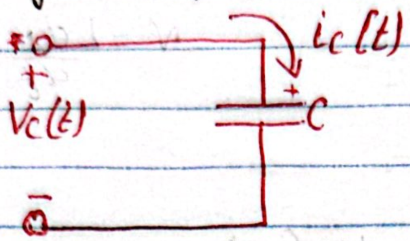
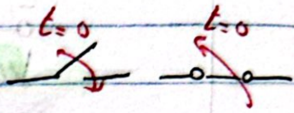
Chapter 7: First order RL, RC circuit:

Some of the important characteristics of a capacitor



$0^+, 0^-, 0$



$$i_c(t) = C\frac{dv_c(t)}{dt}$$

$$\rightarrow v_c(t) = \underbrace{v_c(0^-)}_{\text{initial}} + \frac{1}{C}\int_0^t i_c(t)\,dt \text{, for } t \geq 0.$$

$$\text{at } \boxed{t=0^+} \rightarrow v_c(0) = \underbrace{v_c(0^-)}_{\text{initial condition}} + \frac{1}{C}\cancel{\int_0^{0} i_c(t)\,dt}$$

$$\rightarrow \boxed{v_c(0^+) = v_c(0^-)}$$

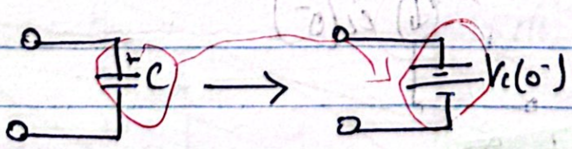
1: the current through a capacitor is zero if the voltage across it is not changing with time → A capacitor is therefore an open circuit to dc.

2: A finite amount of energy can be stored in a capacitor even if the current through the capacitor is zero.

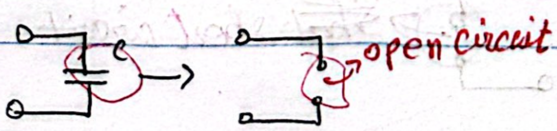
3: the capacitor never dissipate energy, but only store it.

4: It is impossible to change the voltage across a capacitor by a finite amount in zero time, for this requires an infinite current through ~~the capp the capacitor~~ the capacitor.

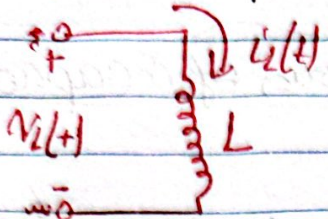
5: At $t=0^+$

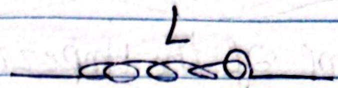


At $t=0^-$ and $t=\infty$



Some of the important characteristics of an inductor





$$v = L\frac{di}{dt}$$

$$v_L(t) = L\frac{di_L(t)}{dt}$$

$$\rightarrow i_L(t) = i_L(0^-) + \frac{1}{L}\int_0^t v_L(t)\,dt, \text{ for } t \geq 0$$

$$\text{at } t=0^+ \rightarrow i_L(0^+) = i_L(0^-) + \cancel{\frac{1}{L}\int_0^{t_0} v_L(t)\,dt}$$

$$\rightarrow \boxed{i_L(0^-) = i_L(0^+)}$$

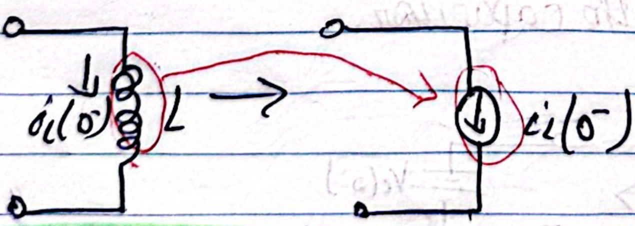
1: There is no voltage across an inductor if the current through it is not changing with time. An inductor is therefore a short circuit to dc.

2: A finite amount of energy can be stored in an inductor even if the voltage across the inductor is zero.

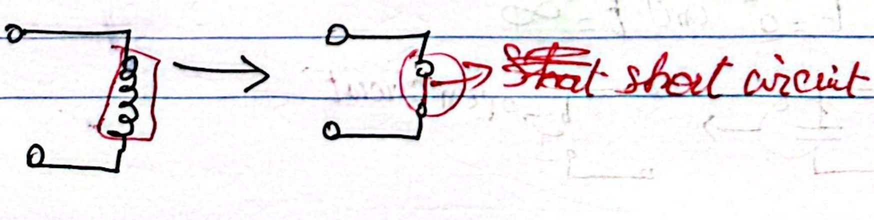
3: the inductor never dissipate energy, but only store it.

→ It is impossible to change the current through an inductor by a finite amount in zero time. for this requires an infinite voltage across the inductor.

At $t=0^+$



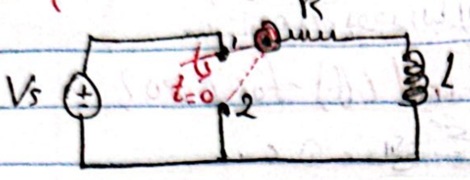
At $t=0^-$ and $t=\infty$

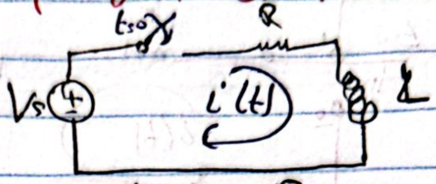


الحل بعد تسكير الويتش ما بكون عندي إي فولتج أو كرنت سورس

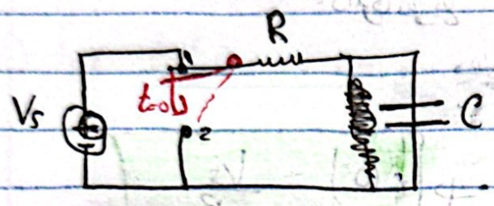
دوائر التفريغ	دوائر شحن (forced)
Natural response at 1^{st} order circuit	**Step response of 1^{st} order circuit**

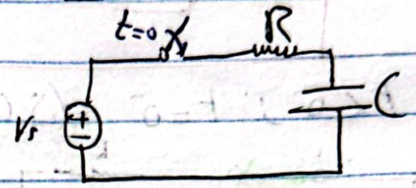
توزيع





كثف

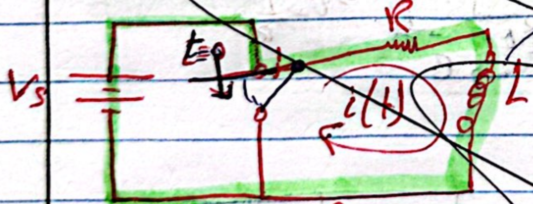




First order Circuit:

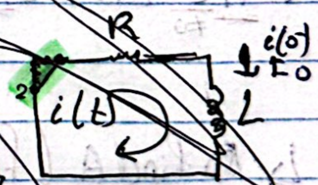
- A first-order circuit can only contain one energy storage element (a capacitor or an inductor) or a combination of capacitors or inductors that can be reduced to one capacitor or inductor.
- The circuit will also contain one or more resistances.
- A first-order circuit is characterized by a first-order diff. eq.

Exp: Natural Response of first-order Circuit



inductor → مخزن تيار

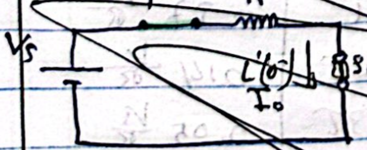
عند (1) دوائر شحن
عند (2) دوائر تفريغ
natural



Find $i(t)$ for $t>0$?

قبل ما يسكر

① for $t<0$; $t=0^-$ s.c تصير



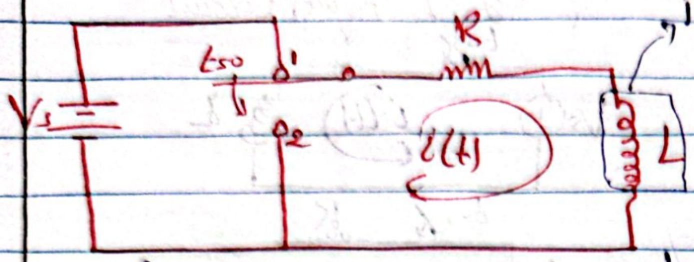
$\boxed{i(0^-)=\frac{V_s}{R}}$

② for $t>0$

KVL → $V_R(t)+V_L(t)=0 \to Ri(t)+L\frac{di(t)}{dt}=0$

$\hookrightarrow i(t)=Ae^{st}$ for $t>0$

Exp: Natural Response of first-order Circuit

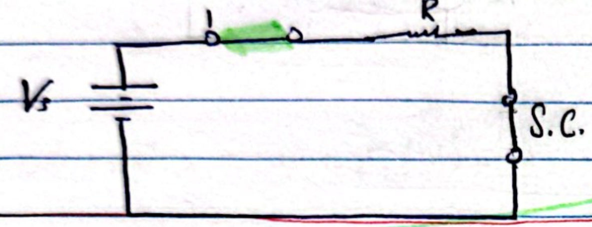


inductor → يخزن تيار
عشان هيك يوجد التيار $i(t)$ -

find $i(t)$ for $t>0$?

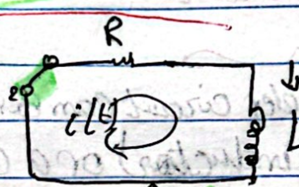
عند ① ← دائرة شحن -
عند ② ← دائرة تفريغ

① for $t<0$; $t=0^-$ → (S.C.) يعني أبيض



$\Rightarrow \boxed{i(0^-) = \frac{V_s}{R}}$

② for $t>0$



$i(0^-) = I_0$

$KVL \rightarrow V_R(t) + V_L(t) = 0 \rightarrow R\,i(t) + L\frac{d\,i(t)}{dt} = 0$

$\hookrightarrow i(t) = A e^{st}$ for $t>0$ — بدي اشتقاق وبعدين عوضه بحل المعادلة diff eq.

نعوض

$\rightarrow R A e^{st} + L A s e^{st} = 0$

$\rightarrow \underbrace{A e^{st}}_{\neq 0}(R + Ls) = 0 \Rightarrow \therefore \boxed{S = -\frac{R}{L}} \rightarrow \frac{-1}{\tau} = \frac{1}{\text{time const.}}$, $\tau = \frac{L}{R}$

$\hookrightarrow e^{-\frac{R}{L}t} \rightarrow e^{-\frac{t}{\tau}}$

$\hookrightarrow$ to find A (from initial conditions) $(0^+, 0^-)$

$i(t) = Ae^{st} \quad t>0$

$\rightarrow i(0^+) = i(0^-) = \frac{V_s}{R}$

then,

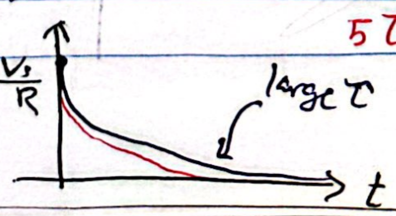
$i(0^+) = \frac{V_s}{R} = A e^{s0} \rightarrow \boxed{A = \frac{V_s}{R}}$

$\therefore i(t) = \frac{V_s}{R} e^{-\frac{R}{L}t} \quad t>0$

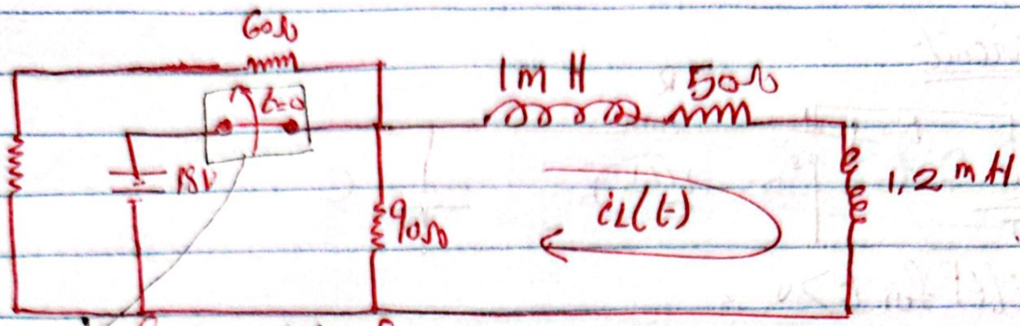
Let $\tau = \frac{L}{R}$

Time cons.

t	$i(t)$
0	$\frac{V_s}{R}$
τ	$0.37\frac{V_s}{R}$
2τ	$0.14\frac{V_s}{R}$
3τ	$0.05\frac{V_s}{R}$
4τ	$0.018\frac{V_s}{R}$
5τ	$0.0067\frac{V_s}{R}$

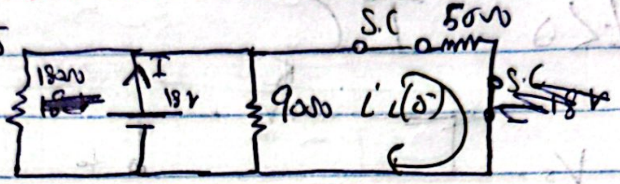


Exp:



كان مسكر صار مفتوح find $i_L(t)$ for $t>0$

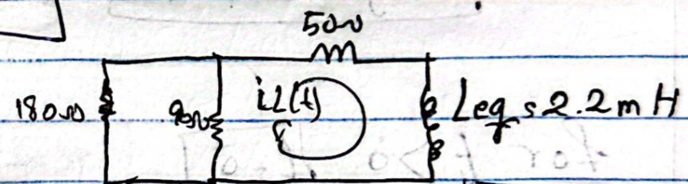
الوتش مسكر ① for $t<0$. $t=0^-$



~~$i_L(0^-) = \frac{(90//180)}{(90//180)+50}$~~

$i_L(0^-) = \frac{18}{50} = \boxed{0.36\ A}$

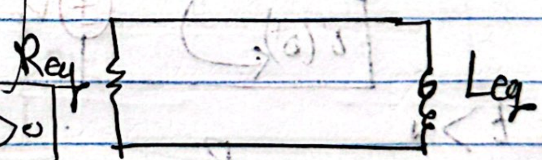
الوتش مفتوح ② for $t>0$



Leq = 1.2m + 1m = $\boxed{2.2mH}$

Req = (180//90) + 50 = $\boxed{110\ \Omega}$

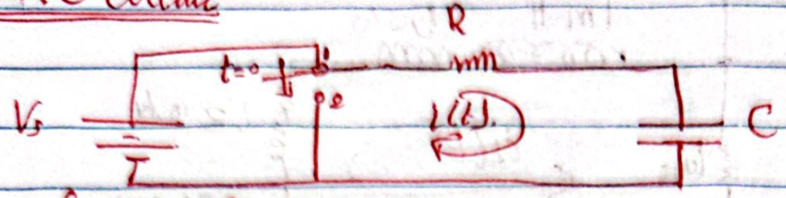
$\tau = \frac{L_{eq}}{R_{eq}} = \frac{2.2}{10} = \boxed{20\ \mu Sec}$



$i_L(t) = A e^{-\frac{t}{\tau}}$ for $t>0$

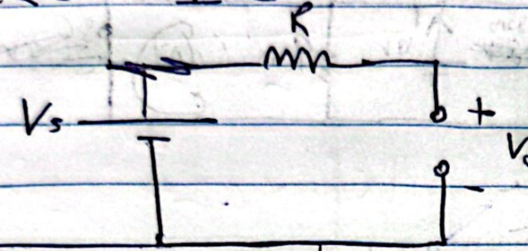
$= \boxed{0.36\ e^{-50000t} \text{ for } t>0}$

RC circuit



find $i(t)$ for $t>0$

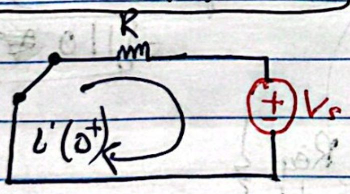
for $t<0$ I C



$\boxed{V_c(0^-) = V_o = V_s}$, $i(0^-) = \text{zero}$

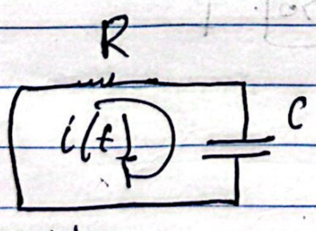
for $t>0$ $\boxed{t = 0^+}$

$\boxed{V_c(0^+) = V_c(0^-) = V_s}$



$i(0^+) = -\dfrac{V_s}{R}$

for $t>0$



$Ri(t) + v_c(t) = 0$

$Ri(t) + v_0 + \dfrac{1}{c}\int_0^t i(t)\,dt = 0$

$R\dfrac{d}{dt}i(t) + \dfrac{1}{c}i(t) = 0 \rightarrow \boxed{i(t) = Ae^{st}}$

→ to find S

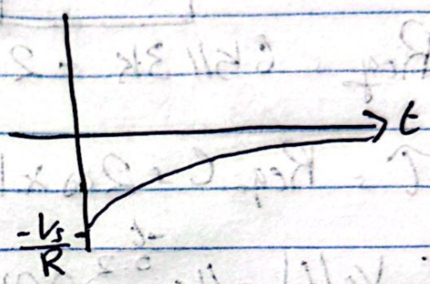
$$RASe^{st} + \frac{1}{C} A e^{st} = 0$$

$$A e^{st} \left(RS + \frac{1}{C} \right) = 0 \longrightarrow \boxed{S = \frac{-1}{RC}} , \boxed{\tau = RC}$$

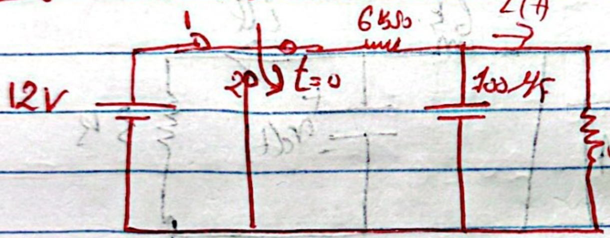
→ to find A

$$t = 0^+ \qquad i(t) = Ae^{st} = Ae^{-\frac{t}{RC}}$$

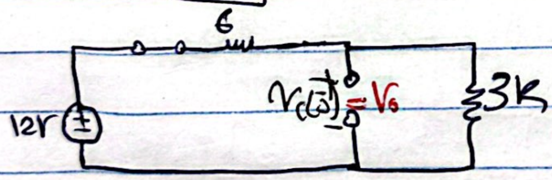
$$\boxed{i(0^+) = -\frac{V_s}{R} e^{-\frac{t}{RC}}}$$



Exp: find $i(t)$ for $t>0$



for $t<0$ $\boxed{t = 0^-}$



$$i(0^-) = \frac{12}{9} \text{ mA}$$

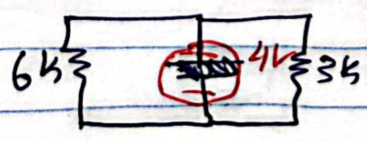
$$v_0 = v_c(0^-) = \frac{3}{3+6} * 12 = \boxed{4 \text{ volt}}$$

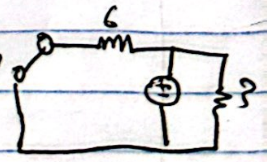
natural response

$$v_c(t) = Ae^{st}$$

to find $A \longrightarrow t = 0^+$

at $t = 0^+ \rightarrow V_c(0^-) = V_c(0^+) = 4V$, $i(0^+) = \frac{4}{3}$ mA



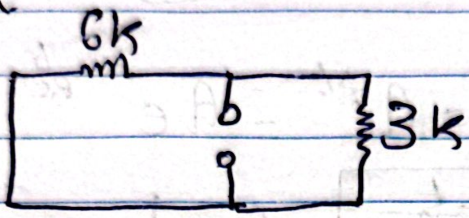


$$\boxed{A = V_c(0^+) = 4V}$$

to find S

$$S = \frac{1}{RC} = \frac{-1}{R_{eq} C}$$

$\rightarrow t = R_{eq} C$



$R_{eq} = 6k \parallel 3k = 2k$

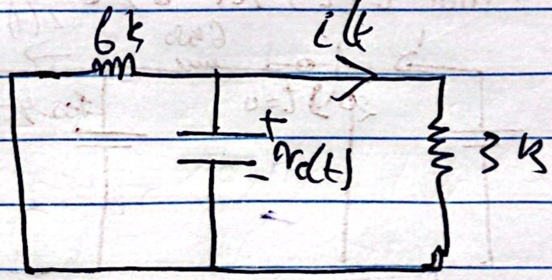
$\tau = R_{eq} C = 200 \times 100 \times 10^{-6} = 0.2$

$\therefore v_c(t) = 4e^{\frac{-t}{0.2}}$ volt , $t > 0$

$$i(t) = \frac{v_c(t)}{3k}$$

$$\boxed{= \frac{4}{3} e^{\frac{-t}{0.2}} \text{ mA}}$$

$t > 0$



step response RL/RC

forced

$$x(t) = \underbrace{x_n(t)}_{\text{natural}} + \underbrace{x_f(t)}_{\text{forced}}$$

Ae^{st} const

general Solution

$$x(t) = x(\infty) + [x(0^+) - x(\infty)]\, e^{-t/\tau}$$

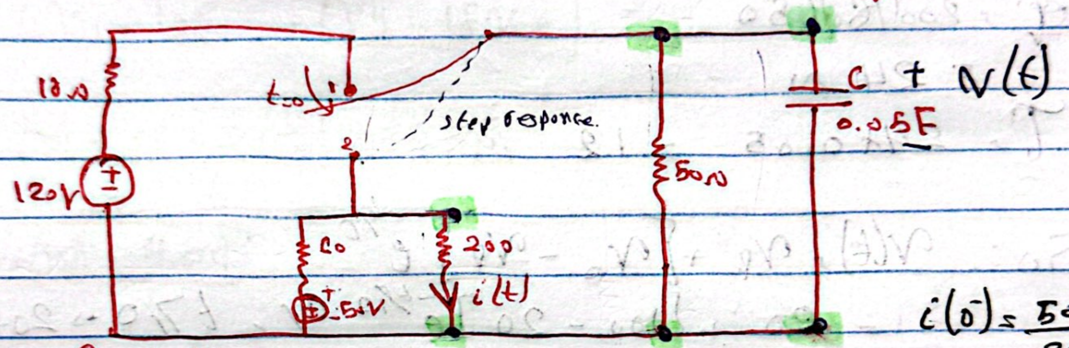
current / voltage $= x_f + [x_0 - x_f]\, e^{-t/\tau}$

τ = Time constant

$\tau = R_{eq} C$

$\tau = \frac{L}{R_{eq}}$

Exp:



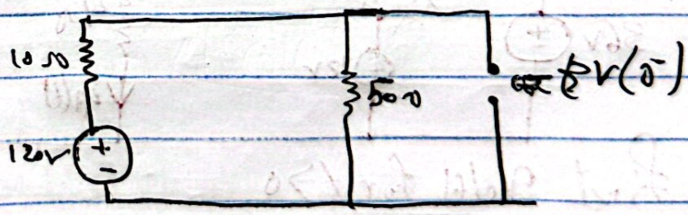
$i(0^-) = \frac{50}{260}$

find $i(t)$ for $t > 0$

$v(t) = v_f + [v_0 - v_f]\, e^{-t/\tau}$

$\therefore i(t) = \frac{v(t)}{R} = \boxed{\frac{v(t)}{200}}$ (parallel) , $v(t) \parallel i(t)$

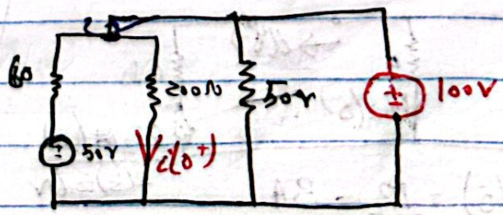
at $t < 0$ $\boxed{t \to 0^-}$



$v(0^-) = \frac{50}{50+10} * 120 = \boxed{100\,V}$

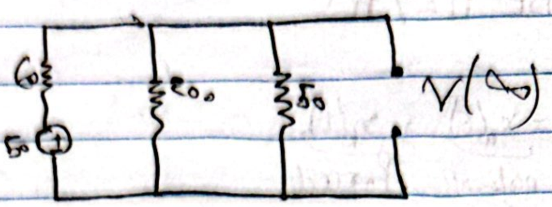
$\boxed{v(0^-) = v(0^+) = 100\,V}$

$\boxed{@\ t = 0^+}$



$i(0^+) = \frac{100}{200}$

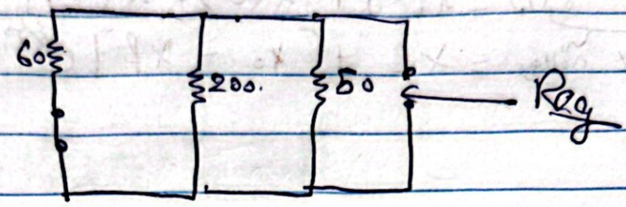
$t > 0 \quad t \to \infty$



$$v(\infty) = \frac{\overset{40}{\boxed{\cancel{200 \| 50}}}}{\underset{100}{\cancel{(200\|50)+60}}} \times 50 = \boxed{20\,V}$$

<u>to find τ:</u>

$\tau = R_{eq}\, C$



$R_{eq} = 200 \| 60 \| 50$

$= 24\ \Omega$

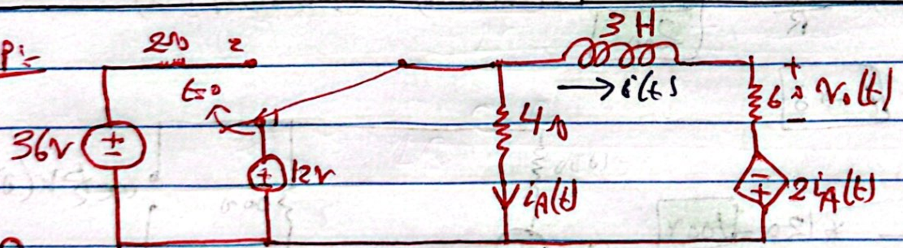
$\tau = 24 \times 0.05 = 1.2$

So: $v(t) = v_f + [v_0 - v_f]\, e^{-t/\tau}$

$= 20 + [100 - 20]\, e^{-t/1.2}\ V,\ t \geq 0 \Rightarrow 20 + 80e^{-t/1.2}$

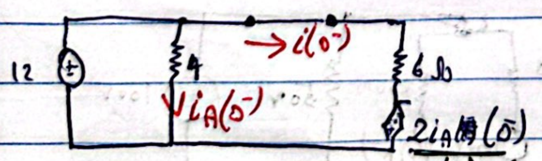
$\therefore i(t) = \frac{v(t)}{200} = \frac{20}{200} + \cancel{\frac{80}{200}} + \frac{80e^{-t/1.2}}{200}\ A,\ t > 0$

Exp:-



find $v_0(t)$ for $t > 0$

for $t < 0$, $\boxed{t \to 0^-}$



$2(3) = 6V$

$i_A(0^-) = \frac{12}{4} = 3A$

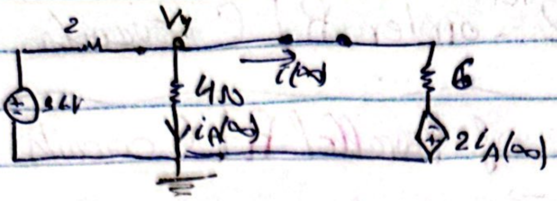
$\cancel{i(0^-)}$, $i(0^-) \to -12 + 6i(0^-) - 6 = 0$

$6i(0^-) = 18 \to i(0^-) = \frac{18}{6} = \boxed{3A}$

OR $\to i(0^-) = \frac{\cancel{12}\ 18}{6} = \boxed{3A}$

$$\boxed{i(0^-) = i(0^+) = 3A}$$

$\boxed{t \to \infty}$



$$\frac{v_y - 36}{2} + \frac{v_y}{4} + \frac{v_y + 2i_A(\infty)}{6} = 0, \quad i_A(\infty) = \frac{v_y}{4}$$

$$6 \times \left(\frac{v_y - 36}{2} + \frac{v_y}{4} + \frac{v_y + 2\frac{v_y}{4}}{6} = 0 \right)$$

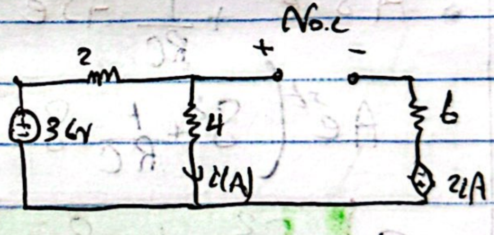
$$3v_y - 108 + \frac{3}{2}v_y + v_y + \frac{1}{2}v_y = 0$$

$$6v_y = 108 \rightarrow \boxed{v_y = 18\,\text{Volt}}$$

$$\hookrightarrow i(\infty) = \frac{18 + 2\left(\frac{18}{4}\right)}{6} = \boxed{4.5A}$$

$$\boxed{i(\infty) = 4.5A}$$

$$\tau = \frac{L}{R_{eq}}, \quad t > 0$$



$$R_{eq} = \frac{v_{oc}}{\underset{i(\infty)}{I_{s.c}}} = \frac{v_{o.c}}{4.5}$$

$$v_{o.c} = v_{4\Omega} + 2\,i(A)$$

$$= \left(\frac{4}{4+2} \cdot 36\right) + \cancel{2A}\ 2\left(\frac{24}{6}\right)$$

$$= 24 + 12 = \boxed{36V}$$

$$\rightarrow R_{eq} = \frac{36}{4.5} = \boxed{8\Omega} \longrightarrow \boxed{\tau = \frac{3}{8}}$$

$$\therefore i(t) = i_f + \{i(0^+) + i_f\}\, e^{-t/\tau}$$

$$i(t) = 4.5 + [3 - 4.5]\, e^{-8/3\,t}\ A, \quad t \geq 0$$

$$\because v(t) = \cancel{L}\ R\,i(t)$$

$$= \boxed{6\,i(t)}$$