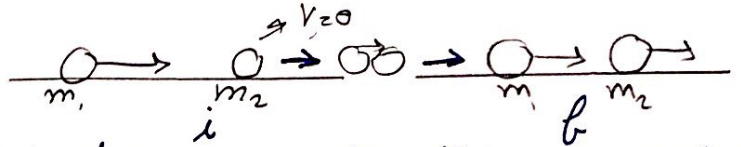
problems with a Projectile and an object :-

• The object (Target) is stationary :- $V_{2i} = 0$

- If $m_1 > m_2 \Rightarrow m_1$ moves forward
- If $m_1 < m_2 \Rightarrow m_1$ bounces (عكس الاتجاه)

- If $m_1 = m_2 \Rightarrow$ body 2

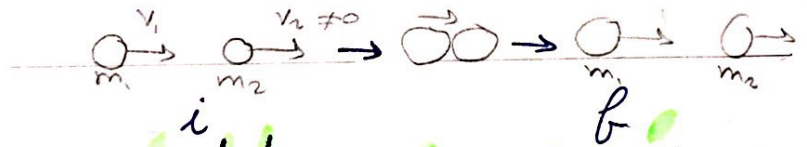
stops after collision and body 1 moves with the same velocity as body 1



• The object (Target) is moving :- $V_{2i} \neq 0$

$$(m_1 v_1 + m_2 v_2)_i = (m_1 v_1 + m_2 v_2)_f$$

$$\left(\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \right)_i = \left(\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \right)_f$$



Systems with Varying Mass : A Rocket

to find a:- $R v_{rel} = \frac{dM}{dt} a_{Total} \Rightarrow T = \frac{dM}{dt} a_{Total}$
 Positive mass rate $= \frac{dM}{dt}$
 Thrust

to find v:- $v_f - v_i = v_{rel} \ln \frac{M_i}{M_f}$

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How to solve Problems

Center of Mass

These are the main ideas

- 1- If the problem is about a system of particles we use

$$X_{com} = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2} \text{ to find the position of COM}$$

- 2- If the system is a uniform body then it's probably in the center

- 3- In some question X_{com} doesn't change cause there is no horizontal or vertical force so we put $X_{com} = 0$

Example : P 17 Page 231



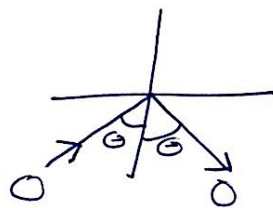
- 4- If 2 bodies are projected the COM will be moving in a projectile motion so we use equation of constant acceleration to find v_f for each body and then we can find v_{com} and a_{com}

Linear Momentum

- 1- If the problem is about finding Δp then we find v_f and v_i

- 2- In these problems you should pay attention to θ 's and

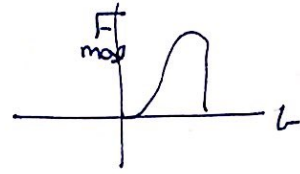
Directions, The main trick is in it



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3- in problems with such plots remember that

Area under (F, t) curve $= \int = \Delta P$



4- in explosions :-

$$\Delta P = 0$$

$$P_i = P_f$$

$$\sum m(v) = m_1 v_1 + m_2 v_2 \dots$$

5- sometimes you need to use $(K+U)_f = (K+U)_i$. Specially if the question is talking about h or when the question is about releasing a ball



Or if the question is talking about a spring

Then $K \neq U$

$$\frac{1}{2} m v^2 = \frac{k x^2}{2}$$

6- in Rockets problems :

you use : $v_f - v_i = v_{rel} \ln \frac{M_i}{M_f}$

$$\text{or } R_{rel} = M_{Tot}$$

$\nwarrow$
 $\frac{dM}{dt}$

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