Chapter 7.1, Problem 27E

Problem

Let *S* be the set of all strings of *a*'s and *b*'s.

a. Define $f : S \to Z$ as follows: For each string *s* in *S*

$$f(s)\begin{cases}\text{the number of } b\text{'s to the left} \\ \text{of the left-most } a \text{ in } s \\ 0 \quad \text{if } s \text{ contains no } a\text{'s.}\end{cases}$$

Find *f (aba), f (bbab)* and *f (b)*. What is the range of *f* ?

b. Define *g*: $S \to S$ as follows: For each string *s* in *S*,

g (s) = the string obtained by writing the characters of *s* in reverse order.

Find *g(aba), g(bbab)*, and *g(b)*. What is the range of *g*?

Step-by-step solution

Step 1 of 3

$f(s) =$ number of *b* variables on the left side of the leftmost *a*

$= 0$ if there are no *a* variables in the string.

Step 2 of 3

(a) $f(aba) = 0$

The leftmost variable in the string is an *a*, and so it has no *b* variables on its left.

$f(bbab) = 2$

There is only one *a,* and so it is the leftmost *a*

To its left, there are 2 *b* variables, consequently $f(bbab) = 2$

$f(b) = 0$, since there are no *a* variables.

The range of such a function *f* is $\{0\} \cup Z^+$

Step 3 of 3

(b) $g(aba) = aba$

By reversing the given string

$g(bbab) = babb$

$g(b) = b$

The range of *S* is the strings *a*'s, *b*'s whose length varies from zero to infinity