

Step 3

$$M_{ult} : M_{u3} = -\frac{1}{4}$$

$$R_4 + \frac{1}{4} R_3 \quad \begin{bmatrix} 2 & 6 & 4 & 3 \\ 0 & -1 & -1 & 2.5 \\ 0 & 0 & 4 & -30 \\ 0 & 0 & 0 & 24 \end{bmatrix}$$

Step	+	-	* , ÷
3	1(1)		1(1), 1

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ 2 & 10 & 1 & 0 \\ -\frac{3}{2} & -10 & -\frac{1}{4} & 1 \end{bmatrix}$$

No Cost
Mult. ~~high~~

Cost of $A^{\#} = LU$

Step	+	-	* , ÷
1	3(3)		3(3) , 3
2	2(2)		2(2) , 2
3	1(1)		1(1) , 1
	14		14 , 6

$$\Rightarrow \boxed{\text{Cost} = 34}$$

$$L \cdot Y = b \Rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ 2 & 10 & 1 & 0 \\ -\frac{3}{2} & -10 & -\frac{1}{4} & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} 22 \\ 13 \\ 20 \\ 6 \end{pmatrix}$$

$$\Rightarrow y_1 = 22$$

$$\frac{1}{2} y_1 + y_2 = 13$$

$$2y_1 + 10y_2 + y_3 = 20$$

$$-\frac{3}{2} y_1 - 10y_2 - \frac{1}{4} y_3 + y_4 = 6$$

$$F.S \quad y_1 = 22$$

$$\frac{1}{2} \text{ ماضية الكاليب} \quad y_2 = 13 - \frac{1}{2} y_1$$

$$y_3 = 20 - 2y_1 - 10y_2$$

$$y_4 = 6 + \frac{3}{2} y_2 + 10y_2 + \frac{1}{4} y_3$$

$$\begin{aligned} \text{Cost of Forward Substitution} &= n^2 - n \\ &= 16 - 4 = 12 \end{aligned}$$

$$y_1 = 22$$

$$y_2 = 2$$

$$y_3 = -44$$

$$y_4 = 48$$

$$u.x = y$$

$$2x_1 + 6x_2 + 4x_3 + 3x_4 = 22$$

$$-x_2 - x_3 + 2.5x_4 = 2$$

$$4x_3 - 30x_4 = +44$$

$$24x_4 = 48$$

$$B.S \Rightarrow \text{Cost} = n^2 = 16$$

Total Cost of Solving a 4×4 system using L.U Fac

$$\Rightarrow 34 + 12 + 16 = 64$$

Now Cost LU Factorization for $n \times n$ system.

$$\text{Cost of } [A] \rightarrow [u] + [y] = b = n^2 - n$$

$$\oplus ux = y = n^2$$

Step	+	-	* , /
1	$(n-k)(n-k)$		$(n-k)(n-k), (n-k)$
...			
k	$(n-k)^2$		$(n-k)^2, (n-k)$
...			
n-1			

$$\sum_{k=1}^{n-1} [2(n-k)^2 + (n-k)]$$

$$\Rightarrow \text{let } t = n-k$$

$$t=1 \Rightarrow$$

$$t=n-1 \Rightarrow$$

$$\sum_{t=1}^{n-1} 2(t)^2 + t = 2 \sum_{t=1}^{n-1} t^2 + \sum_{t=1}^{n-1} t$$

$$= 2 \frac{(n)(n-1)(2n-1)}{6} + \frac{n(n-1)}{2}$$

$$= \frac{4n^3 - 3n^2 - n}{6}$$

$$\text{Total Cost} = \frac{4n^3 - 3n^2 - n}{6} + n^2 - n + n^2$$

$$= \frac{4n^3 + 9n^2 - 7n}{6} \approx \frac{2n^3}{3}$$

* C.E

* نفس

Gauss Jordan Reduction

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$$[A \mid b] \rightarrow [I \mid x] \text{ for system } Ax = b$$

Ex 3x3

$$[A \mid b] = \left[\begin{array}{ccc|c} & & & \\ & & & \\ & & & \end{array} \right]$$

Step	+	-	*	÷
1	(2) 3		(2) 3, 3	$3-1+1$
2	(2) (2)		2(2), 2	$3-2+1$
3	2(1)		2(1), 1	
	12		12, 6	$\Rightarrow 30$

no Mult. حافظات

3 rows

$$\left[\begin{array}{ccc|c} 1 & & & \\ & & & \\ & & & \end{array} \right]$$

n General for nxn

Step	+	-	*	÷
1				
2				
3				
⋮				
k	$(n-1)(n-k+1)$		$(n-1)(n-k+1), (n-k+1)$	

Total Cost of GFR is

$$\sum_{k=1}^n [2(n-1)(n-k+1) + (n-k+1)]$$

$$= \frac{2n^3 + n^2 - n}{2} \approx n^3$$

* Inverse Method

because there is a unique Solution

$$Ax = b$$

① Find A^{-1}

$$[A|I] \rightarrow [I|A^{-1}]$$

② $x = A^{-1} b_{n \times 1}$ Cost = $2n^2 - n$

Ex

$$\left[\begin{array}{ccc|ccc} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{array} \right]_{3 \times 3}$$

Cost of $[A|I] \rightarrow [I|A^{-1}]$

Step	+	-	*	÷
1	2(5)		2(5)	5
2	2(4)		2(4)	4
3	2(3)		2(3)	3
	24		24	12

2 M
3 ⇒ قيمة المتغير الثانية

Cost of $A^{-1} = 60$

Cost of $A^{-1}b = 2(3)^2 - 3 = 15$

Total Cost = $60 + 15 = 75$

For $n \times n$

Step	+, -	*, /
1		
2		
⋮		
K	$(n-1)(2n-K)$	$(n-1)(2n-K), (2n-K)$
⋮		
n		

Cost of A^{-1} is

$$\sum_{k=1}^n (2(n-1)(2n-k) + 2n-k)$$

$$\frac{6n^3 - 5n^2 + n}{2}$$

Total Cost of Inverse Method

$$\frac{6n^3 - 5n^2 + n}{2} + 2n^2 - n = \left(\frac{6n^3 - n - n}{2} \right) \approx 3n^3$$

السرعة ↗

G.E $\frac{2}{3} n^3$

LU Fac $\frac{2}{3} n^3$

G-J-R n^3

Inverse $3n^3$

Cramer's $e(n+1)!$

Slowest

$$M_{21} = \frac{24.14}{1.133} = 21.31$$

التقريب بعد
كل خطوة

$$\left[\begin{array}{cc|c} 1.133 & 5.281 & 6.414 \\ 0 & -113.7 & -113.8 \end{array} \right]$$

✓

درون حباب

$$\begin{aligned} & -1.210 - (21.31)(5.281) \\ & -1.210 - 112.5 \\ \Rightarrow & -113.7 \end{aligned}$$

$$\begin{aligned} & 22.93 - (21.31)(6.414) \\ & 22.93 - 136.7 \\ \Rightarrow & -113.8 \end{aligned}$$

Q5 $X_2 = \frac{-113.8}{-113.7} = 1.001$

$$X_1 = \frac{6.414 - (5.281)(1.001)}{1.133}$$

$$= \frac{6.414 - 5.286}{1.133} = \frac{1.128}{1.133} = 0.9956$$

[2]

$$\left[\begin{array}{cc|c} 24.14 & -1.210 & 22.93 \\ 1.133 & 5.281 & 6.414 \end{array} \right]$$

تبديل كل
الصف

$$M_{21} = \frac{1.133}{24.14} < 1$$

$$\boxed{X_1 = 1} \quad \boxed{X_2 = 1}$$