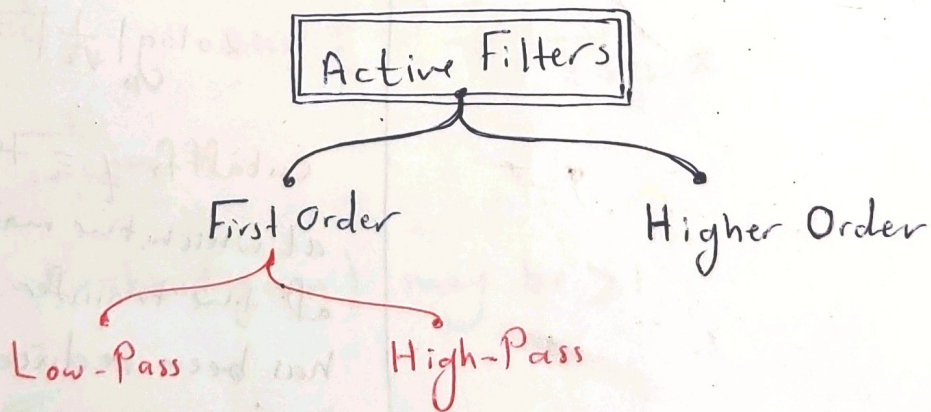


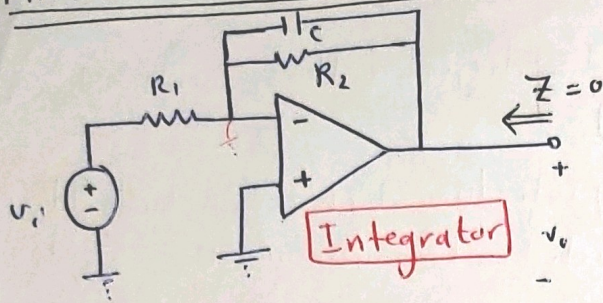
Active Filter Circuits

1. Active filter can produce bandpass and bandreject filters without using inductors (costly, heavy, large, noisy)
 - ↳ may introduce electromagnetic field effects that compromise the desired frequency response characteristics.
2. Filtering + Amplification
Active filters provide a control over amplification not available in passive filter circuits.
3. The cutoff frequency and the passband magnitude of passive filters were altered with the addition of a resistive load at the output of the filter (R_L).

⇒ We use active circuits to implement filter designs when gain, load variation, and physical size are important parameters in the design specifications.



First-Order Low-Pass Active Filter



$\frac{1}{Cs} \Rightarrow$ open

$\omega = 0$ amp with gain $-\frac{R_2}{R_1}$

$\omega = \infty$ $V_o = 0$

$$H(s) = \frac{V_o(s)}{V_i(s)} = - \frac{Z_2(s)}{Z_1(s)}$$

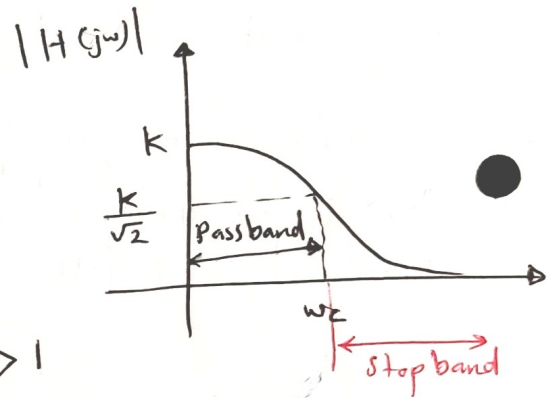
where $Z_2(s) = R_2 \parallel \frac{1}{Cs}$

$$Z_1(s) = R_1$$

$$H(s) = -K \frac{s + \frac{1}{R_2 C}}{s + \omega_c} \quad (\text{low-pass filter})$$

where $K = \frac{R_2}{R_1}$ (Pass band gain) may be > 1
 $\omega_c = \frac{1}{R_2 C}$ (cut off frequency)

In design

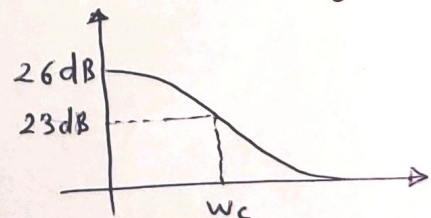


note

$$20 \log_{10} \left| \frac{1}{\sqrt{2}} \right| = -3 \text{ dB}$$

Cutoff freq $\equiv$ the frequency at which the maximum magnitude of the transfer function in dB has been reduced by 3 dB.

example

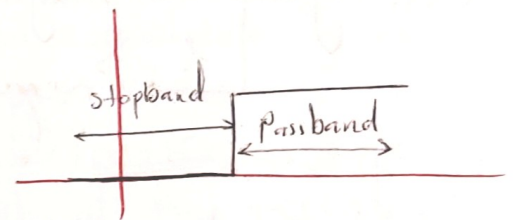
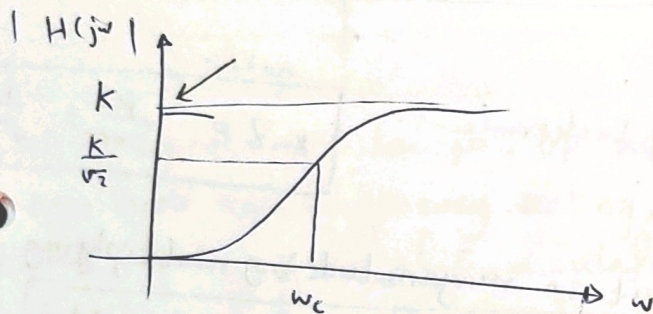
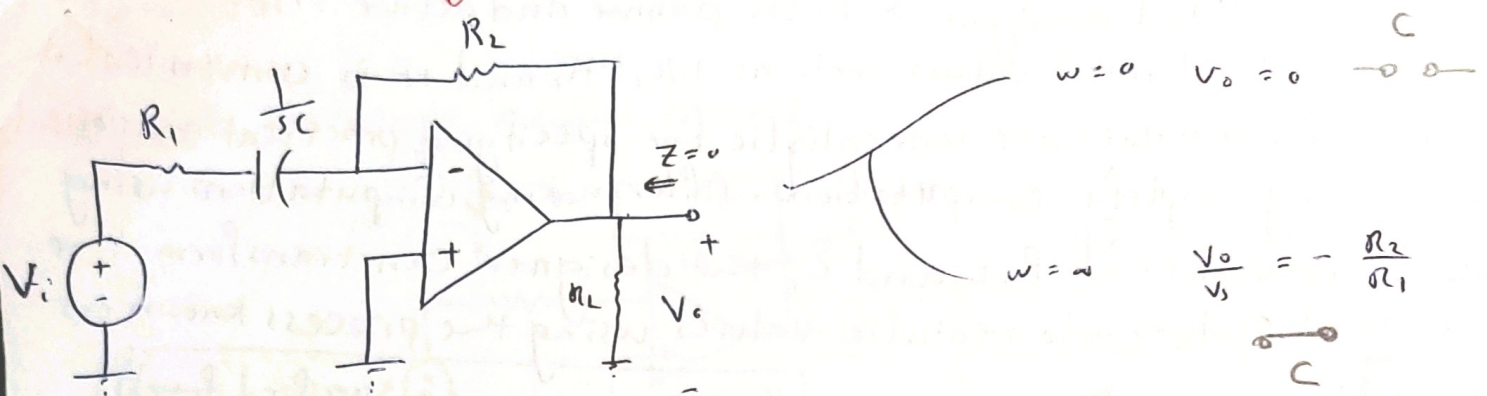


$$26 - 3 = 23 \text{ dB}$$

the mag used to find the cutoff frequency.

$$20 \log x = 26 \Rightarrow x = 19.95$$

[2] First-Order High-Pass Active Filter



$$H(s) = -\frac{Z_2(s)}{Z_1(s)}$$

where

$$Z_2 = R_2$$

$$Z_1 = R_1 + \frac{1}{sC}$$

$$H(s) = -\frac{R_2}{R_1} \frac{s}{s + \frac{1}{R_1 C}}$$

$$H(s) = -k \frac{s}{s + \omega_c}$$

$$k = \frac{R_2}{R_1} \quad (\text{Passband gain}) \text{ may be } > 1$$

$$\omega_c = \frac{1}{R_1 C}$$

In design
 R_L is not required

Scaling

In the design and analysis of both passive and active filter circuits, working with element values such as 1Ω , $1H$, and $1F$ is convenient. Although these values are unrealistic for specifying practical components, they greatly simplify computations. After making computations using convenient values of R , L , and C , the designer can transform the convenient values into realistic values using the process known as Scaling.

((T.F the same + specified freq))

Scaling $\rightarrow$ magnitude $\rightarrow$ Independent $\rightarrow$ Frequency

note:-

$$k_m Z = \frac{k_m}{j\omega C} = \frac{1}{j\omega \frac{C}{k_m}}$$

■ magnitude scaling: We scale ~~the~~ a circuit in magnitude by multiplying the impedance at a given freq. by the scale factor k_m .

$$R' = k_m R, \quad L' = k_m L$$

$$C' = C / k_m$$

$$\omega_c' = \frac{1}{R_2' C'} = \frac{1}{R_2 k_m \cdot \frac{C}{k_m}} = \frac{1}{R_2 C}$$

$$k = \frac{R_2'}{R_1'} = \frac{R_2 k_m}{R_1 k_m} = \frac{R_2}{R_1}$$

k_m : positive real number that can be either less than or greater than 1.
(mag scale factor)

■ Frequency Scaling:-

In freq scaling, we change the CKT parameters so that at the new freq, the impedance of each element is the same as it was at the original frequency.

$$\omega_1 \longrightarrow \omega_2$$

$$\omega_2 = k_f \omega_1$$

$$* R' = R$$

$$* j\omega_2 L' = j\omega_1 L$$

$$jk_f \omega_1 L' = j\omega_1 L$$

$$L' = \frac{L}{k_f}$$

$$\frac{1}{j\omega_2 C'} = \frac{1}{j\omega_1 C}$$

$$\frac{1}{jk_f \omega_1 C'} = \frac{1}{j\omega_1 C}$$

$$C' = \frac{C}{k_f}$$

$$① R' = k_m R$$

$$② L' = \frac{k_m}{k_f} L$$

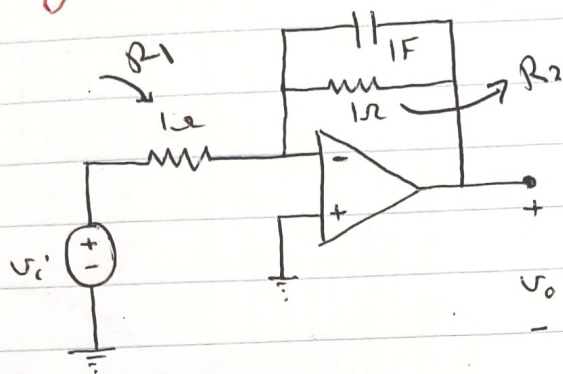
$$③ C' = \frac{1}{k_m k_f} C$$

$$④ k = k$$

$$⑤ \omega_c' = k_f \omega_c, \quad \omega_0' = k_f \omega_0$$

The frequency scale factor k_f is also a positive real number that can be less than or greater than unity.

Example 8 - prototype low-pass filter :-



$$H(s) = -K \frac{\omega_c}{s + \omega_c}$$

$$H(s) = \frac{-1}{s + 1}$$

$$K = \frac{R_2}{R_1}$$

$$\omega_c = 1 \text{ rad/sec} \quad \omega_c = \frac{1}{R_2 C}$$

Use the prototype low-pass opamp filter from example 1, along with magnitude and frequency scaling, to compute the R values for a low-pass filter with gain of 5, a cutoff frequency of 1000 Hz, and a feedback capacitor of 0.01 μ F. Construct a Bode plot of the resulting transfer function's magnitude.

K_m, K_f ?!

$$2\pi f = \frac{\text{rad/sec}}{\omega}$$

$$[1] \omega_c = 1000 \text{ Hz} = 1000(2\pi)$$

$$\omega'_c = K_f \omega_c \Rightarrow K_f = \frac{\omega'_c}{\omega_c} = \frac{2\pi(1000)}{1} = 6283.1853$$

$$[2] C' = \frac{1}{K_m K_f} C \quad C' = 1 \times 10^{-8} \text{ F}$$

$$K_m = \frac{1}{K_f} \frac{C}{C'} = \frac{1}{(6283.1853)(1 \times 10^{-8})} = 15915.5$$

$$[3] R_1 = R_2 = 1 \Omega$$

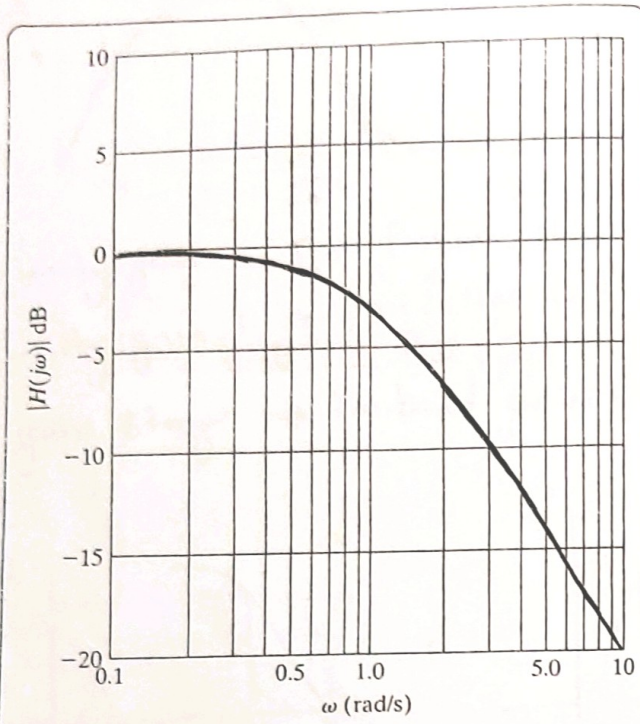
$$R'_1 = R'_2 = K_m R = (15915.5)(1) = 15915.5 \Omega$$

$$[4] K = \frac{R_2}{R_1}, \quad \omega_c = \frac{1}{R_2 C} \quad \text{If we adjust } R_2, \text{ we will change the cutoff frequency.}$$

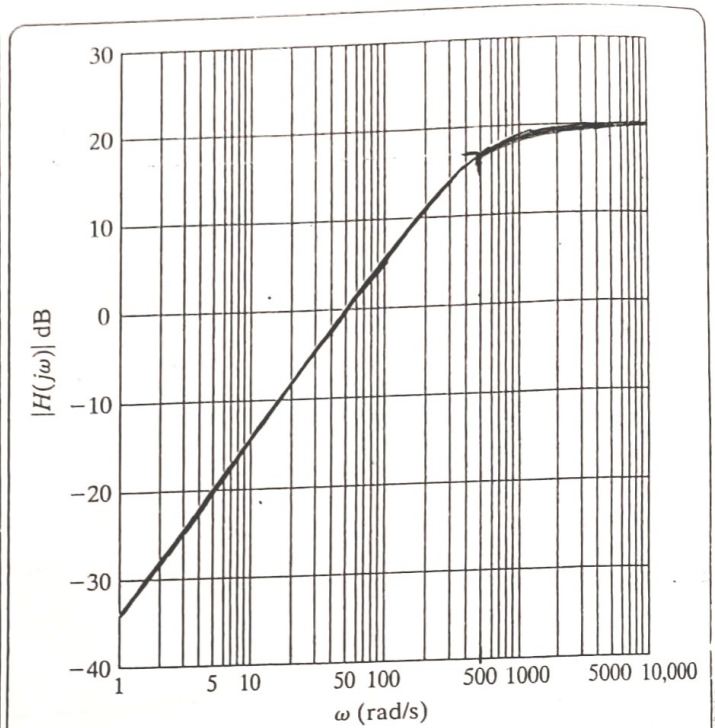
$$R_1 = \frac{R_2}{5} = \frac{15915.5}{5} = 3.183 \text{ k}\Omega$$

$$H(s) = -K \frac{\omega_c}{s + \omega_c}$$

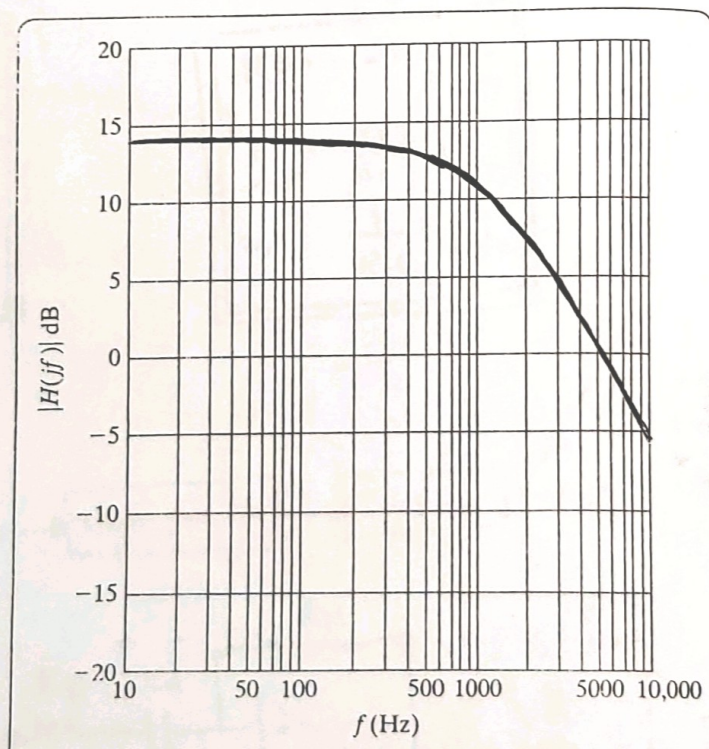
$$R_1 = 3.183 \text{ k}\Omega, \quad R_2 = 15.9155 \text{ k}\Omega, \quad C = 0.01 \mu\text{F} \Rightarrow H(s) = \frac{-31415.93}{s + 6283.185}$$



①

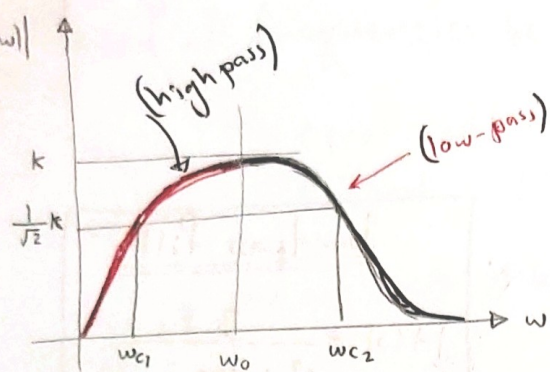


②

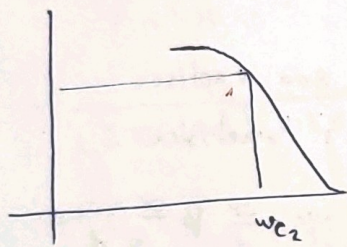


③

3. Active Bandpass Filter :-



1. A unity-gain low-pass filter with cutoff freq ω_{c2} .
2. A unity gain high-pass filter with cutoff freq ω_{c1} .
3. A gain stage $\Rightarrow$ Passband gain $= k$

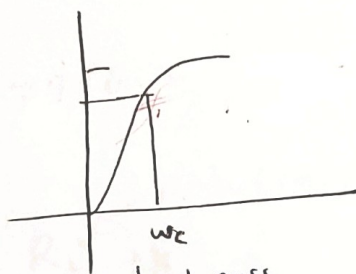


low-pass

$$H(s) = -K \frac{\omega_c}{s + \omega_c}$$

$$K = \frac{R_2}{R_1}$$

$$\omega_c = \frac{1}{R_2 C} = \frac{1}{R_L C_L}$$

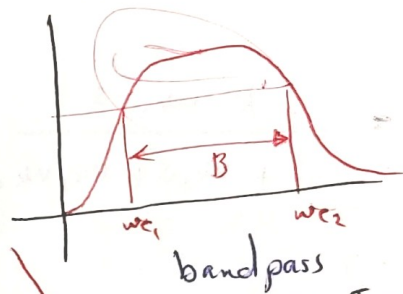


high pass

$$H(s) = -K \frac{s}{s + \omega_c}$$

$$K = \frac{R_2}{R_1}$$

$$\omega_c = \frac{1}{R_1 C} = \frac{1}{R_{HC} C_H}$$



band pass

Important condition.

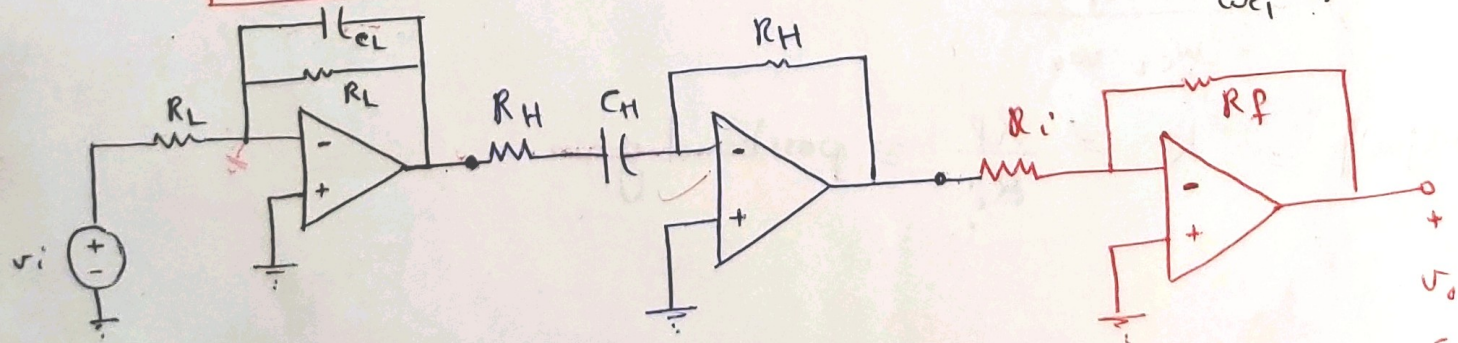
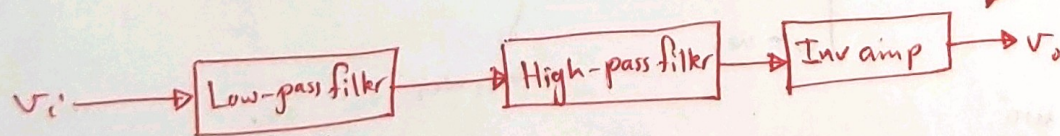
$\omega_{c2} > \omega_{c1}$
broadband bandpass filter

formal defⁿ

$$\frac{\omega_{c2}}{\omega_{c1}} \geq 2$$

The formal definition of a broadband filter requires the two cutoff frequencies to satisfy the equation

$$\frac{\omega_{c2}}{\omega_{c1}} \geq 2$$



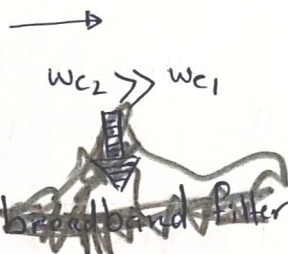
$$H(s) = \frac{V_o(s)}{V_i(s)}$$

$$= \left(\frac{-\omega_c2}{s + \omega_c2} \right) \left(\frac{-s}{s + \omega_c1} \right) \left(-\frac{R_f}{R_i} \right)$$

$\downarrow$ K

$$= \frac{-K \omega_c2 s}{(s + \omega_c1)(s + \omega_c2)}$$

$$= \frac{-K \omega_c2 s}{s^2 + (\omega_c1 + \omega_c2)s + \omega_c1 \omega_c2}$$

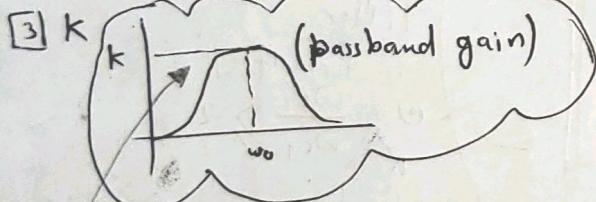


$$(\omega_c1 + \omega_c2) \approx \omega_c2$$

$$s \rightarrow \frac{-K \omega_c2 s}{s^2 + \omega_c2 s + \omega_c1 \omega_c2}$$

$$\textcircled{1} \omega_c2 = \frac{1}{R_L C_L}$$

$$\textcircled{2} \omega_c1 = \frac{1}{R_H C_H}$$



$$|H(j\omega)| = \left| \frac{-K \omega_c2 (j\omega)}{(j\omega)^2 + \omega_c2 (j\omega) + \omega_c1 \omega_c2} \right|$$

$\downarrow$ $\omega = \omega_0$

$$= \left| \frac{-K \omega_c2 \omega_0 j}{-\omega_0^2 + \omega_c2 \omega_0 j + \omega_c1 \omega_c2} \right|$$

$\downarrow$ $-\omega_c1 \omega_c2$

$$= \frac{K \omega_c2 \omega_0}{\omega_c2 \omega_0}$$

$$|H(j\omega_0)| = K = \frac{R_f}{R_i} \equiv \text{passband gain.}$$

bandpass filter

$$H(s) = \frac{\beta s}{s^2 + \beta s + \omega_0^2}$$

$$\omega_0 = \sqrt{\omega_c1 \cdot \omega_c2}$$

$$\beta = \omega_c2 - \omega_c1$$

$$Q = \frac{\omega_0}{\beta}$$

example

$$\omega_c2 = 500 \text{ rad/sec}$$

$$\omega_c1 = 1 \text{ rad/sec}$$

$$\omega_c2 - \omega_c1 \approx \beta \approx \omega_c2$$

example

Design a bandpass filter to provide an amplification of 2 within the band of frequencies between 100 and 10,000 Hz. use 0.2 μ F capacitors.

$$\omega_{c1} = 2\pi(100) \text{ rad/sec}$$

$$\omega_{c2} = 2\pi(10,000) \text{ rad/sec}$$

$$\omega_{c2} = 100\omega_{c1} \Rightarrow \omega_{c2} \gg \omega_{c1}$$

$$K = 2$$

$$\textcircled{1} \quad \omega_{c2} = \frac{1}{R_L C_L} = 2\pi(10000) \Rightarrow R_L = \frac{1}{[2\pi(10,000)](0.2 \times 10^{-6})} \cong 80 \Omega$$

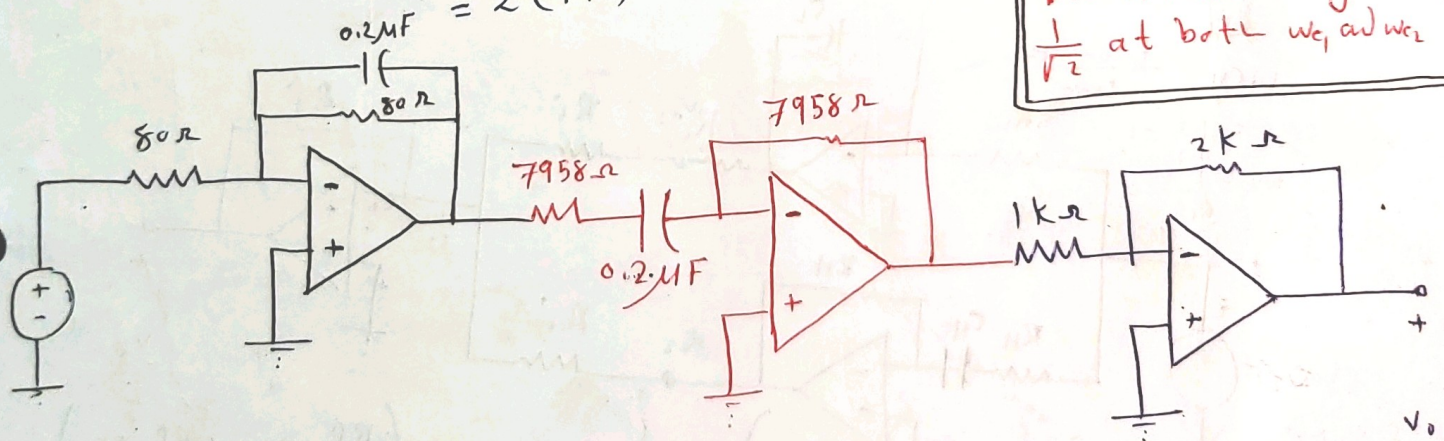
$$\textcircled{2} \quad \omega_{c1} = \frac{1}{R_H C_H} = 2\pi(100)$$

$$R_H = \frac{1}{[2\pi(100)](0.2 \times 10^{-6})} \cong 7958 \Omega$$

$$K = \frac{R_f}{R_i} \Rightarrow R_f = K R_i \quad \text{arbitrarily} \quad 1K$$

$$= 2(1K) = 2k\Omega$$

Verify that the mag. of this CKT ~~TF~~ is reduced by $\frac{1}{\sqrt{2}}$ at both ω_{c1} and ω_{c2}

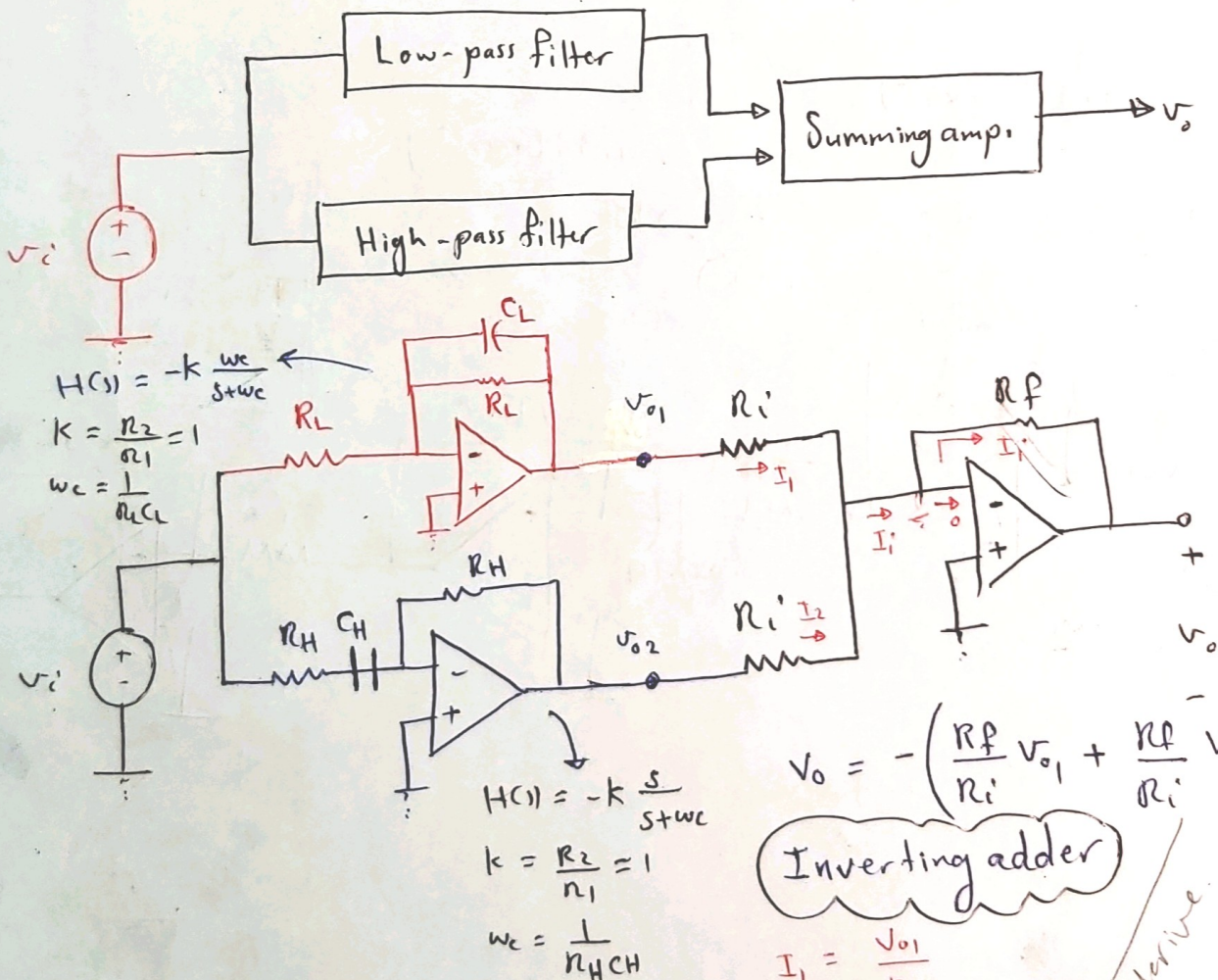
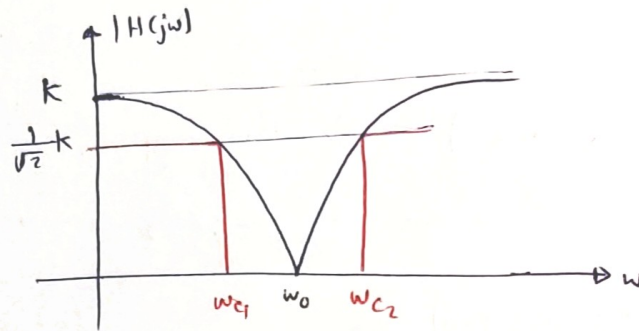


Handwritten notes:
 $\omega_{c1} = 2\pi \times 100$
 $\omega_{c2} = 2\pi \times 10,000$
 $\omega_{c2} = 100\omega_{c1}$
 $\omega_{c2} \gg \omega_{c1}$

4) Active Bandreject Filter :-

1. A unity gain low-pass filter with ω_{c1}
 2. A unity gain highpass filter with ω_{c2}
- $\omega_{c2} \gg \omega_{c1}$

3. gain stage



$$V_o = - \left(\frac{R_f}{R_i} V_{o1} + \frac{R_f}{R_i} V_{o2} \right)$$

Inverting adder

$$I_1 = \frac{V_{o1}}{R_i}$$

$$I_2 = \frac{V_{o2}}{R_i}$$

$$I_i = I_1 + I_2$$

$$V_o = -R_f I_i$$

derive

$$V_o = -R_f \left(\frac{V_{o1}}{R_i} + \frac{V_{o2}}{R_i} \right)$$

$$\begin{aligned}
 H(s) &= -\left(\frac{R_f}{R_i}\right) \left[\frac{-\omega_{c1}}{s + \omega_{c1}} + \frac{-s}{s + \omega_{c2}} \right] \\
 &= \frac{R_f}{R_i} \left[\frac{\omega_{c1}(s + \omega_{c2}) + s(s + \omega_{c1})}{(s + \omega_{c1})(s + \omega_{c2})} \right] \\
 &= \frac{R_f}{R_i} \left[\frac{s^2 + 2\omega_{c1}s + \omega_{c1}\omega_{c2}}{(s + \omega_{c1})(s + \omega_{c2})} \right] \\
 &\quad s^2 + (\omega_{c1} + \omega_{c2})s + \omega_{c1}\omega_{c2}
 \end{aligned}$$

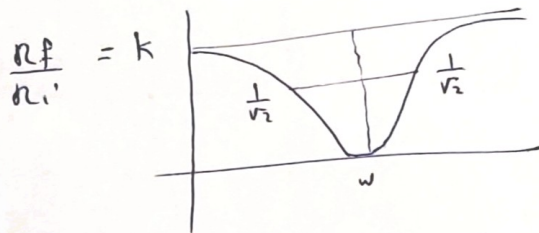
bandreject filter

$$H(s) = \frac{s^2 + \omega_0^2}{s^2 + \beta s + \omega_0^2}$$

$$\omega_0 = \sqrt{\omega_{c1}\omega_{c2}}$$

① $\omega_{c1} = \frac{1}{R_L C_L}$

② $\omega_{c2} = \frac{1}{R_H C_H}$



as $s \rightarrow 0$
 $s \rightarrow \infty$

$$|H(j\omega)|_{\omega=0} = k = \frac{R_f}{R_i}$$

$$|H(j\omega)|_{\omega=\infty} = k = \frac{R_f}{R_i}$$

$$|H(j\omega)|_{\omega=\omega_0} = \frac{R_f}{R_i} \left[\frac{(j\omega_0)^2 + 2\omega_{c1}(j\omega_0) + \omega_{c1}\omega_{c2}}{(j\omega_0)^2 + (\omega_{c1} + \omega_{c2})(j\omega_0) + \omega_{c1}\omega_{c2}} \right]$$

$$= \frac{R_f}{R_i} \cdot \frac{2\omega_{c1}}{\omega_{c1} + \omega_{c2}} \approx \frac{R_f}{R_i} \cdot \frac{2\omega_{c1}}{\omega_{c2}} \approx \text{very small}$$

Design 3- 6 unknowns
 3 eqs

$\subseteq \checkmark \Rightarrow R_H \quad \omega_{c1}$
 $R_L \quad \omega_{c2}$

$$k = \frac{R_f}{R_i} \quad (\text{assume 1})$$