Exercises :

Q32: If $P(C_1) > 0$ and if $C_2, C_3, C_4, \ldots$ are mutually disjoint sets, show that $P(C_2 \cup C_3 \cup \ldots \mid C_1) = P(C_2 \mid C_1) + P(C_3 \mid C_1) + \ldots$.

$$\rightarrow P(C_2 \cup C_3 \cup \ldots \mid C_1) = \frac{P(C_2^* \cap C_3^* \cap \ldots)^c}{P(C_1)}$$

$$= \frac{P((C_2 \cup C_3 \cup \ldots) \cap C)}{P(C_1)}$$

$$= \frac{P(C_2) + P(C_3) + \ldots \cap C}{P(C_1)}$$

$$= \frac{P(C_2 \cap C)}{P(C_1)} + \frac{P(C_3 \cap C)}{P(C_1)} + \ldots$$

$$= P(C_2 \mid C_1) + P(C_3 \mid C_1) + \ldots.$$

Q36: ... 8 pairs of socks. If 6 socks are taken at Random and without replacement compute the prob. that their is at least one matching (M.P) pair among these 6 socks.

↳ prob. that there is Not a matching pair.

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p(event A happening) = 1 − p(event Not happening)

$P(\text{at least } 1 \text{ M.P}) = 1 - P(\text{zero M.P})$

$= 1 - \big(P(\text{pick any sock } 1^{st}) \times P(2^{nd} \text{ sock doesn't match } 1^{st}) \times P(3^{rd} \ldots) \times P(4^{th}) \times P(5^{th}) \times P(6^{th})\big)$

$$= 1 - \left(\frac{16}{16} \times \frac{14}{15} \times \frac{12}{14} \times \frac{10}{13} \times \frac{8}{12} \times \frac{6}{11}\right)$$

$$= \frac{111}{143}.$$

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Q37: A bowl contains 10 chips. 4 are red, 5 are white, 1 is blue. If 3 chips are taken at Random and without replecment. compute the conditional prob. that there is ① 1 chip of each color relative to the hypothesis ② that there exactly 1 red chip among the 3.

C_1: 4 red , C_2: 5 white , C_3: 1 Blue.

① $P(\text{1 chip of each color}) = \dfrac{\binom{4}{1}\binom{5}{1}\binom{1}{1}}{\binom{10}{3}} = \dfrac{4\times5\times1}{120} = \dfrac{1}{6}$.

② $P(\text{exactly 1 red among 3}) = \dfrac{\binom{4}{1}\binom{6}{2}}{\binom{10}{3}} = \dfrac{4\times15}{120} = \dfrac{1}{2}$.

Q38: Bowl I contains 3 Red and 7 Blue chips. Bowl II contains 6 red and 4 blue chips. A bowl selected random and then 1 chip is drawn from this bowl.

a. compute the prob. that this chip is red.

$$P(R) = \underbrace{P(\text{Red from I})}_{P(R|I)} \cdot P(I) + \underbrace{P(\text{Red from II})}_{P(R|II)}\, P(II)$$

$$= \frac{3}{10}\cdot\frac{1}{2} + \frac{6}{10}\times\frac{1}{2} = \frac{9}{20}.$$

b. Relative to the hypothesis that the chip is red, find the conditional prob. that it is drawn from bowl II?

$$P(II|R) = \frac{P(II)\cdot P(R|II)}{P(R)} = \frac{\frac{1}{2}\times\frac{6}{10}}{\frac{9}{20}} = \frac{\frac{6}{\cancel{20}}}{\frac{9}{\cancel{20}}} = \frac{6}{9} = \frac{2}{3}$$

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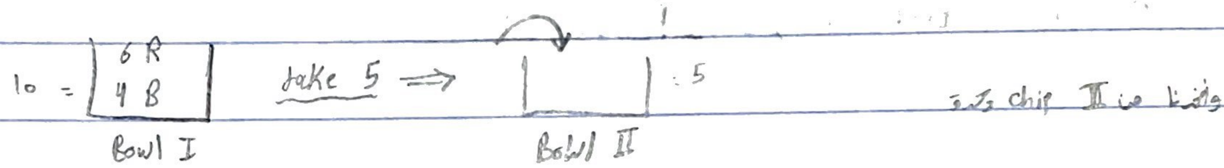
Q39: Bowl I contains 6 red and 4 blue chips. 5 of these 10 chips are selected Random and without replacement and put in bowl II, which was originally empty. one chip is the drawn at Random from bowl II. Relative to the hypothesis that this chip is blue, Find the conditional prob. that 2 Red and 3 Blue chips are transferred From bowl I to bowl II.



① let E_i denote the event that i red chips were initially transferred.

so $P(E_i) = \dfrac{\binom{6}{5-i}\binom{4}{i}}{\binom{10}{5}}$ for $i = 0, 1, 2, 3, 4$.

and let B denote the event that the final chip is blue.

so $P(B \mid E_i) = \dfrac{i}{5}$.

⇒ we want to find $P(E_3 \mid B)$

$$P(E_3 \mid B) = \frac{P(B \mid E_3)\, P(E_3)}{P(E_1)P(B \mid E_1) + P(E_2)P(B \mid E_2) + P(E_3)P(B \mid E_3)}$$

$=$ ______

* $P(B \mid E_3) = \dfrac{3}{5}$
* $P(B \mid E_1) = \dfrac{1}{5}$
* $P(B \mid E_2) = \dfrac{2}{5}$

$P(E_1) = \dfrac{\binom{6}{4}\binom{4}{1}}{\binom{10}{5}} = 15 \times 4$

$P(E_2) = \dfrac{\binom{6}{3}\binom{4}{2}}{\binom{10}{5}} = 20 \times 6$

$P(E_3) = \dfrac{\binom{6}{2}\binom{4}{3}}{\binom{10}{5}} = 15 \times 4$

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Q4o : has 2 boxes of computer disks . box C_1

$\star P(C|C_1) = \frac{3}{10}$

$\star P(C|C_2) = \frac{8}{10}$

• $P(C|C_1) = \frac{3}{10}$, $P(C|C_2) = \frac{8}{10}$

• $P(C) = P(C_1)\,P(C|C_1) + P(C_2)\,P(C|C_2)$

$= \frac{2}{3} \cdot \frac{3}{10} + \frac{1}{3} \cdot \frac{8}{10}$

$= \frac{6}{30} + \frac{8}{30} = \frac{14}{30}$

$\Rightarrow P(C_1|C) = \frac{P(C_1)P(C|C_1)}{P(C)} = \frac{\frac{2}{3} \times \frac{3}{10}}{\frac{14}{30}} = \frac{\cancel{2}^{1}}{\cancel{10}_{1}} \times \frac{\cancel{30}^{3}}{\cancel{14}_{7}} = \frac{3}{7}$

$\Rightarrow P(C_2|C) = \frac{P(C_2)\,P(C|C_2)}{P(C)} = \frac{\frac{1}{3} \times \frac{8}{10}}{\frac{14}{30}} = \frac{8 \div 2}{14 \div 2} = \frac{4}{7}$

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