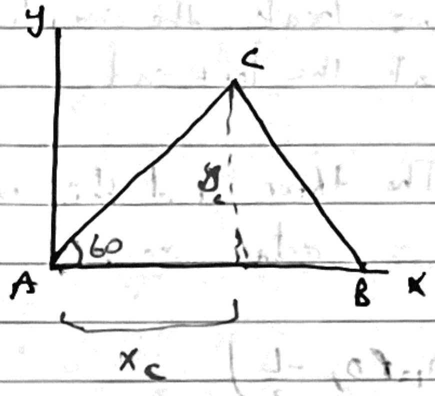
Principles of Physics (141) 10th eddition (Discussion)
Chapter 9: Center of mass and linear momentum

Problems: 1, 4, 12, 17, 37, 46, 59, 68, 76

Prablem 1: Three particles of mass 1.0 kg, 2.0 kg and 3.0 kg are placed at the vertices A, B and C, respectively of an equilateral triangle ABC of edge 1.0 m, Find the distance of their center of mass from A



Sol: Assume A at origin

$$X_{cm} = \frac{1}{M}\sum_{i=1}^{N} m_i x_i$$

$$= \frac{1}{1+2+3}\underbrace{(1\times 0 + 2\times 1 + 3\times 0.5)}_{m_A x_A + m_B x_B + m_C x_C}$$

$$\boxed{X_{cm} = \frac{7}{12}\ m}$$

$\sin 60 = \frac{y_c}{AC} = \frac{y_c}{1}$

$y_c = \sin 60 = \frac{\sqrt{3}}{2}$

$$y_{cm} = \frac{1}{M}\sum_{i=1}^{N} m_i y_i$$

$$= \frac{1}{1+2+3}\left(1\times 0 + 2\times 0 + \frac{3\times\sqrt{3}}{2}\right)$$

$$= \frac{3\times\sqrt{3}}{12}$$

$$\boxed{y_{cm} = \frac{\sqrt{3}}{4}}$$

So coordinate of center of mass $\left(\frac{7}{12}, \frac{\sqrt{3}}{4}\right)$

$\rightarrow r_{cm} = \sqrt{\left(\frac{7}{12}\right)^2 + \left(\frac{\sqrt{3}}{4}\right)^2} = 0.726 \simeq 0.73\ m$

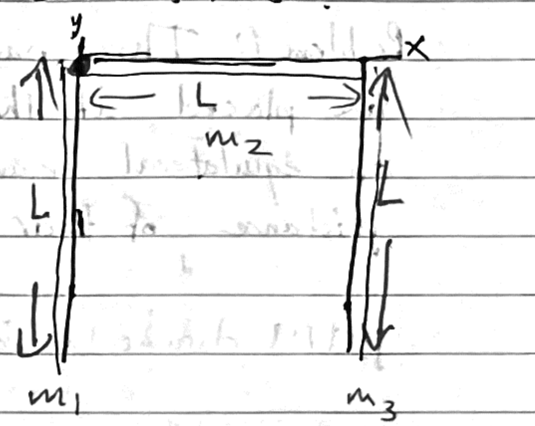
واجب

Problem 4: In fig 9-26: three uniform thin rods, each of length $L = 24\,cm$ form an inverted U. The vertical rods each have a mass of 14 g. the horizontal rod has a mass of 42 g. What are a) the x coordinate and (b) the y coordinate of the system's center of mass?

v: vertical, h: horizontal

sol: $m_1 = m_3 = m_v = 14\,g = M$

$m_2 = m_h = 42\,g = 3M$



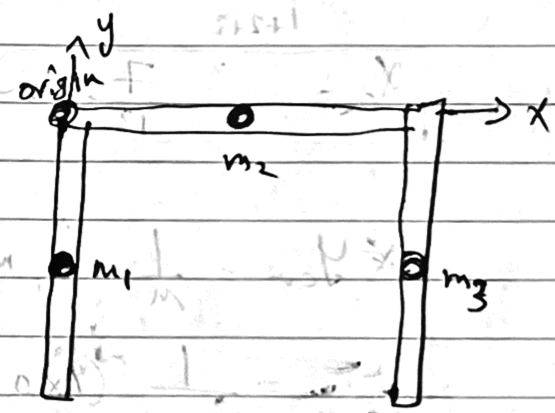
we locate the coordinate origin at the left end

The three rod thin and uniform so, each centers of mass behave as point mass

$m_v = m_1 = (0, \frac{-L}{2})$

$m_h = m_2 = (\frac{L}{2}, 0)$

$m_v = m_3 = (L, \frac{-L}{2})$



a) $X_{com} = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$

$X_{com} = \frac{m_1(0) + m_2(\frac{L}{2}) + m_3(L)}{m_1 + m_2 + m_3}$

$X_{com} = \frac{3M\frac{L}{2} + ML}{M + 3M + M} = \frac{5ML}{2} \div 5M$

$X_{com} = \frac{5ML}{2} \times \frac{1}{5M} = \frac{L}{2} = \frac{24}{2} = 12\,cm$

b) $y_{com} = \frac{\Sigma m_i y_i}{\Sigma m_i} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{M(\frac{-L}{2}) + 3M(0) + M(\frac{-L}{2})}{M + 3M + M}$

$y_{com} = \frac{-ML}{5M} = \frac{-L}{5} = \frac{-24}{5} = -4.8\,cm$

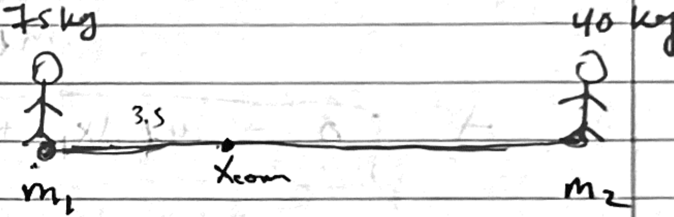
③

متزلجان

Problem 12: Two skaters, one with mass 75 kg and the other with mass 40 kg stand on an ice rink (حلبة التزلج) holding a pole of length 10 m and negligible mass. Starting from the ends (الطرف) of the pole the skaters pull themselves along the pole until they meet. How far does the 40 kg skater move?

sol: both of the skaters will end up at the center of mass

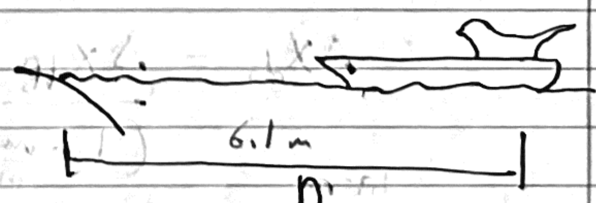
lets find X_{com} with origine at skater 1



$$X_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{75 \times 0 + 40 \times 10}{75 + 40} = 3.47 \simeq 3.5\,m \text{ away from } m_1$$

$$10 - X_{com} = 10 - 3.5 = 6.5\,m \text{ away from } m_2 = 40\,kg$$

Problem 17: In Fig 9.35a, a 4.5 kg dog stands on an 18 kg flatboat at distance D = 6.1 m from the shore (شاطئ). It walks 2.4 m along the boat toward shore and then stops. Assuming no friction between the boat and the water, find how far the dog is then from the shore (see fig 9.35 b)



Sol: No external force (No friction)

$\Rightarrow X_{com}$ constant

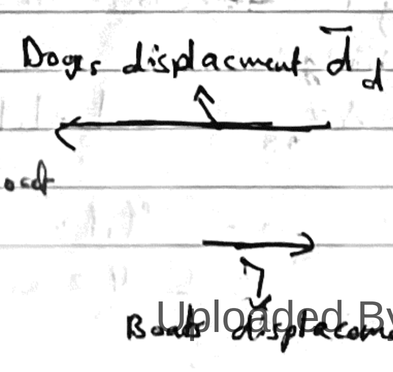
$\Rightarrow \Delta X_{com} = 0$ لعدم وجود مركز الكتلة لن يتغير

$m_d = 4.5\,kg$ mass of dog

$m_B = 18\,kg$ mass of Boat

ΔX_{dB}: displacment of dog relative to Boat

$\Delta X_{dB} = 2.4\,m$



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$$\Delta X_{dB} = \Delta X_{dS} + \Delta X_{SB}$$

ΔX_{dS}: displacement of dog relative to shore

ΔX_{SB}: displacement of shore relative to Boat

$$\boxed{\Delta X_{dB} = \Delta X_{dS} - \Delta X_{BS}} \longrightarrow (1)$$

$$\Delta X_{com} = 0 = \frac{m_d \Delta X_{dS} + m_B \Delta X_{BS}}{m_d + m_B}$$

$$\Rightarrow \boxed{0 = m_d \Delta X_{dS} + m_B \Delta X_{BS}}$$

اقسم طرفي المعادلة على m_B

$$\boxed{0 = \frac{m_d}{m_B} \Delta X_{dS} + \Delta X_{BS}} \longrightarrow (2)$$

Add eq (1) to eq (2)

$$\Delta X_{dB} + 0 = \Delta X_{dS} + \frac{m_d}{m_B} \Delta X_{dS} - \cancel{\Delta X_{BS}} + \cancel{\Delta X_{BS}}$$

$$\Delta X_{dB} = \Delta X_{dS} \left(1 + \frac{m_d}{m_B}\right)$$

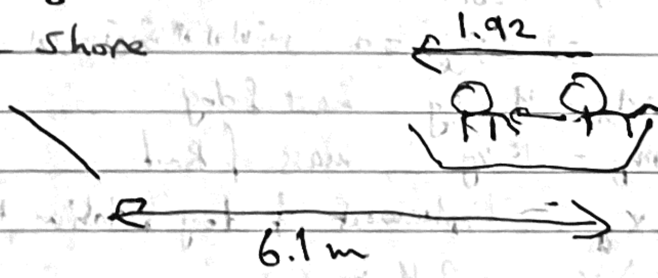
$$\Delta X_{dS} = \frac{\Delta X_{dB}}{\left(1 + \frac{m_d}{m_B}\right)} = \frac{2.4}{\left(1 + \frac{4.5}{18}\right)} = 1.92\, m$$

The dog is 1.92 m closer to the shore than initially

$\Rightarrow D - \Delta X_{dS} = 6.1 - 1.92$

$= 4.18$

$\simeq 4.2\, m$



Problem 37: A particle of unknown mass is acted upon by a force $\vec{F} = (100 e^{-2t} \hat{i})$ N. If at t = 0.00 sec the particle is at rest, for the time interval t = 0.00 sec to t = 2.00 s find (a) the impulse on the particle (b) the average force on the particle.

Sol: a) $\vec{J} = \int_{t_1}^{t_2} \vec{F}(t)\, dt$

$$= \int_0^2 100 e^{-2t}\, dt$$

$$= 100 \int_0^2 e^{-2t}\, dt$$

$$= 100 \left. \frac{e^{-2t}}{-2} \right]_0^2$$

$$= -50 e^{-2t} \Big]_0^2$$

$$= -50 \left[e^{-4} - e^{0} \right]$$

$$= -50 \left[0.0183 - 1 \right]$$

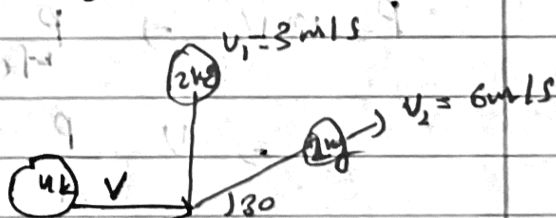
$$= -50 \times 0.98$$

$$\boxed{J = 49.1 \hat{i} \text{ N.sec}}$$

b) $F_{ave} = \dfrac{J}{\Delta t} = \dfrac{49.1}{2} = 24.5 \hat{i}$ N

صندوق ادوات

Problem 46: A 4.0 kg mess kit sliding on a frictionless surface explodes into two 2.0 kg parts: 3 m/s due north and 6.0 m/s 30° north of east. What is the original speed of the mess kit?



Sol: $m_1 = m_2 = m = 2$ kg

The linear momentum for m_1 is $\vec{P}_1$

$\vec{P}_1 = m \vec{v}_1$

$$\boxed{\vec{P}_1 = m v_1 \hat{j}}$$

(b)

Wher $m = 2\,kg$, V_1 = velocity for mass 1 = 3 m/s

P_2 : the linear momentum for mass 2

$$P_2 = m_2\vec{V_2} = m_2(V_{2x}\hat{i} + V_{2y}\hat{j})$$
$$= m(V_2\cos\theta\,\hat{i} + V_2\sin\theta\,\hat{j})$$
$$\boxed{\vec{P_2} = mV_2\cos\theta\,\hat{i} + mV_2\sin\theta\,\hat{j}}$$

Where $m_2 = m = 2\,kg$

V_2 : velocity of mass 2 = 6 m/s

- $\vec{P}_{\text{after exploding}} = \vec{P_1} + \vec{P_2}$

$$= mV_1\hat{j} + (mV_2\cos\theta\,\hat{i} + mV_2\sin\theta\,\hat{j})$$

$$= mV_2\cos\theta\,\hat{i} + mV_1\hat{j} + mV_2\sin\theta\,\hat{j}$$

$$\vec{P}_{after} = (mV_2\cos\theta)\hat{i} + (mV_1 + mV_2\sin\theta)\hat{j}$$

$$\vec{P}_{after} = (2\times 6\cos 30)\hat{i} + (2\times 3 + 2\times 6\times\sin 30)\hat{j}$$
$$= 10.39\,\hat{i} + (6 + (2\times 0.5))\hat{j}$$
$$= (10.39\,\hat{i} + 12\,\hat{j})\ kg.m/s$$

- $\Rightarrow \vec{P}_{after} = (10.39\,\hat{i} + 12\,\hat{j})\ kg\ m/s$

From conservation of ~~energy~~ linear momentum

$$P_{\text{befor exploding}} = P_{\text{after exploding}}$$

$$P_{befor} = P_{after} = \sqrt{P_x^2 + P_y^2} = \sqrt{252} = 15.87\,kg.m/s$$

كتلة الأصلية

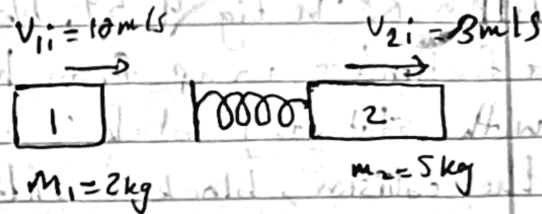
$$P = mV \Rightarrow P_{befor} = \hat{M}V$$

$$\Rightarrow V = \frac{P}{M} = \frac{15.87}{4} = 3.97\ m/s$$

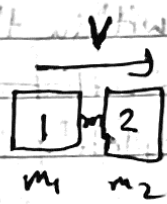
Problem 59: In Fig 9.55, block 1 (mass 2.0 kg) is moving rightward at 10 m/s and block 2 (mass 5.0 kg) is moving rightward at 3.0 m/s. The surface is frictionless, and a spring ~~constant~~ with a spring constant of 1120 N/m is fixed to block 2. When the blocks collide, the compression of the spring is maximum at the instant the blocks have the same velocity. Find the maximum compression.

~~sol:~~

Sol: the velocity v of the system when the spring reaches maximum compression x_m



$$P_i = P_f$$



$$m_1 V_{1i} + m_2 V_{2i} = (m_1 + m_2) V$$

$$\Rightarrow \boxed{V = \frac{m_1 V_{1i} + m_2 V_{2i}}{(m_1 + m_2)}} \rightarrow 1$$

$$\Delta k = k_f - k_i$$

$$= \frac{1}{2}(m_1 + m_2) V^2 - \left(\frac{1}{2} m_1 V_{1i}^2 + \frac{1}{2} m_2 V_{2i}^2\right)$$

$$= \frac{1}{2}(m_1 + m_2)\left(\frac{m_1 V_{1i} + m_2 V_{2i}}{m_1 + m_2}\right)^2 - \frac{1}{2} m_1 V_{1i}^2 - \frac{1}{2} m_2 V_{2i}^2$$

$$= \frac{(m_1 V_{1i} + m_2 V_{2i})^2}{2(m_1 + m_2)} - \frac{1}{2} m_1 V_{1i}^2 - \frac{1}{2} m_2 V_{2i}^2$$

$$= \frac{(2 \times 10 + 5 \times 3)^2}{2(2 + 5)} - \frac{1}{2} \times 2 \times 10^2 - \frac{1}{2} \times 5 \times 3^2$$

$$= 87.5 - 100 - 22.5$$

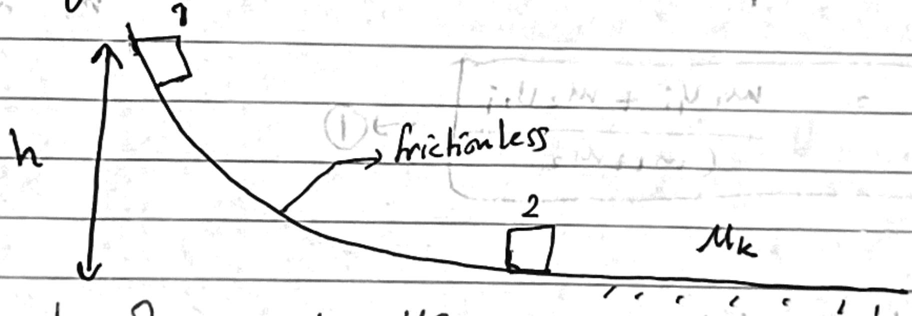
$$\Delta k = -35\,J$$

from the conservation of energy

$$\Delta k = -\frac{1}{2} k x_m^2$$

$$x_m = \sqrt{\frac{-2\Delta k}{k}} = \sqrt{\frac{-2(-35)}{1120}} = 0.25\ m$$

Problem 68: In Fig 9-59, block 1 of mass m_1 slides from rest along a frictionless ramp from height $h = 3.00\ m$ and then collides with stationary block 2, which has mass $m_2 = 2.00\ m_1$. After the collision, block 2 slides into a region where the coefficient of kinetic friction μ_k is 0.45 and comes to stop in distance d within that region. What is the value of distance d ~~within that region. What is the~~ if the collision a) elastic b) completely inelastic



$h = 3\ m \quad \mu_k = 0.45$

sol: a) if the collision is perfectly elastic

$$V_2 = \frac{2m_1}{m_1+m_2} v_{1i} = \frac{2m_1}{m_1+2m_1}\sqrt{2gh} = \frac{2m_1}{3m_1}\sqrt{2gh}$$

$$\boxed{V_2 = \frac{2}{3}\sqrt{2gh}}$$

$$\Delta k = \Delta E_{th}$$

$$k_f - k_i = \Delta E_{th}$$

$k_f = 0$

ايجاد

(9)

~~$k_i = zero$~~ $v_1 = zero$ slid from rest.

$k_f = \Delta E_{th} = f_k d$

$\frac{1}{2} \cancel{m_2} v_2^2 = \mu_k \cancel{m_2} g d$

$\frac{1}{2} v_2^2 = \mu_k g d$

$d = \frac{1}{2} \frac{v_2^2}{\mu_k g}$ $\qquad v_2 = \frac{2}{3} \sqrt{2gh}$ عوض

$= \frac{1}{2} \frac{(\sqrt{2gh})^2}{\mu_k g} \left(\frac{2}{3}\right)^2$

$= \frac{1}{\cancel{2}} \frac{\cancel{2}\cancel{g}h}{\mu_k \cancel{g}} \frac{4}{9}$

$= \frac{4h}{9\mu_k}$

$d = \frac{4 \times 3}{9 \times 0.45} = 2.96 \text{ m}$

b) In completely inelastic collision

$v_2 = \frac{m_1}{m_1+m_2} v_{1i} = \frac{m_1}{m_1+2m_1} v_{1i} = \frac{1}{3} v_{1i}$

$\Rightarrow \boxed{v_2 = \frac{1}{3}\sqrt{2gh}}$

$\frac{1}{2} \cancel{m} v_2^2 = \mu_k \cancel{m} g d$ $\qquad$ عوض بنفس ~~الخطوات~~ الطريقة

$m = m_1 + m_2$

$d = \frac{v_2^2}{2\mu_k g} = \frac{1}{\cancel{2}} \times \frac{1}{9} \frac{\cancel{2}\cancel{g}h}{\mu_k \cancel{g}}$

$\Rightarrow d = \frac{h}{9\mu_k} = \frac{3}{9 \times 0.45} = 0.74 \text{ m}$

problem 76: A 6090 kg space probe moving nose-first toward Jupiter at 120 m/s relative to the sun fires its rocket engine, ejecting 70.0 kg of exhaust at a speed of 253 m/s relative to the space probe. What is the final velocity of the probe?

Sol:

$$V_f - V_i = V_{rel} \ln \frac{M_i}{M_f}$$

$$V_f = V_i + V_{rel} \ln \frac{M_i}{M_f}$$

$$= 120 + 253 \ln \frac{6090}{(6090-70)}$$

$$= 120 + 253 \ln \left(\frac{6090}{6020}\right)$$

$$= 120 + 253 \times 0.01156$$

$$= 120 + 2.92$$

$$= 122.92$$

$$\simeq 123 \text{ m/s}$$