جامعة بيرزيت

BIRZEIT UNIVERSITY

ANSWER BOOKLET

Student: Digital	Number 2
Course: Department:	Number:
Division:	Instructor:
Date: Day	Month / Year

Dr. Abdellatif Abu Issa

For Instructor's Use

Question	Grade
1	
2	
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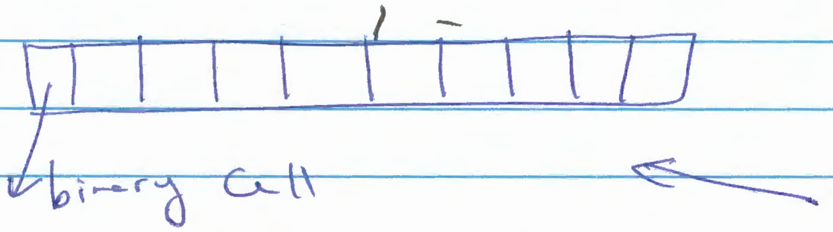
10	
11	
12	
Total	

Registers.



⇒ 2^n different combinations.

Binary Logic:

~~Based on logic, can~~

* True, false
  yes, no
  (1, 0) → bits

* 3- basic logical operations:
* AND, OR, NOT

1. AND e.g $a = b.c$ or $a = bc$

   $a = 1$ if $b = 1$ && $c = 1$
   otherwise $a = 0$

② OR: $a = b + c$

$a = 0$ if $b = 0$ && $c = 0$

otherwise $a = 1$

③ NOT $a = \bar{b}$

→ $a = 1$ if $b = 0$

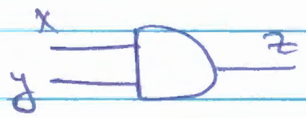
$a = 0$ if $b = 1$

* These are logical operation not arithmetic operations.

$1 + 1 = 10$ (arithmetic).

$1 + 1 = 1$ (logic)

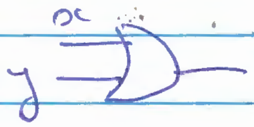
④ Logic Gates



$(z = x \cdot y)$

x	y	z
0	0	0
0	1	0
1	0	0
1	1	1

*



$z = x + y$

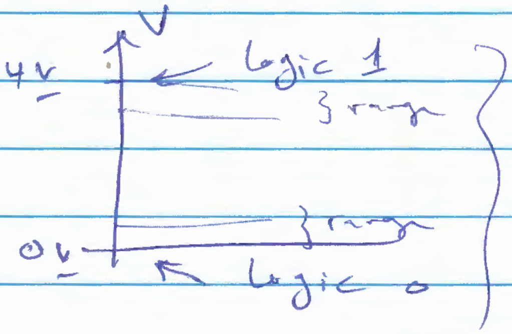
x	y	z
0	0	0
0	1	1
1	0	1
1	1	1

*

z $\quad z = \bar{x}$

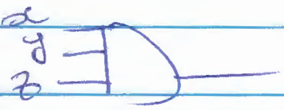
x	z
0	1
1	0

*



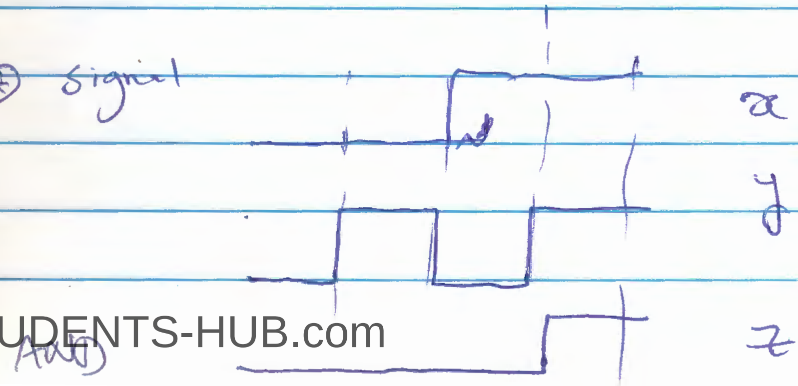
not alway

* gates may have multiple input (AND + OR)



w $\quad (w = x \cdot y \cdot z \cdot)$

* signal



CHAPTER 2

Boolean Algebra & Logic Gates

- A set of elements is any collection of objects having a common property.

- If S is a set, and x and y are certain objects, then $x \in S$ denotes that x is a member of the set, S, and $y \notin S$ denotes that y is not an element of S.

- The most common postulates used to formulate various algebraic structures are:-

1. Closure: A set S is closed with respect to a binary operator if, for every pair of elements of S, the binary operator specifies a rule for obtaining a unique element of S.

e.g natural Number $N = \{1, 2, 3 \dots\}$ is closed for binary operation $(+)$ because for any $a, b \in N \Rightarrow c = a + b \in N$

eg: it is not closed for $(-)$ since

~~2-3 = -1 ∉~~ $2 - 5 = -3 \notin S$

2. Associative law: A binary operator $*$ on a set S is said to be associative if

$$(x*y)*z = x*(y*z) \text{ for all } x,y,z \in S$$

e.g. in N $\times$ is Associative

3. Commutative law: A binary operator $*$ on a set S is said to be commutative whenever

$$x*y = y*x \quad \text{for all } x,y \in S$$

in N $\times$ is commutative ($-$ is not).

4. Identity element: A set S is said to have an identity element with respect to a binary operation $*$ on S if there exists an element $e \in S$ with the property

$$e*x = x*e = x \quad \text{for all } x \in S.$$

e.g. 0 is identity for $+$ on N

1 is identity for $\times$ on N.

5. Inverse: A set S having the identity element e with respect to a binary operator $*$ is said to have an inverse whenever, for every $x \in S$, there exists an element $y \in S$ such that

$$x*y = e.$$

e.g. in $S = \{\ldots, -3, -2, -1, 0, 1, 2 \ldots\}$

the inverse of a is $-a$

6- Distributive law: if $*$ and $\cdot$ are two binary operators on a set S, $*$ is said to be distributive over $\cdot$ if

$$x * (y \cdot z) = (x * y) \cdot (x * z)$$

& fractions

E.g The set of real numbers $(\dots -1, 0, 1, \dots)$ with the binary operator $+$ & $\cdot$, form the field of real number. The operators and postulates have the following meanings:-

(1) $+$ binary operator defines addition

- closure ✓
- Associative $(5+2)+3 = 5+(2+3)$
- commutative $(5+7) = 7+5$
- Identity element 0 $(5+0=0+5=5)$
- Inverse ; a inverse is $-a$

$5+(-5) = (-5)+5 = 0$ (identity)

(2) $\cdot$ binary op defines multiplication

- closure
- Associative
- commutative
- Identity
- Inverse

→ Distributive law : $(\cdot)$ is distributive over $(+)$

$$a \cdot (b+c) = (a \cdot b)+(a \cdot c)$$

$$a+(b \cdot c) \neq (a+b) \cdot (a+c)$$

Axiomatic definition of boolean algebra

- Boolean algebra is an algebraic structure defined by a set of elements, B, together with two binary operators, + and ·, provided that the following postulates are satisfied

1- (a) Closure with respect to (+).
(b) " " " " (·)

2- (a) An identity element with respect to +, designated by 0: $x+0=0+x=x$
(b) An identity element with respect to (·), designated by 1 : $x.1=1.x=x$.

3- (a) commutative with respect to +: $x+y=y+x$
(b) " " " " ·: $x.y=y.x$

4- (a) (·) is distributive over +: $x.(y+z)=(x.y)+(x.z)$
(b) + " " " (·): $x+(y.z)=(x+y).(x+z)$

5- for every $x\in B$, there is $x'\in B$ (complement of x) such that
(a) $x+x'=1$
(b) $xx'=0$

(6) There exists at least two elements $x,y\in B$ such that $x\neq y$.

notes

- Distributive law for (·) over (+) & (+) over (·) in Boolean algebra (not valid for ordinary algebra).

- Boolean algebra doesn't have additive or multiplicative inverses; therefore, there are no subtraction or division operations

- complement is not available in ordinary algebra.

- B is defined as a set with only two elements, 0 & 1 while ordinary algebra deals with the real numbers, ~~which~~ constitute an infinite set of elements.

* Two-Valued Boolean Algebra

* ~~This algebra~~ It is defined on a set of two elements, $B = \{0, 1\}$ and two binary operators (+) & (·).

The following Table shows the operations

x	y	$x \cdot y$	$x+y$
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

x	x'
0	1
1	0

* As shown in the table, these rules are exactly the same as the AND, OR, and NOT operations, respectively.

The postulates are valid

1- closure : The result is either 0 or 1
$\Rightarrow$ the result always $\in B$.

2- commutative $0+1 = 1+0 = 1$
$(0).(1) = (1).(0) = 0$

3- identity element
- $0+1 = 1+0 = 1$
$0+0 = 0 \Rightarrow 0$ is identity element for (+)

~~1.0~~

- $1.0 = 0.1 = 0$
$1.1 = 1 \Rightarrow 1$ is identity elem for (.)

4. distributive $x.(y+z) = x.y + x.z$

x	y	z	$y+z$	$x.(y+z)$	$x.y$	$x.z$	$(x.y)+(x.z)$
0	0	0					
0	0	1					
0	1	0					
0	1	1					
1	0	0					
1	0	1					
1	1	0					
1	1	1					

als. distributive law of + over (.)

$x+(y.z) = (x+y).(x+z)$

5- complement

$x+x' = 1$ } $\rightarrow$ demo by tables

$x.x' = 0$

6- two distinct element 1 & 0 $(1 \neq 0)$.

* Basic Theorems And Properties of Boolean Algebra

* Duality principle

Every algebraic expression deducible from the postulates of Boolean algebra remains valid if the operators and identity elements are interchanged.

e.g. $0+0=0 \xrightarrow{\text{by duality}} 1.1=1$

$0+1=1 \longrightarrow 1.0=0$

& the same thing for variables.

Basic Theorems

1a) $x + x = x$

Proof

$$x + x = (x + x) \cdot 1 \qquad (2b)$$
$$= (x + x) \cdot (x + x') \qquad (5a)$$
$$\cancel{= xx + xx + xx' + xx'}$$
$$= x + xx' \qquad (4b)$$
$$= x + 0 \qquad (5b)$$
$$= x$$

(1b) $x \cdot x = x$

$$x \cdot x = xx + 0 \qquad (2a)$$
$$= xx + xx' \qquad (5b)$$
$$= x(x + x') \qquad (4a)$$
$$= x \cdot 1 \qquad (5a)$$
$$= x$$

Theorem

(2a) $x + 1 = 1$

$$x + 1 = 1 \cdot (x + 1)$$
$$= (x + x')(x + 1)$$
$$= x + x' \cdot 1$$
$$= x + x' = 1$$

(2b) $x \cdot 0 = 0$ (by duality)

$$x \cdot 0 = \cancel{x \cdot (xx')}\; 0 + (x \cdot 0)$$
$$= (xx') + (x \cdot 0)$$
$$= x(x' + 0)$$
$$= xx' = 0$$

③ $(x')' = x$

④a $x + (y + z) = (x + y) + z$
④b $x(yz) = (xy)z$
} associative

⑤a $(x + y)' = x' \cdot y'$
⑤b $(xy)' = x' + y'$
} DeMorgan

⑥a $x + xy = x$
⑥b $x(x + y) = x$
} absorption

Truth Table proof

$x + xy = x$

x	y	xy	$x+xy$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

$\Rightarrow x + xy = x$

x	y	$x+y$	$x(x+y)$
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

$\Rightarrow x(x+y) = x.$

all possible combinations

* Proof that $(xy)' = x' + y'$ using truth table

x	y	xy	$(xy)'$	x'	y'	$x'+y'$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

identical for all possible cases

$\Rightarrow (xy)' = x' + y'$

* Operator Precedence

1) Parentheses
2) NOT (complement)
3) AND
4) OR

e.g $x'y + z$

1) complement x
2) make x' & y
3) make $x'y + z$

Boolean Functions :-

- If x, y and z are binary variables $(0,1)$, then $F_1 = x + y'z$ is an example of a Boolean function.

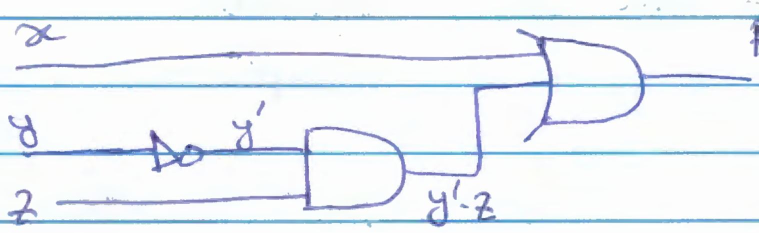
$$F_2 = xyz \quad , \quad F_3 = x'yz' \ \ldots \text{etc.}$$

- A Boolean function can be represented in a truth table. A truth table is a list of combinations of 1's and 0's assigned to the binary variables and a column that shows the value of the function for each binary combination. The number of rows in the truth table is 2^n, where n is the number of variables (inputs) in the function.

Ex. $F1 = x + y'z$

x	y	z	y'	$y'z$	$x + y'z = F1$
0	0	0	1	0	0
0	0	1	1	1	1
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	1	0	1
1	0	1	1	1	1
1	1	0	0	0	1
1	1	1	0	0	1

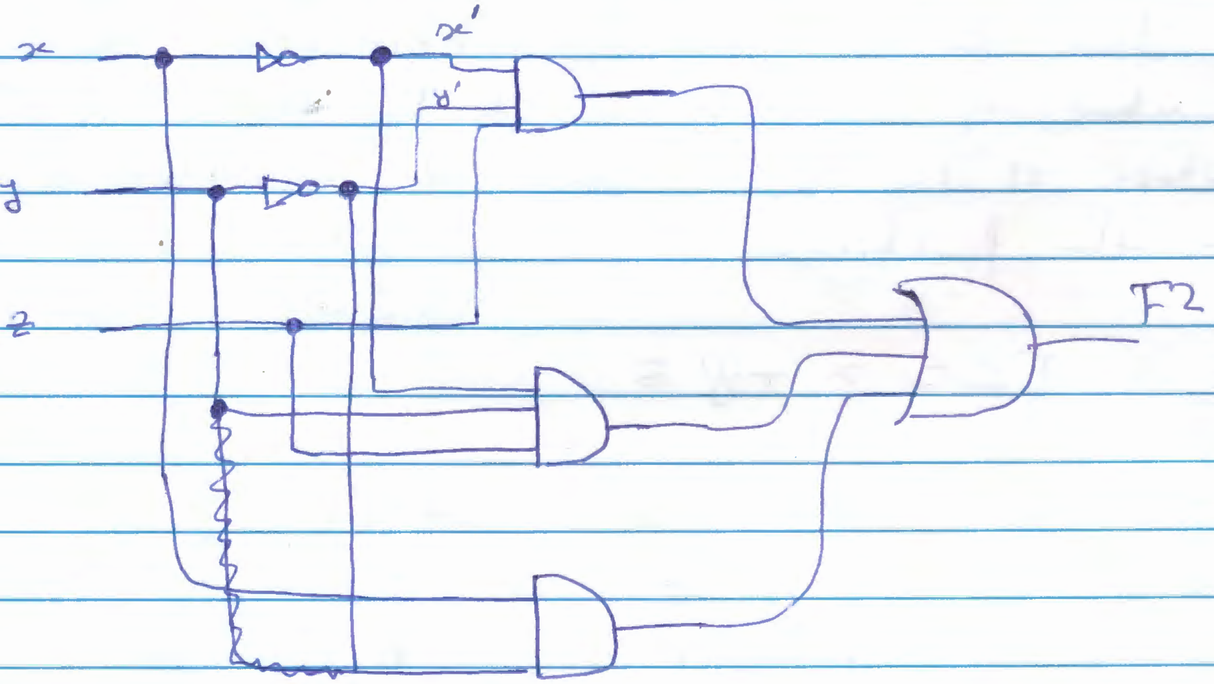
- Gate implementation of F_1



$F_1 = x + y' \cdot z$

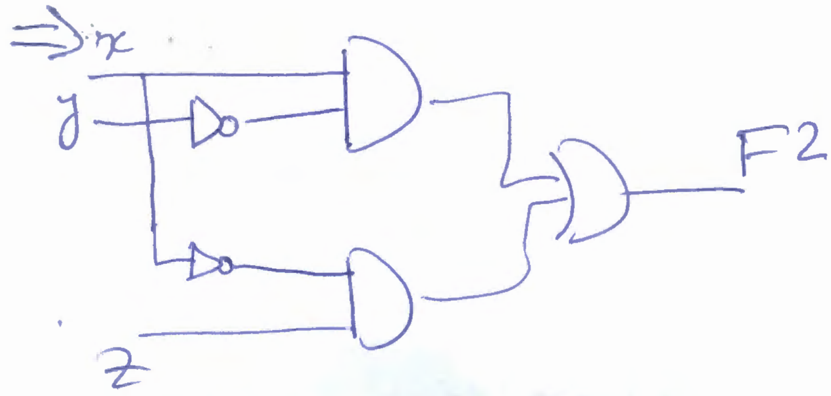
Ex. 2

$$F2 = x'y'z + x'yz + xy'$$



The question: Is it possible to make a simpler implementation??

$$F2 = x'y'z + x'yz + xy'$$
$$= x'z(y + y') + xy'$$
$$= x'z + xy'$$



~~t~~ sim

* This is a simpler implementation more than the previous one.
* Algebraic Manipulation was used to acheive the simple formula.
* Other methods for simplification will be described later.