7.1 and 7.2

Saturday, April 24, 2021 11:05 AM

7.1 Q 21

- $f(x) \to y$
- solve for x
- $y \leftrightarrow x$
- $y \to f^{-1}(x)$

$y = \boxed{f(x) = x^3 - 1}$ Find $f^{-1}(x)$

$y = x^3 - 1$

$y + 1 = x^3$

$(x^3)^{\frac{1}{3}} = (y+1)^{\frac{1}{3}}$

$x = \sqrt[3]{y+1}$

$y = \sqrt[3]{x+1}$

$\boxed{f^{-1}(x) = \sqrt[3]{x+1}}$

$D(f^{-1}) = \mathbb{R} = R(f)$

$D(f) = \mathbb{R}$

$= R(f^{-1})$

$D(f) = R(f^{-1})$

$R(f) = D(f^{-1})$

Th

$f : D \longrightarrow R$

$a \longrightarrow b = f(a)$

$f' : D \longrightarrow R$

$f'' : D \longrightarrow R$

$\vdots$

$f(a), f'(a), f''(a) \ldots$

$f^{-1} : R \longrightarrow D$

$b \longrightarrow a = f^{-1}(b)$

$(f^{-1})' : R \longrightarrow D$

$(f^{-1})'' : R \longrightarrow D$

$b = f(a)$

$f^{-1}(b) = \cancel{f^{-1}}(\cancel{f}(a)) = a$

$f^{-1}(b) = a$

$(f^{-1})'(b), (f^{-1})''(b), \ldots$

$f'(b)$ V

$f'(b)$ X

$(f^{-1})'(a)$ X

Th

$f(x)$ 1-1 $\Rightarrow$ $f^{-1}(x)$ if diff

$b = f(a)$

$a = f^{-1}(b)$

$$\left.\frac{df^{-1}}{dx}\right|_{x=\boxed{b}} = \frac{1}{f'(\underset{\overset{\uparrow}{f^{-1}(b)}}{a})}$$

Q44 $y = g(x)$ has inverse $g^{-1}(x)$ $\Rightarrow$ g is 1-1

$\boxed{g(0)=0}$

$g'(0) = 2$

Find $\left.\frac{dg^{-1}}{dx}\right|_{x=0}$ $\rightarrow b$

$b = g(a)$

$\boxed{0 = g(a)}$ $\rightarrow$ $\boxed{a=0}$

$$\left.\frac{dg^{-1}}{dx}\right|_{x=\underset{b}{0}} = \frac{1}{g'(\underset{\uparrow}{a})}$$

$$= \frac{1}{g'(0)} = \frac{1}{2}$$

7.2 Q14 $y = (\ln x)^3$

$$y' = 3(\ln x)^2 \frac{1}{x}$$ ✓

Q24 $y = \ln\left[\underbrace{\ln(\ln x)}_{g(x)}\right]$

$$y = \ln g(x)$$

$$y' = \frac{g'(x)}{g(x)}$$

$$g(x) = \ln(\underbrace{\ln x}_{f})$$

$$g'(x) = \frac{\overset{f'}{\frac{1}{x}}}{\underset{f}{\ln x}}$$

$$= \frac{1}{x \ln x}$$

$$y' = \frac{g'(x)}{\ln(\ln x)}$$

$$= \frac{\frac{1}{x \ln x}}{\ln(\ln x)}$$

$$= \frac{1}{x \ln x \ln(\ln x)}$$

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(41) $\int_0^{\pi} \frac{\sin t}{2-\cos t}\, dt$ ✓

$u = 2 - \cos t$

$du = \sin t \, dt$

$t = 0 \Rightarrow u = 2 - 1 = 1$

$t = \pi \Rightarrow u = 2 - -1 = 3$

$$\int^3 \frac{du}{u}$$

$$\int_{1} \frac{\quad}{u}$$

$$\ln|u| \Big|_{1}^{3} = \ln 3 - \ln 1 = \ln 3 - 0 = \boxed{\ln 3}$$

7.1 Q 32 $f(x) = y = \frac{\sqrt{x}}{\sqrt{x}-3}$ Find

$\boxed{x \geq 0}$

$\sqrt{x} - 3 \neq 0$

$\sqrt{x} \neq 3$

$\boxed{x \neq 9}$

$\mathbb{R} \setminus \{9\}$ ×

$(0, \infty) \setminus \{9\}$ ×

✓ $\boxed{[0,9) \cup (9, \infty)}$

① $D(f) = [0,9) \cup (9, \infty)$

$= R(f^{-1})$

② $R(f^{-1}) = D(f) = [0,9) \cup (9, \infty)$

③ $f^{-1}(x)$ $y = \frac{\sqrt{x}}{\sqrt{x}-3}$

- $f(x) \to y$
- solve for x

$y(\sqrt{x} - 3) = \sqrt{x}$

$y\sqrt{x} - 3y = \sqrt{x}$

$y\sqrt{x} - \sqrt{x} = 3y$

$\sqrt{x}(y-1) = 3y$

$\sqrt{x} = \frac{3y}{y-1}$

$y - 1$

$$x = \left(\frac{3y}{y-1}\right)^2$$

- $y \leftrightarrow x$

$$y = \left(\frac{3x}{x-1}\right)^2$$

- $y \rightarrow f^{-1}(x)$

$$f^{-1}(\overset{\uparrow}{x}) = \left(\frac{3\overset{b}{x}}{\underset{\uparrow b}{x}-1}\right)^2 \to a \Rightarrow \cancel{D(f^{-1}) = \mathbb{R} \setminus \{1\}} \times \quad X$$

④ $\underline{\underline{D(f^{-1})}} = \underline{\underline{R(f)}}$

$f : D \to R$ | $f^{-1} : R \to D$

$a \to b$ | $\boxed{b} \to a$

المدخل

$\Downarrow$

f المدى

$$\overset{f(x)}{\underset{\uparrow}{y}} = \frac{\sqrt{x}}{\sqrt{x}-3} \qquad D(\underline{\underline{f}}) = \underline{\underline{[0,9)}} \cup \underline{\underline{(9,\infty)}} = R(f^{-1})$$

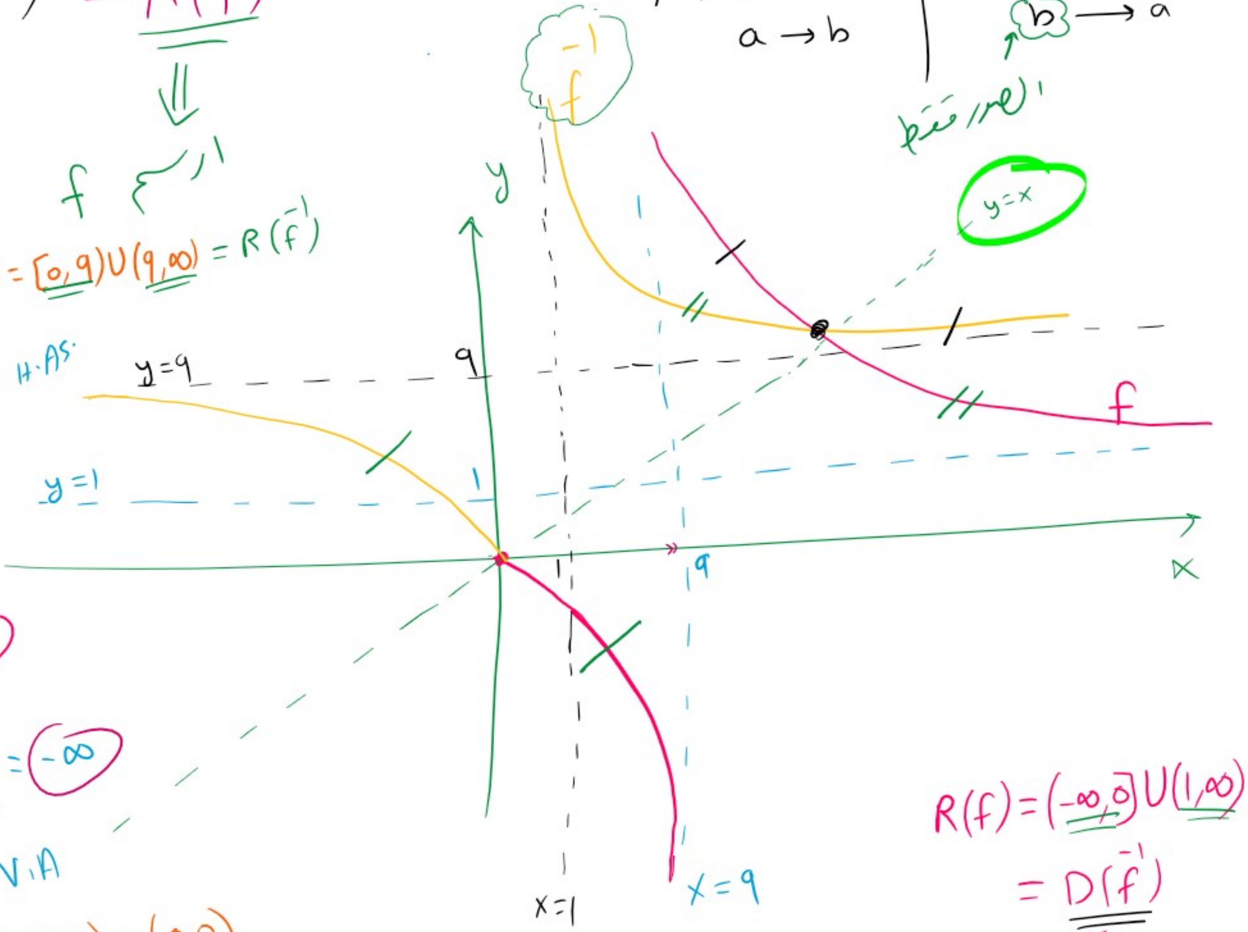
$$\lim_{x\to+\infty} f(x) = 1 \Rightarrow y = 1 \text{ H.AS.}$$

$$\lim_{x\to 9^+} f(x) = \lim_{x\to 9^+} \frac{\sqrt{x}}{\sqrt{x}-3} = \frac{3}{\text{small}+} = \infty$$

$$\lim_{x\to 9^-} f(x) = \frac{1}{\text{small}-} = -\infty$$

$\boxed{x = 9}$ V.A

Keypoint $(0, f(0)) = (0,0)$



$R(f) = (\underline{\underline{-\infty,0}}] \cup (\underline{\underline{1,\infty}})$
$= \underline{\underline{D(f^{-1})}}$

$f^{-1}(x) = \left(\frac{3x}{x-1}\right)^2$

⑤ sketch f, f^{-1}

$$f^{-1}(x) = \left(\frac{3x}{x-1}\right)$$

$$\lim_{x \to \pm\infty} f^{-1}(x) = \left(\frac{3}{1}\right)^2 = 9 \Rightarrow \boxed{y=9}$$

H. Asy.

$$\lim_{x \to 1^+} f^{-1}(x) = \left(\frac{3}{\text{small} +}\right)^2 = \infty$$

$$\lim_{x \to 1^-} f^{-1}(x) \quad \underline{\text{DNE}}$$