Trigonometric Identities

- $\sin^2 x + \cos^2 x = 1$ ... *
- $\tan^2 x + 1 = \sec^2 x$ Divide * by $\cos^2 x$
- $1 + \cot^2 x = \csc^2 x$ Divide * by $\sin^2 x$
- $\sin 2x = 2 \sin x \cos x$
- $\cos 2x = \cos^2 x - \sin^2 x$
  $= 2\cos^2 x - 1 \Rightarrow \cos^2 x = \frac{1+\cos 2x}{2}$ ✓
  $= 1 - 2\sin^2 x \Rightarrow \sin^2 x = \frac{1-\cos 2x}{2}$ ✓
- $\cos(A+B) = \cos A \cos B - \sin A \sin B$
- $\sin(A+B) = \sin A \cos B + \cos A \sin B$

Exp $\sin(x+2\pi) = \sin x \cos(2\pi) + \cos x \sin(2\pi) = \sin x$ ✓
$\sin(x+\pi) = \sin x \cos\pi + \cos x \sin \pi = -\sin x$
$\cos(x+\pi) = \cos x \cos \pi - \sin x \sin \pi = -\cos x$
$\cos(x+\frac{\pi}{2}) = \cos x \cos\frac{\pi}{2} - \sin x \sin\frac{\pi}{2} = -\sin x$

- **Even function** defined on interval I is symmetric about y-axis and satisfy $f(-x) = f(x)\ \forall x \in I$.

  Exp $f(x) = x^2$, $y = x^4$, $g(x) = x^6$, $h(x) = |x|$, $r(x) = \cos x$, $m(x) = \sec x$ ... are even

- **Odd function** defined on interval I is symmetric about origin $(0,0)$ and satisfy $f(-x) = -f(x)\ \forall x \in I$.

  Exp $f(x) = x$, $y = x^3$, $g(x) = x^5$, $h(x) = \frac{1}{x}$, $r(x) = \sin x$, $m(x) = \csc x$, ... are odd

Exp ① show that $f(x) = \frac{x}{x^2-1}$ is odd function

$$f(-x) = \frac{(-x)}{(-x)^2-1} = \frac{-x}{x^2-1} = -\frac{x}{x^2-1} = -f(x)$$

② show that $g(x) = \frac{1}{x^2-1}$ is even function

$$g(-x) = \frac{1}{(-x)^2-1} = \frac{1}{x^2-1} = g(x)$$

- Composition $(f \circ g)(x) = f(g(x))$

Exp $f(x) = \sqrt{x}$, $g(x) = x^2$ ① Find $f \circ g$ and its domain
$D(f) = [0, \infty)$ $D(g) = \mathbb{R}$ ② Find $g \circ f$ and its domain

① $(f \circ g)(x) = f(g(x)) = f(x^2) = \sqrt{x^2} = |x| \Rightarrow D = \mathbb{R}$ ✓

② $(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 = x \Rightarrow D = [0, \infty)$ ✓

- $y = A \sin(B(x+c)) + D$

$|A|$: Amplitude

$$\text{period} = \frac{2\pi}{B}$$

c : Horizontal shift $\begin{cases} \text{to the left if } c>0 \\ \text{to the right if } c<0 \end{cases}$

D : Vertical shift $\begin{cases} \text{up if } D>0 \\ \text{down if } D<0 \end{cases}$
